Production of electricity from geothermal heat – efficiency calculation and ideal cycles

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Abstract

Production of electricity from heat requires a heat engine working between two heat reservoirs, a heat source and a heat sink. The Carnot efficiency is defined for the reversible heat engine working between two reservoirs of infinite heat capacity and constant temperature. Geothermal heat source is a stream of water, either in the liquid phase or as liquid-steam mixture of any quality. The heat capacity of the heat source is thus finite, and the source temperature is not constant in the heat addition process of the heat engine. Same applies for the heat rejection of the heat engine, usually the cooling fluid available has some strict limitation on the possible temperatures, and rejection to a sink at a constant temperature is practically almost impossible. This paper analyzes electricity production when the heat source and the heat sink do not have a constant temperature. The maximum production efficiency is defined and calculated, both for pure electricity production (E) and Combined Heat and Power (CHP) generation. The maximum obtainable efficiencies are calculated for typical values for geothermal application. It is stated, that the Carnot efficiency should not be used as a reference for such power production, but the real maximum efficiency based on the process.

***Keywords**: geothermal, electricity production, heat engine, Carnot efficiency.*

1 Introduction

Energy can be transferred in two ways, as work and as heat. Geothermal utilization involves conversion of the energy source, geothermal heat, into the more valuable form of work or electricity. The second law of thermodynamics limits the conversion of heat into work. The most efficient work-producing engine possible is the reversible heat engine, or the Carnot engine.

The highest conversion efficiency possible is thus the Carnot efficiency. But the question of the definition of the efficiency arises, which parts of the rejected heat are to be considered lost, and which are products, heat that can be sold from the power station?

Figure 1 is an input-output diagram for a power plant.

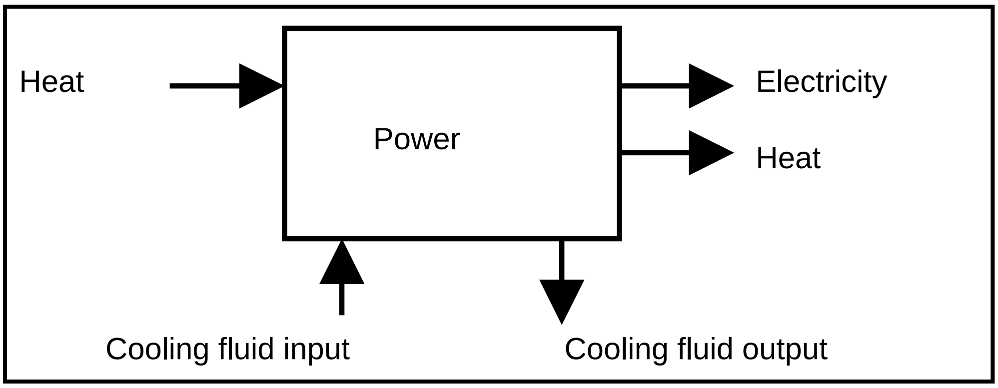

Figure 1: Block diagram of a generic power plant.

The temperature of the entering cooling fluid is taken to be the environmental temperature, the lowest temperature which can be obtained, as well as defining the thermal sink temperature for the Carnot engine efficiency.

This process can be seen as a non-conserving heat exchange process between the source stream and the cooling fluid stream. Figure 2 is a block diagram for a non-conserving heat exchanger representing a power plant.

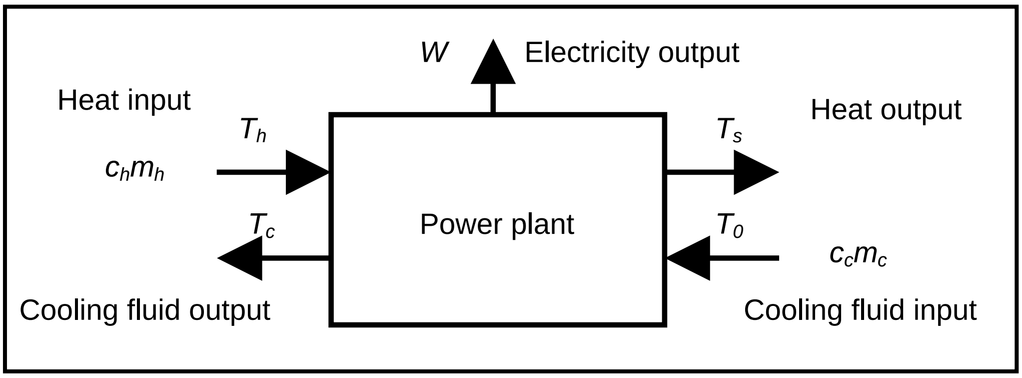

Figure 2: Block diagram of a heat exchanger equivalent to a power plant.

c_h	Source fluid heat capacity
m_h	Flow rate of source fluid
T_h	Source fluid inlet temperature
T_s	Source fluid outlet temperature
c_c	Cooling fluid heat capacity
m_c	Cooling fluid flow rate
T_c	Cooling fluid outlet temperature
T_0	Cooling fluid inlet temperature (Environmental temperature)

In the following this system will be analyzed in order to gain a better understanding of the conversion of low temperature heat into electricity.

2 Analysis

The streams in and out_of the system have four flow properties: mass, heat capacity, enthalpy and exergy. The mass conservation is obvious, no mixing of the source and cooling streams is assumed. The heat capacity is important for the characteristics of the heat conversion, and will be treated here as a heat capacity flow, the product of fluid heat capacity and flow rate. The product of the enthalpy relative to the environmental temperature and the flow rate defines the heat flow in and out of the system. The exergy will give information on the work producing potential of the system, and is calculated in the same way as the enthalpy. Reference textbooks such as Cengel (2002) give basic information on exergy and its definition, but here the analysis is as well based on Kotas (1985) and Szargut (1988). Þórólfsson (2002) and Valdimarsson (2002)-apply these methods on specific geothermal applications. In this paper the focus is on the general conversion efficiency calculation.

2.1 Flows

The heat ($\dot{Q}$) and exergy ($\dot{X}$) flows are given by:

$$\dot{Q}_h = c_h \dot{m}_h (T_h - T_0) \tag{1}$$

$$\dot{Q}_s = c_h \dot{m}_h (T_s - T_0) \tag{2}$$

$$\dot{Q}_c = c_c \dot{m}_c (T_c - T_0) \tag{3}$$

$$\dot{X}_h = c_h \dot{m}_h \left[(T_h - T_0) - T_0 \ln\left(\frac{T_h}{T_0}\right) \right] = \dot{Q}_h - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_0}\right) \tag{4}$$

$$\dot{X}_s = c_h \dot{m}_h \left[(T_s - T_0) - T_0 \ln\left(\frac{T_s}{T_0}\right) \right] = \dot{Q}_s - c_h \dot{m}_h T_0 \ln\left(\frac{T_s}{T_0}\right) \tag{5}$$

$$\dot{X}_c = c_c \dot{m}_c \left[(T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right) \right] = \dot{Q}_c - c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right) \tag{6}$$

The energy (1. law) and exergy (2. law) balances are:

$$\dot{Q}_h - \dot{Q}_s - \dot{Q}_c = \dot{W} \tag{7}$$

$$\dot{X}_h - \dot{X}_s - \dot{X}_c = \dot{W}_{rev} \quad \text{or}$$

$$\dot{Q}_h - \dot{Q}_s - \dot{Q}_c - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right) = \dot{W}_{rev} \tag{8}$$

The energy balance is valid for all processes, ideal and real. The exergy balance gives only information on the reversible work, or the largest amount of work that can be obtained from the power plant.

If the power plant is ideal, then:

$$\dot{W}_{rev} = \dot{W} \quad or \quad -c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right) = 0 \tag{9}$$

Then the heat capacity flow ratio for a reversible power plant is:

$$C_{rev} = \left.\frac{c_c \dot{m}_c}{c_h \dot{m}_h}\right|_{rev} = \frac{\ln\left(\frac{T_h}{T_s}\right)}{\ln\left(\frac{T_c}{T_0}\right)} \tag{10}$$

2.2 Efficiency

Efficiency is the ratio of benefit to cost. In order to be able to define efficiency, the inputs (cost) and outputs have to be defined. In a low temperature heat conversion process, two cases regarding the stream $\dot{m}_s$ are possible, depending on if the heat contained in that stream can be sold to a heat consuming process.

Case E

Electricity is the only output of the power plant. The heat contained in the stream $\dot{m}_s$ is rejected to the surroundings.

$$\begin{aligned} \text{Product}: &\quad \dot{W} \\ \text{Input}: &\quad \dot{Q}_h \\ \text{Rejected}: &\quad \dot{Q}_s \textit{ and } \dot{Q}_c \end{aligned}$$

First law efficiency:

$$\eta_{I,E} = \frac{\dot{W}}{\dot{Q}_h} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{Q}_h} = \frac{(T_h - T_s) - C(T_c - T_0)}{T_h - T_0} \qquad (11)$$

First law maximum efficiency:

$$\eta_{I,\max,E} = \frac{\dot{W}_{rev}}{\dot{Q}_h} = \frac{\dot{X}_h - \dot{X}_s - \dot{X}_c}{\dot{Q}_h} = 1 - \frac{\dot{Q}_s + \dot{Q}_c + c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) - c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right)}{\dot{Q}_h}$$

$$= \frac{(T_h - T_s) - T_0 \ln\left(\frac{T_h}{T_s}\right)}{(T_h - T_0)} - C\frac{(T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right)}{(T_h - T_0)}$$

$$= \frac{\ln\left(\frac{T_c}{T_0}\right)(T_h - T_s) - \ln\left(\frac{T_h}{T_s}\right)(T_c - T_0)}{\ln\left(\frac{T_c}{T_0}\right)(T_h - T_0)} \qquad (12)$$

Second law efficiency:

$$\eta_{II,E} = \frac{\dot{W}}{\dot{W}_{rev}} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{X}_h - \dot{X}_s - \dot{X}_c} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right)}$$

$$= \frac{(T_h - T_s) - C(T_c - T_0)}{(T_h - T_s) - T_0 \ln\left(\frac{T_h}{T_s}\right) - C\left((T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right)\right)} \qquad (13)$$

Case CHP

Combined heat and power production. The stream $\dot{m}_s$ has to be kept at a given temperature, required by the consumer. Each of the products (heat and power) has its own load characteristics and duration. The heat contained in the stream $\dot{m}_s$ cannot be sold completely, as the heat consuming process has its own efficiency. But this is the matter of the heat consuming process, and is therefore not a matter in this study of the primary power plant. Seen from the power plant, the whole stream $\dot{m}_s$ is sold to the heat consuming process, and it is the responsibility of the heat consumer to utilize that heat as far down as possible, preferably all the way down to T_0.

The efficiencies for the CHP process are thus electrical power generation efficiencies. All the heat contained in the stream $\dot{m}_s$ is considered a by-product, and does not enter the efficiency calculation.

$$\begin{aligned} \text{Product:} &\quad \dot{W}\ and\ \dot{Q}_s \\ \text{Input:} &\quad \dot{Q}_h - \dot{Q}_s \\ \text{Rejected:} &\quad \dot{Q}_c \end{aligned}$$

First law efficiency:

$$\eta_{I,CHP} = \frac{\dot{W}}{\dot{Q}_h - \dot{Q}_s} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{Q}_h - \dot{Q}_s} = \frac{(T_h - T_s) - C(T_c - T_0)}{T_h - T_s} \tag{14}$$

First law maximum efficiency:

$$\eta_{I,\max,CHP} = \frac{\dot{W}_{rev}}{\dot{Q}_h - \dot{Q}_s} = \frac{\dot{X}_h - \dot{X}_s - \dot{X}_c}{\dot{Q}_h - \dot{Q}_s} = 1 - \frac{\dot{Q}_c + c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) - c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right)}{\dot{Q}_h - \dot{Q}_s}$$

$$= \frac{(T_h - T_s) - T_0 \ln\left(\frac{T_h}{T_s}\right)}{(T_h - T_s)} - C\frac{(T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right)}{(T_h - T_s)}$$

$$= 1 - \frac{\ln\left(\frac{T_h}{T_s}\right)(T_c - T_0)}{\ln\left(\frac{T_c}{T_0}\right)(T_h - T_s)} \tag{15}$$

Heat ratio is the ratio of sellable heat to work produced:

$$r_{heat} = \frac{\dot{Q}_s}{\dot{W}} = \frac{\dot{Q}_s}{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c} = \frac{(T_s - T_0)}{(T_h - T_s) - C(T_c - T_0)}$$

$$r_{heat,rev} = \frac{(T_s - T_0)\ln\left(\frac{T_c}{T_0}\right)}{(T_h - T_s)\ln\left(\frac{T_c}{T_0}\right) - (T_c - T_0)\ln\left(\frac{T_h}{T_s}\right)} \qquad (16)$$

Second law efficiency is the same in both cases:

$$\eta_{II,E} = \frac{\dot{W}}{\dot{W}_{rev}} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{X}_h - \dot{X}_s - \dot{X}_c} = \frac{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c}{\dot{Q}_h - \dot{Q}_s - \dot{Q}_c - c_h \dot{m}_h T_0 \ln\left(\frac{T_h}{T_s}\right) + c_c \dot{m}_c T_0 \ln\left(\frac{T_c}{T_0}\right)}$$

$$= \frac{(T_h - T_s) - C(T_c - T_0)}{(T_h - T_s) - T_0 \ln\left(\frac{T_h}{T_s}\right) - C\left((T_c - T_0) - T_0 \ln\left(\frac{T_c}{T_0}\right)\right)} \qquad (17)$$

3 Results

To illustrate the preceding analysis, an example is taken of a power plant, where cooling fluid is available at T_0=30°C. The source temperature T_h is varied between 100 and 150°C. The source fluid outlet temperature is assumed to be T_s=80°C, and is varied in order to see its influence. Similarly the cooling fluid outlet temperature is assumed to be T_c=45°C, and is varied as well to see its influence.

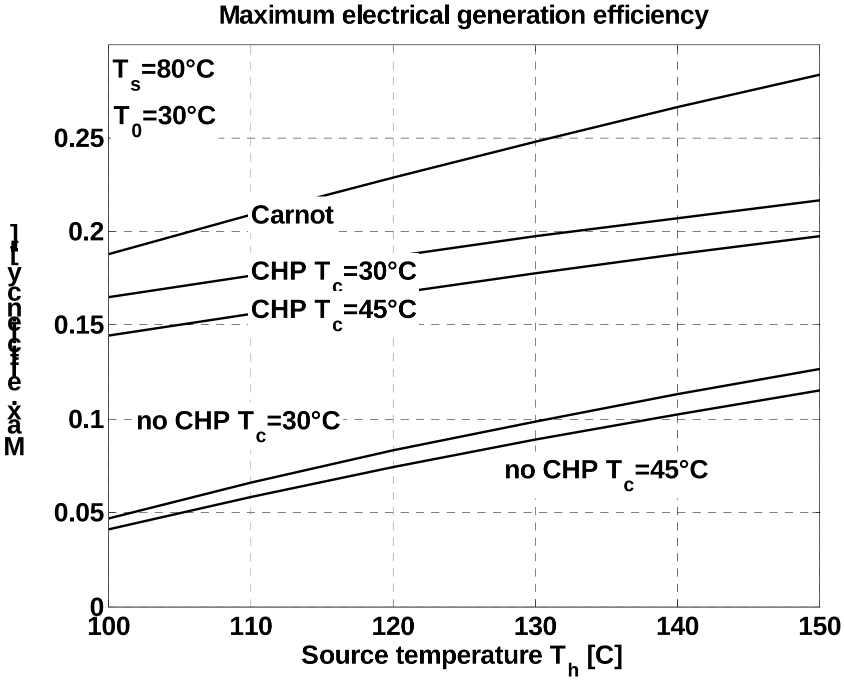

Figure 3: Maximum electrical generation efficiency for T_0=30°C and T_s=80°C.

The CHP case shows that the maximum generation efficiency decreases by around 2% by having realistic cooling fluid temperature, as compared to the ideal case, where the temperature is constant. The increase in the maximum efficiency is only around

0.5-1% for the other case, but then all heat contained in the stream $\dot{m}_s$ is considered wasted.

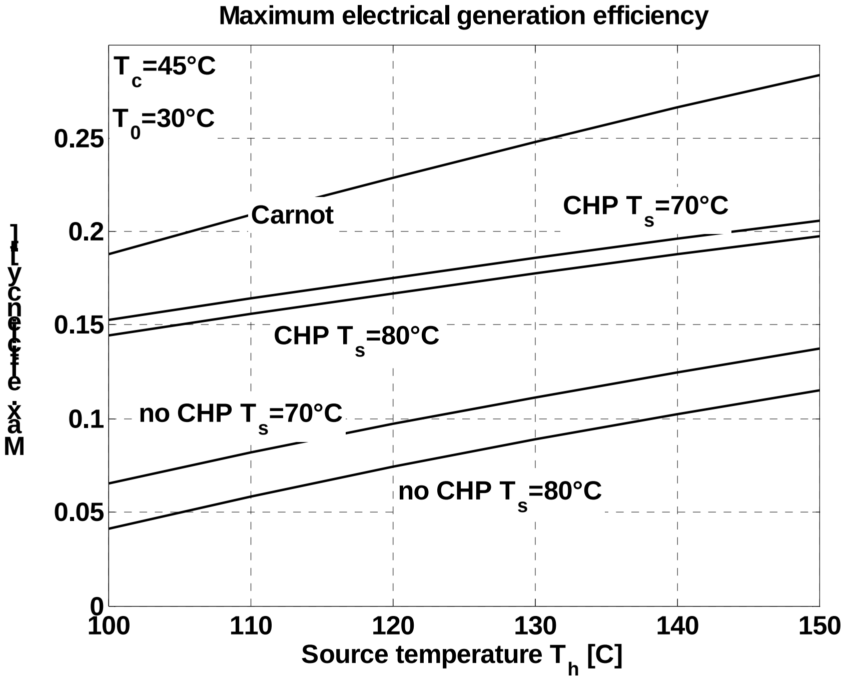

Figure 4: Maximum electrical generation efficiency for T_0=30°C and T_c=45°C.

The influence of the temperature of the heat extraction stream has greater influence on the case with no CHP. This is obvious, as this heat is lost and worthless in that case. Already a reduction from 80°C down to 70°C increases the efficiency by some 2.5%.

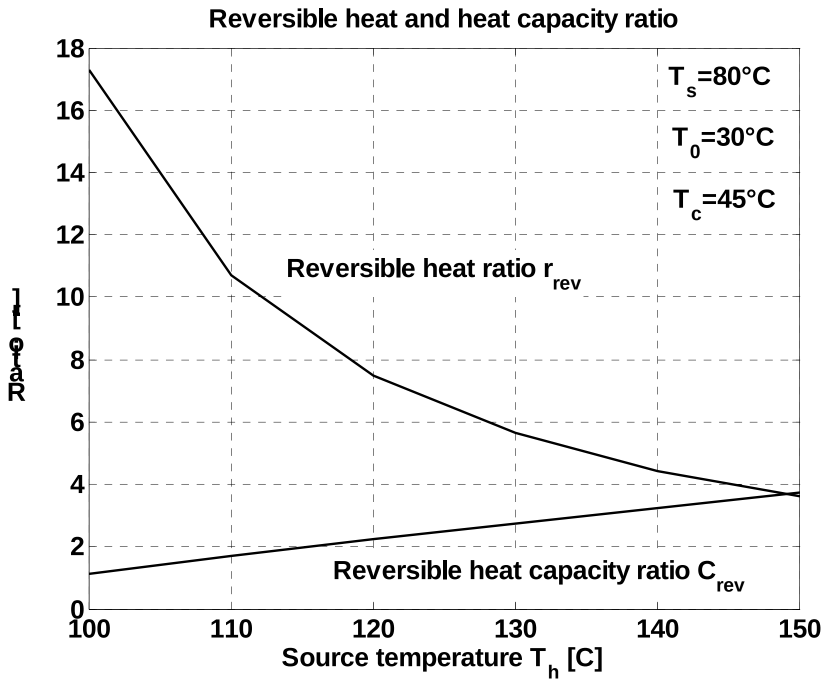

Figure 5: Reversible heat and capacity ratios for T_0=30°C and T_c=45°C.

The ratio of the cooling fluid heat capacity flow to the primary fluid heat capacity flow is calculated here assuming that the conversion process is reversible. As the source temperature gets higher, the more flow of the cooling fluid is needed in order to stay reversible. At low temperatures the heat ratio tends towards infinity, indicating that no useable work can be produced.

4 Conclusion

The analysis has shown that the Carnot efficiency is not valid as a reference for electrical power generation from low temperature sources. The energy and exergy contained in the stream $\dot{m}_s$ has to be considered waste, if this energy cannot be sold as heat. The only valid reference for the efficiency of a pure electricity plant is thus the maximum first law efficiency for Case E. If the heat in the stream $\dot{m}_s$ can be sold, the whole stream is a by-product of the power plant, and how that stream is utilized by the customer is no matter of the power plant itself. The maximum electrical production efficiency in this case is presented as Case CHP.

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