# Rannsóknamiðstöð Háskólans í Reykjavík 

## Viðskiptadeild

# Individual quotas, discarding and stock size 

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This paper analyses how successful an individual transferable quota (ITQ) management system is in bringing a fishery from the open access level to the optimum in the presence of discarding. The model developed extends previous work in this area by including long run effects, firstly to assess whether an ITQ system can lead to a reduction in, or even depletion of, fish stocks through excessive discarding, and secondly, to analyse the effect of discarding on the number of fishers operating in the fishery. It is shown that in an ITQ fishery more fish will be discarded than is optimal and that too many fishers will operate in the fishery. A significant result is that an ITQ system will result in a larger stock size, as compared to open access, even with excessive discarding. This suggests that the combination of ITQS and excessive discarding will not result in stock depletion.

Keywords: Fisheries management, discarding, individual transferable quotas

## 1 Introduction

This paper aims to add to the understanding of the interaction between discarding of fish and fisheries management. Fishers have in recent years faced accusations of wasting valuable resources by excessviely discarding low value catches, and individual transferable quota (ITQ) management is believed to exacerbate this problem.

In terms of theoretical economic analysis of discarding and highgrading behaviour the field is not large. During the mid 1990s a few papers analysing discarding formally appeared $[4,2,8,9]$, but the interest seems to have faded since then. Some of the findings of this work were that discarding may be optimal in fisheries with different grades of fish, for example, because of hold constraints. Also, ITQS tend to generate an excessive incentive for discarding catch as compared to a socially optimal fishery. An empirical study from a shrimp fishery in Greenland finds that discarding may exist without any management of the fishery, and that individual quotas increase the incentive to discard [10]. An interesting aspect of the Greenland study is that a non-transferable quota system had different discarding incentives than a transferable system. In the ITQ case the incentive was found to depend on the quota price. If the price is below the shadow value of the quota in the non-transferable case then the incentive to discard falls. However, there is no attempt made to include the quota price in the model.

In the current literature there are a number of gaps. Firstly, all the above analyses assume a short run fishery, i.e., the biomass is fixed in the models. This is unfortunate because one of the biggest concerns of discarding, among politicians, fishers and the general public, is the negative effect it may have on fish stocks. The argument is that excessive discarding and highgrading will lead to increased fishing mortality, perhaps to such an extent that the fish stock in question may come under threat of depletion. This is such an important concern that models looking at discarding should be capable of analysing this stock effect.

Secondly, the short run nature of these papers will only give a partial analysis of the impact on the effort level in the fishery. Effort changes come, not only from the change in behaviour of the individual fisher, but also through the entry and exit of fishers. Entry and exit may offset or amplify impacts from changes in the behaviour of the individual fisher.

Thirdly, there is no attempt made in these papers - most of which aim to analyse individual transferable quotas and discarding - to explicitly model the quota market and the interaction
between the price of quota and discarding. There have been suggestions [10] that the price of quota may be linked to the incentive to discard, and therefore it seems a worthwhile exercise to bring the determination of quota prices into the models.

The purpose of this paper is to address these problems to see if and how that will change the conclusions of previous work on discarding, and what additional insights can be gained regarding discarding behaviour. The paper is organised as follows. First, the model is introduced and the social optimum derived. Then the open access situation is presented, following which ITQS are introduced and compared to the other situations. Finally, the conclusions of the analysis are summarised.

## 2 The model

Discarding, in the current model, can occur because the fish stock is differentiated; there are high quality fish and low quality fish. The difference between high and low quality fish is that catches of high quality fish fetch a price, $p_{1}$, which is higher than the price that catches from the low quality part of the fish stock fetch, $p_{2}$. Fishers sell their catches to fish markets that have substitutes available, thus prices are not influenced by the supply from this particular fish stock. Why the price differential exists is not important for the purposes of the current analysis. For a curious reader, one possibility is that some fish are larger than other fish and that the market is willing to pay a premium for large fish. What is important, however, is that this differential exists, that fishers are aware of it, and it is crucial that fishers are able to recognise high quality fish from low quality fish while at sea.

Fish can be harvested by the individual fisher according to the function

$$
h\left(e^{i}, x\right), \quad i=1, \ldots, n
$$

implying that all fishers have access to the same harvest technology. The effort of the $i$-th fisher is denoted $e^{i}$, while $x$ is the total biomass of the fish stock. The stock size affects all fishers, and therefore enters the harvest function of each and every fisher. The harvest function is assumed
to have the following properties

$$
\begin{array}{lll}
h_{e^{i}}>0 & h_{x}>0 & \forall i \\
h_{e^{i} e^{i}} \leq 0 & h_{x x} \leq 0 & \forall i \\
h_{e^{i} x} \geq 0 & h_{e^{i} x}=h_{x e^{i}} & \forall i
\end{array}
$$

These standard assumptions regarding production functions imply diminishing marginal returns of both inputs. The cross derivative $h_{e^{i} x}$ is assumed positive implying a search fishery [6].

In the model, the catch of high quality fish is a certain fraction, $\alpha$, of the total harvest and the remainder, $(1-\alpha)$, is the catch of low quality fish. As in other work analysing the economics of discarding, this production function restricts fishers to harvest a fixed composition of the stock. ${ }^{1}$ In a model such as the one developed here, where the focus is on comparing long run equilibria, this restriction may not be very serious. The determination of $\alpha$ comes from the fishers themselves. They experiment with different fishing techniques in order to find the one that maximises their profits. This experimentation includes gear type, mesh size, tow time, distance of gear from bottom etc. By varying these techniques fishers can change the selectivity of their harvesting operations. It is only reasonable to expect that in the long run fishers choose fishing techniques that maximise their return, i.e., the techniques that equate the marginal benefit and marginal cost of improving selectivity. It is assumed that this decision is not affected in any significant way by the choice of management policy and therefore $\alpha$ is considered fixed in the analysis.

In this model fishers may not necessarily land their full harvest of low quality fish. Fishers might throw overboard less valuable fish, in either the hope of exchanging it for more valuable fish or simply because the fisher judges the price not sufficient to cover the landings cost of the fish. Landings of an individual fisher, $y^{i}$, are given by

$$
y^{i}=\alpha h\left(e^{i}, x\right)+(1-\alpha) h\left(e^{i}, x\right)-d^{i}=h\left(e^{i}, x\right)-d^{i}
$$

where $d^{i}$ represents the amount of low quality fish discarded by fisher $i$.
Total landings in the fishery are the sum of the landings of individual fishers, i.e.,

$$
\sum_{i=1}^{n} y^{i}=\sum_{i=1}^{n}\left(h\left(e^{i}, x\right)-d^{i}\right)
$$

[^0]It is assumed that discarded fish do not survive. If all discarded fish would survive, then discarding is not a problem as discarded fish would grow to become bigger and more valuable and could be harvested at a later stage. The problem with discarding arises because fish that are discarded will not contribute to the productivity of the fish stock. How much of discarded fish survives depends on many factors and some species, lobster being one, may survive quite well [1]. However, in this model complete mortality of discarded fish is assumed. It would not be difficult to include a parameter in the model that would reflect the survival rate of discarded fish [4]. However, it will not add significantly to the qualitative analysis undertaken here and is therefore not pursued.

The cost of fishing comes from three sources. Firstly, is the cost of fishing effort and, secondly, the cost of landing the fish. The latter cost includes cost from any fish processing done aboard the fishing vessel, in addition to the cost of physically landing the fish. Thirdly, is the cost of discarding. For instance, in order to discard fish of lower quality, someone has to keep an eye out for such fish, grab them and throw them over the side. This may use up time and energy that could be used for other activities. The total cost of the individual fisher is expressed as

$$
c^{i}\left(e^{i}, d^{i}, y^{i}\right), \quad i=1, \ldots, n
$$

indicating that cost depends on effort, discarding and landings. The following assumptions are made regarding the cost function

$$
\begin{array}{lll}
c_{e^{i}}^{i}>0 & c_{e^{i} e^{i}}>0 & \forall i \\
c_{y^{i}}=\gamma & c_{y^{i} y^{i}}=0 & \forall i \\
c_{d^{i}}>0 & c_{d^{i} d^{i}}>0 & \forall i \\
c_{j k}=c_{k j}=0 & j, k=e^{i}, d^{i}, y^{i} ; \quad j \neq k
\end{array}
$$

It is assumed that the cost of effort and the cost of discarding are increasing at an increasing rate. The marginal cost of landings, $c_{y^{i}}$, is assumed constant and the same across all fishers. The cost of landings will to a large extent be outside the control of the fisher. For instance harbour dues and the cost of physically landing the fish will often by decided by the port authorities and will not differ among fishers. Therefore, this seems a reasonable assumption. The last condition implies that the cost function is additively separable in its arguments. For instance, if the price of fuel increases, raising the cost of effort, that does not affect the marginal cost of discarding.

Also, since effort, discarding, and landings do not occur at the same time, they do not affect each other's unit cost. Finally, the average cost of a fisher as a function of harvest,

$$
\mathrm{AC}^{i}=\frac{c^{i}\left(e^{i}, d^{i}, y^{i}\right)}{h\left(e^{i}, x\right)}, \quad i=1, \ldots, n
$$

is assumed to have the traditional U-shape, with minimum average cost denoted as $\tilde{A C}^{i}$. Individual fishers may have different cost functions, implying heterogeneity of fishers.

Now the stage has been set and all the ingredients needed to analyse discarding have been introduced. Next it is time to look at the socially optimal situation.

## 3 Social optimum

In order to maximise the value of the fishery to society, the management authority, or social planner, chooses the level of effort, discarding, and stock size, taking into account that it is not possible to discard more than is harvested of low quality fish. This maximisation problem can be expressed formally as

$$
\begin{aligned}
\max _{\forall e^{i}, \forall d, i, x} V= & \sum_{i=1}^{n}\left(p_{1} \alpha h\left(e^{i}, x\right)+p_{2}\left((1-\alpha) h\left(e^{i}, x\right)-d^{i}\right)-c^{i}\left(e^{i}, d^{i}, y^{i}\right)\right) \\
\text { s.t. } \quad & (1-\alpha) h\left(e^{i}, x\right)-d^{i} \geq 0, \quad \forall i \\
& F(x)-\sum_{i=1}^{n} h\left(e^{i}, x\right)=0
\end{aligned}
$$

where the role of the first $n$ constraints is to ascertain that only low quality fish are discarded, while the last constraint assures a sustainable fishery limiting harvest to the surplus growth of the fish stock, $F(x)$. The Lagrangian of the maximisation problem is the following

$$
\begin{aligned}
\mathcal{L}= & \sum_{i=1}^{n}\left(p_{1} \alpha h\left(e^{i}, x\right)+p_{2}\left((1-\alpha) h\left(e^{i}, x\right)-d^{i}\right)-c^{i}\left(e^{i}, d^{i}, y^{i}\right)\right) \\
& +\sum_{i=1}^{n} \eta^{i}\left((1-\alpha) h\left(e^{i}, x\right)-d^{i}\right)+\lambda\left(F(x)-\sum_{i=1}^{n} h\left(e^{i}, x\right)\right)
\end{aligned}
$$

where $\lambda$ represents the shadow value of the fish stock. The other Lagrangian multipliers, $\eta^{i}$, are the shadow values of discarding for the individual fishers. Differentiating the Lagrangian with respect to the choice variables results in $3 n+2$ Kuhn-Tucker conditions, solving for the optimal effort, discarding, and shadow values of discarding for the $n$ fishers, in addition to the optimal
biomass level and the shadow value of the stock. From these, the marginal effort condition for fisher $i$ can be expressed as

$$
\begin{equation*}
\left(p_{1} \alpha+\left(p_{2}+\eta^{i}\right)(1-\alpha)-\lambda\right) h_{e^{i}}\left(e^{i}, x\right)=c_{e^{i}}^{i}\left(e^{i}, d^{i}, y^{i}\right)+\gamma h_{e^{i}}\left(e^{i}, x\right) \tag{1}
\end{equation*}
$$

This simply states that marginal benefits from changing effort must equal marginal costs of the effort change. The marginal cost has two components, first the direct cost from the effort increase, $c_{e^{i}}^{i}$, and second the increased cost in landing the additional catch, $\gamma$, multiplied with the additional harvest arising from the effort increase. The marginal benefit reflects that the fish stock consists of two types of fish, high and low quality. Each additional effort unit will bring in a certain quantity of high quality fish, $\alpha h_{e^{i}}$, which are priced at $p_{1}$. In addition, some low quality fish are caught, $(1-\alpha) h_{e^{i}}$, which are valued at $p_{2}$ plus whatever shadow value is associated with discarding. Account must also be taken of the fact that the fish input does not carry a market price and therefore the marginal benefit is reduced by the shadow value of the additional harvest, $\lambda$.

From the first order conditions a marginal discarding condition can also be derived for a representative fisher as

$$
\begin{equation*}
\gamma \leq p_{2}+c_{d^{i}}^{i}\left(e^{i}, d^{i}, y^{i}\right)+\eta^{i}, \quad d^{i} \geq 0, \quad d^{i} \mathcal{L}_{d^{i}}^{i}=0 \tag{2}
\end{equation*}
$$

where the marginal benefits from discarding - avoided landings costs - are on the left-hand side (LHS) of the inequality sign and the marginal costs on the right-hand side (RHS). The Lagrangian multiplier, $\eta^{i}$, is the shadow value of discarding, showing how much society values a marginal increase in the harvest of low quality fish for fisher $i$, keeping everything else constant. This increase in value results from being able to discard the marginal catch. Notice, that when cost functions differ among fishers, the shadow value of discarding will differ among them.

Since discarding is expressed as an inequality constraint, there are some cases to keep in mind, depending on whether the constraint is binding or not. In figure 1 the discarding constraint is partially drawn in $\left(e^{i}, d^{i}\right)$ space. The area above $d^{i}=(1-\alpha) h\left(e^{i}, x\right)$ is non-feasible since there a fisher would be discarding more than was caught of low quality fish. The feasible set is anywhere on or below the curve. If located on the curve the fisher is discarding all the low value catch, while points below the curve indicate that some low value catch is retained and landed. Three cases need consideration. The first is when profit maximisation leads fishers to operate on the


Figure 1: The discarding constraint of an individual fisher
horizontal axis, e.g., point $A$. In that case, $e^{i}>0$ and $d^{i}=0$. The second case is when the optimum is somewhere along the constraint, e.g., point $C$. The constraint is binding in that case, and $e^{i}>0$ and $d^{i}=(1-\alpha) h\left(e^{i}, x\right)$. The third case is when the optimum is somewhere inside the feasible set, e.g., point $B$. There, $e^{i}>0$ and $0<d^{i}<(1-\alpha) h\left(e^{i}, x\right)$. Often it is beneficial to use the third case in the analysis as discarding is present and fishers can react to exogenous changes without restrictions.

The final marginal condition of interest here is the marginal stock condition, which can be written as

$$
\begin{equation*}
(\hat{p}-\gamma) \sum_{i=1}^{n} h_{x}\left(e^{i}, x\right)=-\lambda\left(F^{\prime}(x)-\sum_{i=1}^{n} h_{x}\left(e^{i}, x\right)\right)-\sum_{i=1}^{n} \eta^{i} h_{x}\left(e^{i}, x\right) \tag{3}
\end{equation*}
$$

where $\hat{p}=p_{1} \alpha+p_{2}(1-\alpha)$. The condition simply states that biomass must be kept at a level where the marginal cost of increasing the stock, i.e., the foregone profit from leaving the marginal fish in the ocean, be equal to the marginal benefit of doing so, namely the increase in the value of the resource from adding one fish to the stock.

As stated above, the first order conditions determine the optimum levels of effort, discarding and stock size, as well as the shadow values of discarding and the stock. In addition to these variables, the number of fishers is endogenous to the model and needs to be determined. The marginal fisher will earn zero profits and his profit function looks like

$$
(\hat{p}-\lambda) h\left(e^{n}, x\right)-p_{2} d^{n}-c^{n}\left(e^{n}, d^{n}, y^{n}\right)=0
$$

indicating that fishers enter this fishery until

$$
\begin{equation*}
\tilde{\mathrm{AC}}^{n} \leq \hat{p}-\lambda-\frac{p_{2} d^{n}}{h\left(e^{n}, x\right)}<\tilde{\mathrm{AC}}^{n+1} \tag{4}
\end{equation*}
$$

where $\tilde{\mathrm{AC}}^{i}$ is the minimum average cost for fisher $i$. This condition shows that the level of discarding will affect the number of fishers, as the optimal number of fishers depends on the average revenue of the marginal fisher, including loss from discarding and adjusted for the shadow value of the stock.

## 4 Open access

The next step is to analyse an open access fishery and compare to the social optimum. In the analysis it is assumed that fishers behave as if they do not realise that their harvesting decisions affect stock size. Perhaps, this assumption gives fishers less credit than is due. Many fishers are knowledgeable about the biology and behaviour of fish stocks, and realise that their harvest affects the size of the fish stock. It has been argued that the most reasonable assumption when modelling the behaviour of fishers is rationality where fishers take all relevant variables into account, including the stock constraint and effort levels of other fishers [3]. Also, surveys indicate that fishers have an understanding of the effects that too many fishers can have on the resource [5]. However, it is possible to show that as soon as the number of fishers begins to increase, each fisher only has a miniscule effect on the overall fishery, thus behaving as not recognising stock effects. This is a result of a common pool externality where the benefits of one fisher reducing effort is spread across all fishers. Therefore, in the current model, each fisher is assumed to choose the level of effort and discarding that maximises his individual profits, without taking the stock constraint into account. Of course, each fisher cannot discard more than is caught of low quality fish. Formally this is expressed as

$$
\begin{gathered}
\max _{e^{i}, d^{i}} \pi^{i}=p_{1} \alpha h\left(e^{i} ; x\right)+p_{2}\left((1-\alpha) h\left(e^{i} ; x\right)-d^{i}\right)-c^{i}\left(e^{i}, d^{i}, y^{i}\right) \\
\text { s.t. }(1-\alpha) h\left(e^{i} ; x\right)-d^{i} \geq 0
\end{gathered}
$$

Since the fisher does not recognise the impact harvesting has on the fish stock, there is no stock constraint. The Lagrangian is

$$
\mathcal{L}^{i}=p_{1} \alpha h\left(e^{i} ; x\right)+p_{2}\left((1-\alpha) h\left(e^{i} ; x\right)-d^{i}\right)
$$

$$
-c^{i}\left(e^{i}, d^{i}, y^{i}\right)+\mu^{i}\left((1-\alpha) h\left(e^{i} ; x\right)-d^{i}\right)
$$

where $\mu^{i}$ is the shadow value of discarding for the individual fisher.
From the corresponding Kuhn-Tucker conditions for maximisation, the marginal effort condition can be shown as

$$
\left(p_{1} \alpha+\left(p_{2}+\mu^{i}\right)(1-\alpha)\right) h_{e^{i}}\left(e^{i} ; x\right)=c_{e^{i}}^{i}\left(e^{i}, d^{i}, y^{i}\right)+\gamma h_{e^{i}}\left(e^{i} ; x\right)
$$

Comparison to equation (1) shows the expected result, that since the fisher does not take the shadow value of the stock into account, marginal benefit would exceed marginal cost if effort was chosen at the optimal level. Therefore, more effort will be used in open access than is optimal.

In addition to the increased effort of the individual fisher in open access as compared to the optimal situation, more fishers will enter the fishery. This is seen from the entry condition, which now is

$$
\tilde{\mathrm{AC}}^{n} \leq \hat{p}-\frac{p_{2} d^{n}}{h\left(e^{n}, x\right)}<\tilde{\mathrm{AC}}^{n+1}
$$

This is very similar to equation (4) which shows the socially optimal entry condition. The only difference is that now $\lambda$ is missing - fishers do not recognise the shadow value of the fish stockincreasing the average cost at which the marginal fisher enters, indicating that more fishers will be operating in the fishery than is socially optimal. This can also be looked at from the revenue side. Since fishers do not recognise the shadow value, their private landed value of the catch is higher than the social landed value. Therefore the marginal fisher can enter at a higher average cost than if the shadow value is included.

To compare the stock size between the socially optimal situation and open access, rewrite the socially optimal marginal stock condition (equation 3) as

$$
(\hat{p}-\lambda) \sum_{i=1}^{n} h_{x}\left(e^{i}, x\right)+\sum_{i=1}^{n} \eta^{i} h_{x}\left(e^{i}, x\right)=-\lambda F^{\prime}(x)+\gamma \sum_{i=1}^{n} h_{x}\left(e^{i}, x\right)
$$

The lhs of this equation measures the marginal benefits from reducing the size of the fish stock, while the RHS is the marginal cost of the reduction. Since in open access the landed value is higher than is optimal, it implies that the marginal benefit of reducing the stock, for any given effort level, is higher in open access than in the socially optimal situation. As a result, at the optimal stock size fishers in open access will have marginal benefits from reducing the stock greater than the marginal cost. They will fish the stock further down and consequently the stock size in open access will be lower than is socially optimal.

## 5 Individual transferable quotas

The management authority sets a total allowable catch (TAC) in the fishery and each fisher receives an allocation of $\hat{q}^{i}$. Each fisher may also purchase or sell quota. Notice, that enforcement takes place by comparing the quota against the amount of fish landed, not harvested fish. The problem of the individual fisher is now to

$$
\begin{aligned}
\max _{e^{i}, d^{i}, q^{i}} \pi= & p_{1} \alpha h\left(e^{i} ; x\right)+p_{2}\left((1-\alpha) h\left(e^{i} ; x\right)-d^{i}\right)-c^{i}\left(e^{i}, d^{i}, y^{i}\right)-a q^{i} \\
\text { s.t. } & (1-\alpha) h\left(e^{i} ; x\right)-d^{i} \geq 0 \\
& \hat{q}^{i}+q^{i}-y^{i}=\hat{q}^{i}+q^{i}-h\left(e^{i} ; x\right)+d^{i} \geq 0
\end{aligned}
$$

where the second constraint is the quota constraint. The total quota of the fisher is the sum of his allocation and his purchases, $q^{i}$, of additional quota. If $q^{i}$ is a positive number, that indicates that the fisher purchased additional quota at the unit price of $a$. If $q^{i}$ is negative, on the other hand, then the fisher sold some of the initial allocation, also at a price of $a$. Landings of fish may not exceed the sum of quota allocated and net purchases of quota. If harvest exceeds the quota of a fisher, then some fish must be discarded.

The Lagrangian for the individual fisher is now

$$
\begin{aligned}
\mathcal{L}^{i}= & p_{1} \alpha h\left(e^{i} ; x\right)+p_{2}\left((1-\alpha) h\left(e^{i} ; x\right)-d^{i}\right)-c^{i}\left(e^{i}, d^{i}, y^{i}\right)-a q^{i} \\
& +\mu^{i}\left((1-\alpha) h\left(e^{i} ; x\right)-d^{i}\right)+\rho^{i}\left(\hat{q}^{i}+q^{i}-h\left(e^{i} ; x\right)+d^{i}\right)
\end{aligned}
$$

where $\rho^{i}$ is the shadow value of quota.
Assuming that the discarding constraint is non-binding and $d^{i}>0$, the marginal discarding condition under the ITQ system can be derived from the Kuhn-Tucker conditions as

$$
\gamma+a=p_{2}+c_{d^{i}}^{i}\left(e^{i}, d^{i}, y^{i}\right)
$$

Comparing this with the socially optimal condition from equation $(2)^{2}$, there is an extra term on the marginal benefit side, namely the quota price, $a$. This is because discarding fish not only saves landings cost but also less quota is used and that quota may be sold. The quota price therefore has an impact on discarding behaviour, as suggested in [10]. Unless $a=0$, the quota price will increase the incentive to discard as compared to the optimal situation. The more

[^1]restrictive the quota, the more valuable it is, and the greater is the discarding incentive. Only when the quota constraint is not binding will $a=0$, but in that case the ITQ system has no impact on fishers' behaviour anyway.

To compare the ITQ fishery with the open access situation, let's derive the comparative statics results of the model when the quota allocation is changed. In that exercise the discarding contraint is assumed nonbinding, while the quota constraint is binding. The relevant first order conditions are

$$
\begin{aligned}
\mathcal{L}_{e^{i}} & =\left(\hat{p}-c_{y^{i}}-a\right) h_{e^{i}}\left(e^{i} ; x\right)-c_{e^{i}}^{i}\left(e^{i}, d^{i}, y^{i}\right)=0 \quad \forall i \\
\mathcal{L}_{d^{i}} & =-p_{2}-c_{d^{i}}^{i}\left(e^{i}, d^{i}, y^{i}\right)+\gamma+a=0 \quad \forall i \\
\mathcal{L}_{\rho^{i}} & =\hat{q}^{i}+q^{i}-h\left(e^{i} ; x\right)+d^{i}=0 \quad \forall i
\end{aligned}
$$

Notice, that differentiating the Lagrangian with respect to $q^{i}$ results in the condition that $a=\rho^{i}$ for every fisher. This condition means that quota trading will take place until all fishers have the same shadow value for quota, equal to $a$. If that was not the case, and two or more fishers have different shadow values then there are gains from trade to be made between those fishers. In the above equations, $a$ has been substituted in for $\rho^{i}$ to facilitate the calculations.

In addition to these equations, two more conditions need to be taken into account. The first is

$$
\sum_{i=1}^{n} q^{i}=0
$$

which sets a limit on the supply of quota, recognising that fishers cannot buy, or sell, unlimited amounts of quota. What one buys, another must sell, and this equation is the market clearing condition for the quota market.

The last equation to be added to the system is the ubiquitous stock condition

$$
\theta\left(e^{1}, \ldots, e^{n}, x\right)=F(x)-\sum_{i=1}^{n} h\left(e^{i}, x\right)=0
$$

ensuring that the harvest level is sustainable.
All these $3 n+2$ equations together solve for the endogenous variables in the system, $e^{i}$, $d^{i}, q^{i}, x$, and $a$ for all $i$. The equation system can be differentiated with respect to all the endogenous variables and the exogenous variables of interest, $\hat{q}^{i}$. The resulting equation system
can be expressed as

$$
\left[\begin{array}{ccccc}
\mathbf{L}_{\mathbf{e}^{\mathbf{i}}} & \mathbf{0} & \mathbf{0} & -\vec{h}_{e^{i}} & \overrightarrow{\mathcal{L}}_{e^{i} x}^{T} \\
\mathbf{0} & \mathbf{L}_{\mathbf{d}^{\mathbf{i}}} & \mathbf{0} & \overrightarrow{1}^{T} & \overrightarrow{0}^{T} \\
-\mathbf{h}_{\mathbf{e}^{\mathbf{i}}} & \mathbf{I} & \mathbf{I} & \overrightarrow{0}^{T} & -\vec{h}_{x}^{T} \\
\overrightarrow{0} & \overrightarrow{0} & \overrightarrow{1} & 0 & 0 \\
-\vec{h}_{e^{i}} & \overrightarrow{0} & \overrightarrow{0} & 0 & \theta_{x}
\end{array}\right]\left[\begin{array}{c}
\overrightarrow{d e^{i}} \\
\overrightarrow{d d^{T}} \\
\overrightarrow{d q^{i}} \\
d a \\
d x
\end{array}\right]=\left[\begin{array}{c}
\overrightarrow{0}^{T} \\
\overrightarrow{0}^{T} \\
-\overrightarrow{1}^{T} \\
0 \\
0
\end{array}\right] d \hat{q}^{i}
$$

where $\mathcal{L}_{e^{i} e^{i}}^{i}=(\hat{p}-\gamma-a) h_{e^{i} e^{i}}-c_{e^{i} e^{i}}<0, \mathcal{L}_{e^{i} x}^{i}=(\hat{p}-\gamma-a) h_{e^{i} x}>0, \mathcal{L}_{d^{i} d^{i}}^{i}=-c_{d^{i} d^{i}}<0$, and $\theta_{x}=F^{\prime}(x)-\sum_{i=1}^{n} h_{x}<0$. The coefficient matrix, denoted $\mathbf{D}$, has been partitioned into smaller matrices. The boldfaced elements of $\mathbf{D}$ refer to $n \times n$ diagonal matrices. For instance, the diagonal of $\mathbf{L}_{\mathbf{e}^{\mathbf{i}}}$ consists of the second derivatives of the Lagrangian with respect to the effort level of the corresponding fisher. That is

$$
\mathbf{L}_{\mathbf{e}^{\mathbf{i}}}=\left[\begin{array}{cccc}
\mathcal{L}_{e^{1} e^{1}}^{i} & 0 & \cdots & 0 \\
0 & \mathcal{L}_{e^{2} e^{2}}^{i} & & \vdots \\
\vdots & & \ddots & 0 \\
0 & \cdots & 0 & \mathcal{L}_{e^{n} e^{n}}^{i}
\end{array}\right]
$$

$\mathbf{L}_{\mathbf{d}^{i}}$ is the same except the diagonal elements are differentiated with respect to $d^{i}$. The matrix $\mathbf{I}$ is the $n \times n$ identity matrix, while $\mathbf{0}$ is $n \times n$ matrix with all elements equal to zero, $\overrightarrow{0}$ is a $1 \times n$ vector of zeros, and $\overrightarrow{0}^{T}$ is the transpose of $\overrightarrow{0}$. The vectors, in general, are $1 \times n$, containing the elements indicated. For instance,

$$
\begin{aligned}
\vec{h}_{e^{i}} & =\left[\begin{array}{llll}
h_{e^{1}} & h_{e^{2}} & \cdots & h_{e^{n}}
\end{array}\right] \\
\overrightarrow{d e^{i}} & =\left[\begin{array}{llll}
d e^{1} & d e^{2} & \cdots & d e^{n}
\end{array}\right] \\
\overrightarrow{1} & =\left[\begin{array}{llll}
1 & 1 & \cdots & 1
\end{array}\right]
\end{aligned}
$$

This notation simplifies considerably some of the matrix equations that need to be analysed.
Turning back to the model, the determinant of $\mathbf{D}$ can be shown to equal ${ }^{3}$

$$
|\mathbf{D}|=\prod_{i=1}^{n} \mathcal{L}_{e^{i} e^{i}}^{i} \mathcal{L}_{d^{i} d^{i}}^{i}\left(F^{\prime} \sum_{i=1}^{n} \frac{h_{e^{i}}^{2}}{\mathcal{L}_{e^{i} e^{i}}^{i}}+\left(\left(F^{\prime}-\sum_{i=1}^{n} h_{x}\right)+\sum_{i=1}^{n} \frac{h_{e^{i}} \mathcal{L}_{e^{i} x}^{i}}{\mathcal{L}_{e^{i} e^{i}}^{i}}\right) \sum_{i=1}^{n} \frac{1}{\mathcal{L}_{d^{i} d^{i}}^{i}}\right)
$$

[^2]This determinant is difficult to sign. However, assuming that $F^{\prime}<0$, makes the whole determinant positive. ${ }^{4}$

Starting with the quota price, the effects an increase in the quota allocation has can be found by utilising the following determinant

$$
\left|\mathbf{D}_{a}\right|=n \prod_{i=1}^{n} \mathcal{L}_{e^{i} e^{i}}^{i} \mathcal{L}_{d^{i} d^{i}}^{i}\left(\left(F^{\prime}-\sum_{i=1}^{n} h_{x}\right)+\sum_{i=1}^{n} \frac{h_{e^{i}} \mathcal{L}_{e^{i} x}^{i}}{\mathcal{L}_{e^{i} e^{i}}^{i}}\right)<0
$$

which implies

$$
\frac{\partial a}{\partial \hat{q}^{i}}=\frac{\left|\mathbf{D}_{a}\right|}{|\mathbf{D}|}<0
$$

If there is a greater supply of quotas, i.e., quota allocations are increased, the price of one quota unit falls. If less is supplied, then the price increases. This is not surprising at all; it simply indicates a downward sloping demand for quotas.

Looking next at the effect a change in the quota allocation has on discarding, it is clear in the model. Since the determinant

$$
\left|\mathbf{D}_{d^{i}}\right|=-n \mathcal{L}_{e^{i} e^{i}}^{i} \prod_{j \neq i}^{n} \mathcal{L}_{e^{j} e^{j}}^{i} \mathcal{L}_{d^{j} d^{j}}^{i}\left(\left(F^{\prime}-\sum_{i=1}^{n} h_{x}\right)+\sum_{i=1}^{n} \frac{h_{e^{i}} \mathcal{L}_{e^{i} x}^{i}}{\mathcal{L}_{e^{i} e^{i}}^{i}}\right)
$$

is negative, the implication is that

$$
\frac{\partial d^{i}}{\partial \hat{q}^{i}}=\frac{\left|\mathbf{D}_{d^{i}}\right|}{|\mathbf{D}|}<0
$$

This means that if the management authority allocates more quota, the individual fisher will reduce discarding. With a higher allocation the quota constraint is not as restrictive as before. As shown above, the value of quota falls and with it the marginal benefit of discarding low value fish. If, on the other hand quotas are reduced, the opposite occurs. Quotas will be in greater demand, and consequently their value increases. The marginal benefit of discarding rises and each fisher will have an incentive to increase the amount of fish that is discarded.

Turning the attention to the effect an increase in the quota allocation has on stock size, it can be shown to equal

$$
\frac{\partial x}{\partial \hat{q}^{i}}=\frac{\left|\mathbf{D}_{x}\right|}{|\mathbf{D}|}=\frac{n \prod_{i=1}^{n} \mathcal{L}_{e^{i} e^{i}}^{i} \mathcal{L}_{d^{i} d^{i}}^{i}\left(\sum_{i=1}^{n} \frac{h_{e^{i}}^{2}}{\mathcal{L}_{e^{i} e^{i}}^{i}}\right)}{|\mathbf{D}|}<0
$$

[^3]which states that as the quota allocation is increased the stock size falls. Reducing the quota allocation, even in the presence of discarding will therefore have conservatory effects on the fish stock. This is of considerable interest, of course, since a frequently heard argument against an ITQ system is the destructive impact that discarding may have on the stock. This model, on the other hand, predicts that the biomass will always increase following a reduction in quota levels. The rationale is that it will never pay to increase the effort level when you are allowed to land less fish. It increases marginal costs while marginal revenues are falling. Since stock size falls when more quota is allocated, overall effort must increase. On the other hand, when quota allocations are reduced, effort must fall since the biomass becomes larger.

The optimal number of fishers will be reduced, as compared to open access, when a quota is introduced. Remember, that the quota is restricting the fisher from the open access situation by giving him a quota that is less than he would want to harvest. Therefore the quota constraint is binding. The profits of the $n$-th fisher will now be

$$
\pi^{n}=\hat{p} h\left(e^{n} ; x\right)-p_{2} d^{n}-c^{n}\left(e^{n}, d^{n}, y^{n}\right)-a q^{n}=a \hat{q}^{n}
$$

where $a \hat{q}^{n}$ is the profit that can be made by selling the quota allocation and leaving the fishery. In other words, the profit of the marginal fisher of staying in the fishery must be equal to the profit he can make if he leaves. From this equation the entry condition in the ITQ situation can be expressed as

$$
\tilde{\mathrm{AC}}^{n} \leq \hat{p}-a+\frac{a d^{n}}{h\left(e^{n} ; x\right)}-\frac{p_{2} d^{n}}{h\left(e^{n} ; x\right)}<\tilde{\mathrm{AC}}^{n+1}
$$

As long as $a$ is positive, the minimum average cost of the $n$-th fisher will be lower than in open access. Consequently fewer fishers will operate in the fishery after individual quotas have been introduced. This is logical, since low cost fishers that value quota more will buy quota from high cost fishers. Some high cost fishers will opt to sell all their quota and leave the fishery for greener pastures elsewhere. It is worth emphasising that these fishers leave the fishery at their own free will and receive the value of their quota allocation when they leave.

It is of interest to compare the ITQ entry condition with the optimal entry condition as given by equation (4). If an ITQ system is to be optimal, the correct number of fishers must operate in the fishery. However, when discarding is present, the two entry conditions differ. Even if $a=\lambda$, which results in an optimal fishery in the absence of discarding [3] the ITQ entry condition has an
extra term that is not present in the optimal condition, i.e., $\lambda d^{n} / h\left(e^{n} ; x\right)$. This term represents the fact that the individual fisher does not recognise the loss to society caused by the reduction in the biomass due to discarding. Therefore, the minimum average cost of the $n$-th fisher is higher with ITQS than is optimal, indicating that too many fishers will operate in the fishery.

## 6 Conclusion

The presence of discarding means that an ITQ system will not lead to an optimal situation. There will be excessive discarding and too many fishers. However, an ITQ system will nonetheless be an improvement from the open access situation. The biomass will be larger and rents will be generated in the fishery, represented by the quota price. Whether fishers capture these rents depends on how the government allocates quotas. If allocated for free, then the rents accrue to fishers, but if the government charges for the allocation then the government receives a part or all of the rents.

A striking result from the analysis is that even if an ITQ system is suboptimal in the presence of discarding, it leads to an increase in biomass as compared to the open access situation. This is an important finding as it suggests that even if restrictive management policies such as ITQS increase the incentive to discard fish, they will not lead to an increase in effort. A fisher whose quota is reduced will reduce effort even if he discards more fish. The reduction in effort will be less than if discarding was not possible, but it will still be a reduction. Since effort is falling, the size of the fish stock will increase. This suggests that it is not the level of discards that is important from a conservation point of view, but the overall effort level and the associated fishing mortality of the stock. Therefore conservation policies should focus on effort, aiming to reduce fishing mortality, rather than a reduction in discards. This is not to say that the excessive discarding that may result from a management policy is harmless. However, the harm is on the economic side, not on the biological side. Excessive discarding leads to a loss for society in terms of wasted economic resources, but according to the model presented here it is not expected to result in the depletion of fish stocks.

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[^0]:    ${ }^{1}$ However, this allows a focus on the essentials of discarding in a relatively straightforward manner [2, 8].

[^1]:    ${ }^{2}$ Since the discarding constraint is assumed non-binding, $\eta^{i}=0$ in the socially optimal situation.

[^2]:    ${ }^{3}$ To calculate this determinant, and the others that follow, use was made of the mathematical computer programme Maple V created by the University of Waterloo. The number of fishers was set to three and Maple asked to calculate the determinant. Once the outcome had been simplified, it was expanded to the case of $n$ fishers.

[^3]:    ${ }^{4}$ This assumption follows [7] who claims that it: "... is reasonable for a fishery comprised of a large number of entrepreneurs." This assumption is actually stronger than needed as $\mathbf{D}$ will be positive for some range of positive values of $F^{\prime}$

