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Signal-noise Decomposition in Financial Markets: An Empirical Stochastic Process Analysis for Infrequent Trading

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Abstract

The observed transaction prices on a stock market at discrete time points are assumed to be a sample from a continuous time-value process. The theory of an efficient market is used as motivation for a random-walk type model. The fact that bid-ask spread and other microstructure phenomena exist is accounted for by adding a noise term to the model. Models for elementary detrending based on stochastic processes in continuous time are set up. For empirical analysis, they are formulated in state-space form, and calculations are performed with the Kalman-filter recursions. The result is an analytical way of decomposing the observed transaction price change into a value innovation and a market noise component. The respective innovation standard deviations and market noise standard deviation are easily interpreted. Some alternative stochastic structures are considered and applied to data from the Iceland Stock Exchange.

Keywords: Diffusion processes, state-space models, financial data, infrequent trading.

1 Introduction

The value of a financial asset is of interest, and up-to-date information is needed for optimal decision-making. Cash or money in the bank is immediately transferable to goods in the store so information on the value can be obtained at each time point by counting without any uncertainty involved. In highly active financial markets, assets with high liquidity have similar properties, the holder of the asset follows the reported statistics with certainty similar to that of a statement from a bank. The goal of this paper is to develop and test tools for reporting the status of an asset traded on a financial market characterized by "infrequent trading".

The nature of a financial market is that trading takes place at discrete time points. If the time intervals between trading tend to be long, the trading is viewed as infrequent. The problem of infrequent trading describes a situation of uncertainty about the value of assets. This uncertainty calls for some estimation of the value between trades. The estimation is essentially a kind of a forecasting exercise. Based on history, we calculate at any point in time an estimate of the value. When trading takes place, we obtain a measure of the quality of this estimate. Official data reported from financial markets are typically time-series data, containing information of closing prices, last bid/ask, highest/lowest price of the day, etc. So, when closing prices are reported, it is essentially a forecast, stating that the last transaction is an objective measure of the value, and that nothing has happened in the time from last trade to closing time.

The theory of efficient markets describes the behavior of prices under ideal conditions. By definition of an efficient market, one should not be able to make a profit by forecasting future prices. The efficient market hypothesis implies a kind of martingale probability structure. The simplest model of that type for returns in a financial market is the random-walk model. It captures the non-forecast-ability feature of the ideal conditions. The quoting of last trading prices as the current value is essentially just an application of this model. Therefore, the fact that the observed auto- and cross-correlation of observed returns seem to yield scope for profit has puzzled many authors. Explanations have been suggested, such as that this is somehow due to infrequent trading, non-synchronized trading, bid-ask spread, asymmetric information, etc. (Campbell et al., 1997; Glosten and Milgrom, 1985). Campbell et al. (1997) give a review of how such phenomena could possibly affect the sample autocorrelation function estimated from reported closing time prices. Some authors have developed computational procedures to correct for such microstructure effects, perhaps the most well-known is the Scholes and Williams (1977) method for calculating betas in CAPM models under non-synchronized trading. The market microstructure literature is full of empirical examples showing that empirical returns have significant autocorrelation structure, seemingly contradicting the efficient market hypothesis. For further discussion on the efficient market hypothesis, see, e.g., Cuthbertson (1996). The market microstructure literature deals with explaining the difference in theoretical and observed phenomena of the market. A review of market microstructure theory is given in O'Hara (1995).

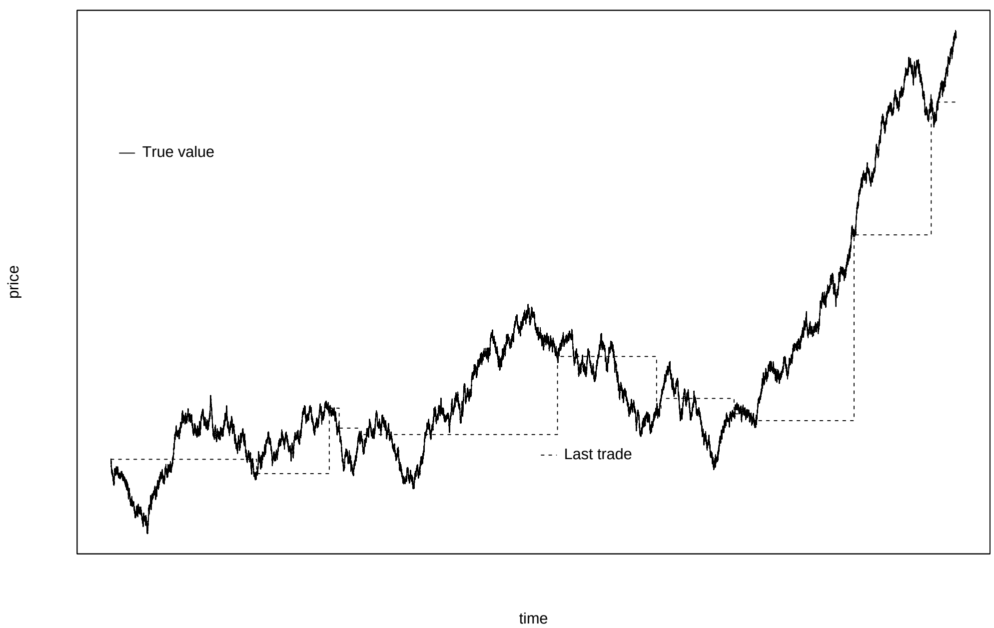

Figure 1: True value process and observed process.

Ait-Sahalia (1998) shows a mathematical model with a monopolist specialist market maker and agents, where all participants in the market optimize their respective utility functions. The resulting optimal behavior generates an observed process with a correlation-structure different from that of a random walk. The optimization problems of agents in the market are also discussed by O'Hara (1995).

It is not realistic to assume that this trend is constant over time.

In this paper, the optimizing problem of agents in the market is not treated nor is the theory of efficient market contested. The focus is on defining a simple stochastic model capturing the features of observed data resulting from infrequent trading. The phenomena of infrequent trading is illustrated in Figure 1. The jagged curve represents the value at each time point, whereas the jumping one is a continuous extrapolation of last-observed transactions. The aim is to get an up-to-date, on-line estimate of the value of a financial asset. The base will be a random-walk model, which is augmented in a simple way. The key idea is to add a noise component to the random walk model so that the observed transaction price is composed of a market signal describing value dynamics and a noise term capturing microstructure phenomena. The interpretation of the noise component is that it is an estimate of the size of all microstructure phenomena. Typically the bid-ask spread would contribute to such a component. For markets with infrequent trading, data on bids and asks are also likely be infrequent. For assets where there are no market makers some of the time, the quality of bid-ask data as well as statistical modeling of it might also raise difficulties. The setup of the model as market signal plus noise also allows for simultaneous transactions at different prices. The standard deviation of the noise term gives an objective measure of the spread of such transactions. Graphical inspection of many financial time series often reveals a persistent trend. Assets on a financial market compete with money in the bank, and the fact that interest rates are generally positive suggests that a realistic model for the development of value in time should contain a drift term. The random-walk model consistently underestimates trend so it could be argued that for a credible signal-noise analysis, some kind of detrending is necessary. Four types of trends in a continuous-time stochastic process are considered: first, no drift; second, a constant drift; third, a random walk drift and fourth, a mean-reverting stationary drift. Detrending is performed by estimating a model containing a given drift structure.

The fact that transactions only take place at discrete time points calls for an adaption of the continuous-time model to a discrete one. The popularity of continuous-time stochastic process models in finance has increased in recent years due to the work of people like the Nobel prize winner Merton, see, e.g., Merton (1990) and others. From a statistical viewpoint, it is not clear how to deal with the estimation of drift and diffusion of a continuous-time process where it is not even possible to perform approximately sampling in continuous time. The basis of the approach taken here is to assume a very simple form of the drift and diffusion terms and set up a state-space model, distinguising between the observed price and the true value. The state-space model consists of a state-equation, describing the dynamics of the true unobservable state and a measurement equation, describing the composition of state and noise. A feature of this approach is that it incorporates the fact that at the same time point, it is possible to have two or more observed prices, whereas it is assumed that value is unique, i.e., the approach is that observed prices are noisy measurements of true value.

The model considered will assume that the log of observed prices consists of a random-walk signal, analogous to the efficient market hypothesis, with added noise independent of the signal. The size of the signal, i.e., daily innovations, is related to the size of the noise by the ratio of the respective standard deviations.

The goal of this paper is to design descriptive measures for market development in thin, newly developed markets with infrequent trading and low volumes. The slow trading represents rare sampling of a continuous value process at discrete time points. The estimated standard deviations for the signal and the noise, respectively, give an objective measure of market behavior of individual stocks. Some simple continuous-time stochastic process models are defined in Section 2. The choice of models is such that the traditional way of reporting, i.e., reporting the last transaction as an estimate of value, is a special case of the model considered. The scope for very complex models is not feasible when data are sparse. The observational models for the discretely observed empirical process are defined in Section 3. The model is naturally represented in a state-space form, which is ideal for computational purposes. The state-space setup together with the Kalman-filter yields on-line estimates of the state vector, i.e., value and trend, at each point in time together with an uncertainty measure of these estimates. Together with the estimated standard deviation of the noise, they can be used to calculate a probability-based interval for the price in an eventual transaction. The interpretation of these quantities for the state of the market is straightforward. Authorities on stock markets monitor trading in order to detect eventual foul play. It is, for example, possible to use the methods presented here to give objective benchmarks. In Section 4, the models are applied to Icelandic stock market data, and parameters are estimated by numerically maximizing the normal likelihood. The choice of the normal likelihood is partly based on computational reasons. The results could be interpreted as quasi-maximum-likelihood. Section 5 contains discussion of the methods and future research.

2 The models

Modeling the dynamics of stocks and other financial assets is often based on some form of Brownian motion. The simplest one is perhaps a pure random-walk model, i.e:

$$dS(t) = \sigma dW(t) \tag{1}$$

where $W(t)$ is standard Brownian motion, $(W(0) = 0, V(W(t)) = t)$. A popular version assumes that the relative prices (log-prices) follow Brownian motion, i.e., that the absolute prices follow geometric Brownian motion. The best known case is perhaps the Black-Scholes model.

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{2}$$

$$dB(t) = rB(t)dt \tag{3}$$

where $S(t)$ denotes the value of a certain stock at time t, and $B(t)$ denotes the value of a bank account at time t. Model based on equation systems such as equations (2) and (3) play an important role in the option pricing literature (Björk, 1998). The modern mathematical finance literature treats more general forms of diffusion models than equation (2) (Merton, 1990; Shiryaev, 1999).

The traditional approach, e.g., the use of last observed prices as a proxy for value at closing time, is essentially forecasting future value with the last observed price, which corresponds to using equation (1) on equidistant time spaces.

In the present paper, just as in equation (2), the approach is based on geometric Brownian motion. The motivation is the author's belief that behavior of relative price changes is closer to Brownian motion than that of absolute price changes. The initial model considered is:

$$d\alpha(t) = \sigma dW(t) \tag{4}$$

where $\alpha(t) = log(\text{value at time } t)$. The notation of using α, rather than $log(S)$, is to emphasize that the value is interpreted as a latent variable. The market microstructure literature suggest that the empirical performance of equation (4) is not satisfactory. A simple, straightforward extension of the model is to add a noise term. The observations on the market are then given by

$$y(t) = \alpha(t) + \varepsilon(t) \tag{5}$$

where $y(t) = log(\text{observed prices at time } t)$. The second term in equation 5, $\varepsilon(t)$, can be interpreted as some kind of market noise, e.g., due to bid-ask spread, or how different observed prices at the same time-point can be. The variance of $\varepsilon(t)$, σ_ε^2, is a measure of this variability.

Equation (4) together with equation (5) represents a simple extension of the model, "today's value=best prediction for tomorrow". This extension represents a random-walk model with added noise linking together the concepts of transaction price and true value. The fact that a pure random-walk model underestimates trend, often observed in financial data, suggests that a drift parameter, like μ, in equation (2) should be included. The simplest case is to add a constant drift. The notion of constant drift may not be realistic. As the cycles in a financial market are conceivably steeper than those of a random-walk, it may be preferable to let the drift itself be a stochastic process. A straightforward extension is to assume that the drift itself is a random-walk.

$$d\alpha_1(t) = \alpha_2(t)dt + \sigma_1 dW_1(t) \tag{6}$$

The function $\alpha_2(t)$ is the drift and is assumed to be stochastic and to follow its own diffusion:

$$d\alpha_2(t) = \sigma_2 dW_2(t) \tag{7}$$

On vector form equations (6) and (7) can be written:

$$d\boldsymbol{\alpha}(t) = A\boldsymbol{\alpha}(t)dt + Cd\boldsymbol{W}(t)$$

where $\boldsymbol{\alpha}(t) = (\alpha_1(t), \alpha_2(t))'$, $\boldsymbol{W}(t) = (W_1(t), W_2(t))'$ and:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and } C = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \tag{8}$$

A model of this type is more flexible and conceivably picks up more persistent movements than those of the random walk.

A property of this model's forecast function might be a drawback from an infrequent trading point of view. In the period of no observations, the point forecast is the last estimated value prolonged with the last estimate of the drift. A possible remedy is to modify the model so that the long horizon forecast is more conservative. One possibility for achieving this is to assume that the drift follows a mean-reverting process, e.g., of Ornstein-Uhlenbeck type:

$$\begin{aligned} d\alpha_1(t) &= \alpha_2 dt + \sigma_1 dW_1(t) \\ d\alpha_2(t) &= -\gamma(\alpha_2(t) - r)dt + \sigma_2 dW_2(t) \qquad \gamma \geq 0 \end{aligned} \tag{9}$$

The parameter r represents the mean for the $\alpha_2(t)$ process, and γ represents the speed of reversion to the mean. In matrix form it is:

$$d\boldsymbol{\alpha}(t) = A\boldsymbol{\alpha}(t)dt + \boldsymbol{b}dt + Cd\boldsymbol{W}(t) \tag{10}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\gamma \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ \gamma r \end{bmatrix} \tag{11}$$

and, as before, the observed log-price is

$$y(t) = \alpha_1(t) + \varepsilon(t) \tag{12}$$

The difference between equation (5) and equation (12) is that $\alpha_1(t)$ in equation (12) contains drift. The model in its most general form, equations (10) and (12), includes the other models in the section as special cases. The parameter vector $\theta = \sigma_\varepsilon, \sigma_1, \sigma_2, \gamma, r$ is easily interpretable as well as estimatable. The models are labeled $M_1, \ldots, M_4$, see Table 1.

Table 1 summarizes the model structures involved. The theoretical financial literature typically treats $\sigma_\varepsilon = 0$ (Björk, 1998; Merton, 1990; Shiryaev, 1999) which does not allow simultaneous transactions at different prices. The interpretation of the parameters and the states is easy. The state α_1 represents level of value; the state α_2 represents trend of value; σ_ε represents standard deviation of market noise; and σ_1 and σ_2 represent standard deviation of input signals.

Model	Type
M1	Brownian motion with added noise
M2	Brownian motion with added noise and deterministic trend
M3	Brownian motion with added noise and Brownian motion drift
M4	Brownian motion with added noise and Ornstein-Uhlenbeck drift

Table 1: Model types analyzed

3 Dealing with discrete data

Continuous sampling is only possible in theoretical situations, and the possibility of approximating continuous sampling by decreasing the sampling interval is only an option when the sampling can be controlled. In the case of financial market data, the sampling process cannot be controlled, and in the case of infrequent trading, the time interval between sampling points can become considerable. In the this paper, the technical problem of discrete sampling is cast in the framework of state-space analysis for time series. The notation is based on Harvey (1989). Data on log-observed prices $y(t)$ are obtained at discrete time points $t_1, t_2, \ldots$. The models in Section 2 are linear stochastic differential equations. Solving these at time point t_i given t_{i-1} gives:

$$\boldsymbol{\alpha(t_i)} = e^{A\delta_i}\boldsymbol{\alpha(t_{i-1})} + \int_{t_{i-1}}^{t_i} e^{A(s-t_{i-1})}\boldsymbol{b}ds + \int_{t_{i-1}}^{t_i} e^{A(s-t_{i-1})}Cd\boldsymbol{W(s)} \tag{13}$$

where $\delta_i = t_i - t_{i-1}$, and A is given by equation (11). Equation (13) represents the dynamics of the unobservable state vector between observation time points. Equation (13) together with the observation equation, (12) fits directly into a state-space representation. With some algebraic manipulations, shown in appendix A, equation (13) can be written as:

$$\boldsymbol{\alpha}(t_i) = T(t_i, t_{i-1})\boldsymbol{\alpha}(t_{i-1}) + \begin{bmatrix} \delta_t - g_1(\delta_i, \gamma) \\ \gamma g_1(\delta_i, \gamma) \end{bmatrix} + \boldsymbol{\xi}(t_i, t_{i-1}) \tag{14}$$

with the matrix controlling the transition between time t_i and t_{i-1} given by

$$T(t_i, t_{i-1}) = \begin{bmatrix} 1 & g_1(\delta_i, \gamma) \\ 0 & e^{-\gamma\delta_i} \end{bmatrix}$$

The variance of the innovation from time point t_{i-1} to time point t_i, $\boldsymbol{\xi}(t_i, t_{i-1})$ is given by

$$\begin{gathered} \boldsymbol{Q}(t_i, t_{i-1}) = V(\boldsymbol{\xi}(t_i, t_{i-1})) = \\ \int_{t_{i-1}}^{t_i} \begin{bmatrix} \sigma_1^2 + \frac{\sigma_2^2}{\gamma^2}(1 - e^{-\gamma(s-t_{i-1})})^2 & \frac{\sigma_2^2}{\gamma}(1 - e^{-\gamma(s-t_{i-1})}) \\ \frac{\sigma_2^2}{\gamma}(1 - e^{-\gamma(s-t_{i-1})}) & \sigma_2^2 \end{bmatrix} ds = \\ \begin{bmatrix} \sigma_1^2\delta_i + \sigma_2^2 g_3(\delta_i, \gamma) & \sigma_2^2 g_2(\delta_i, \gamma) \\ \sigma_2^2 g_2(\delta_i, \gamma) & \sigma_2^2 g_1(\delta_i, 2\gamma) \end{bmatrix} \end{gathered} \tag{15}$$

where (see appendix A)

$$\begin{aligned} g_1(\delta, \gamma) &= \frac{1}{\gamma}(1 - e^{-\gamma\delta}) \\ g_2(\delta, \gamma) &= \frac{1}{\gamma}(g_1(\delta, \gamma) - g_1(\delta, 2\gamma)) \\ g_3(\delta, \gamma) &= \frac{1}{\gamma^2}(\delta - 2g_1(\delta, \gamma) + g_1(\delta, 2\gamma)) \end{aligned}$$

These formulas fit directly into the Kalman-filter recursions (Harvey, 1989). Conditional on information available at time t_{i-1}, $\boldsymbol{\alpha}(t|t_{i-1})$, denotes the estimate of the state vector at time t. The corresponding covariance-matrix of this estimate is $\boldsymbol{P}(t|t_{i-1})$. The are calculates with equations (16) and (17).

$$\boldsymbol{\alpha}(t|t_{i-1}) = T(t, t_{i-1})\boldsymbol{\alpha}(t_{i-1}|t_{i-1}) + \begin{bmatrix} t - t_{i-1} - g_1(t - t_{i-1}, \gamma) \\ \gamma g_1(t - t_{i-1}, \gamma) \end{bmatrix} \tag{16}$$

$$\boldsymbol{P}(t|t_{i-1}) = T(t, t_{i-1})\boldsymbol{P}(t_{i-1}|t_{i-1})T(t, t_{i-1})' + \boldsymbol{Q}(t, t_{i-1}) \tag{17}$$

Given that a transaction takes place at time t_i, the predicted value, $\hat{y}_(t_i)$ and the variance of the corresponding prediction error are given, respectively, by equations (18) and (19).

$$\hat{y}(t_i) = [1 \quad 0]\boldsymbol{\alpha}(t_i|t_{i-1}) \tag{18}$$

$$f(t_i) = [1 \quad 0]\boldsymbol{P}(t_i|t_{i-1})[1 \quad 0]' + \sigma_\varepsilon^2 \tag{19}$$

The first term in $f(t_i)$ is due to the uncertainty in the estimation of the state, and the second term is due to the existence of the noise term. (These terms are analogous to the terms in the variance of the prediction in regression analysis.) On the arrival of new information, i.e., a transaction, $y(t_i)$, at time point t_i, the estimate of the state vector and its corresponding covariance matrix are updated with the updating equations (20) and (21).

$$\boldsymbol{\alpha}(t_i|t_i) = \boldsymbol{\alpha}(t_i|t_{i-1}) + \boldsymbol{P}(t_i|t_{i-1})(y(t_i) - \hat{y}(t_i|t_{i-1}))/f(t_i) \tag{20}$$

$$\boldsymbol{P}(t_i|t_i) = \boldsymbol{P}(t_i|t_{i-1}) - \boldsymbol{P}(t_i|t_{i-1})[1 \quad 0]'[1 \quad 0]\boldsymbol{P}(t_i|t_{i-1})/f(t_i) \tag{21}$$

The parameters $(\sigma_\varepsilon, \sigma_1, \sigma_2, \gamma, r)$ may be obtained by maximizing the normal likelihood function.

$$\begin{aligned} & log(L(\sigma_\varepsilon, \sigma_1, \sigma_2, \gamma, r|y_{t_1}, \ldots, y_{t_n})) \\ = -\frac{n}{2} \quad & log(2\pi) - \frac{1}{2}\sum_{i=1}^{n} log(f(t_i)) - \frac{1}{2}\sum v(t_i)^2/f(t_i) \end{aligned} \tag{22}$$

The likelihood is calculated with the Kalman-filter, using the formulas above.

Some remarks on interpretation: At each point in time, t, one can calculate an estimate of the value of a stock conditional on the information gained by the last transaction at time t_l, i.e., $\alpha_1(t|t_l)$. If $\sigma_2 > 0$, then $\alpha_2(t|t_l)$ is an estimate of the drift. The values of $\boldsymbol{P}(t|t_l)$ can also be calculated on-line and give an estimate of the uncertainty of the value and drift estimates. On-line calculation of the prediction-variance $f(t|t_l)$ is also straightforward. The on-line calculation of $f(t|t_l)$ gives an objective measure of how far from the predicted value an eventual transaction is expected. It is conceivable that authorities at the stock exchange could use such an objective measure for monitoring and perhaps controlling eventual foul play in the market.

The approach given above results in the traditional way of reporting, i.e., reporting closing prices, as a special case. Applying this model with $\sigma_\varepsilon = \sigma_2 = 0$ gives the same result as reporting last transaction price as value. The assumption $\sigma_\varepsilon = 0$ is obviously unrealistic in markets where it is possible to trade simultaneously at two different prices. Using $\sigma_\varepsilon > 0$ may also be realistic when transactions are never simultaneous, e.g., to interpret bid-ask spread and other microstructure phenomena. The assumption of a fixed drift, $(\sigma_2 = 0, r > 0)$, is unrealistic for stocks but might be appropriate for bonds. Relaxing the assumption $\sigma_2 = 0$ will introduce a stochastic drift and allows for more persistent movements than those of the random walk. This would however mean that there is a local drift, which perhaps is more likely to fit a bond market than a stock market. When $\sigma_2 > 0$, putting $\gamma > 0$ sets a bound on the drift term so it cannot wander away. In practical situations, the fit might be similar to the case $\gamma = 0$. The Kalman-filter prediction equation gives an on-line estimate of the value and an uncertainty measure. The estimated value of σ_ε gives an idea of the size of market noise. The recursive nature of the Kalman-filter requires some initial values for the state vector $\boldsymbol{\alpha}(t_0|t_0)$ and the corresponding covariance matrix $\boldsymbol{P}(t_0|t_0)$. The Kalman-filter recursions are repeated application of the Bayes rule, and these initital values are needed to represent an inital guess of the state and the uncertainty of that guess. A natural way of quantifying the situation of knowing little about the state is to assign large values to the diagonal elements of $\boldsymbol{P}(t_0|t_0)$. Another possibility would be to estimate the initial values directly from data, i.e., by adding them as parameters to the likelihood function(equation (22)).

4 Some empirical results

These ideas were tested on data from the Iceland Stock Exchange (ISE). The ISE is an automated, computerized market, which opened in 1991. Active trading started in 1993 and has gradually increased, in both the number of companies registered and trading intensity. By the year 2000, over 70 companies were registered. The accessible data consist of every registered transaction since the opening of the ISE. The distribution of the transactions is tabulated in Table 2. For the years 1995-1999, the extreme quantiles seem pretty much constant, whereas there might be some upward drift in the center of the distribution. The 1 percent quantile in this period is very close to the tax-deduction limit. There seems to be an upward trend in the center of the distribution.

Data for all transactions on the market were available from the opening till February 24th 2000. The variables observed were, stock number, price per share, volume in number of shares and time of transaction. The dates of each companies' annual meeting of shareholders were also available. A total of 76 stocks were traded in a little over 76000 transactions with the least traded stock only traded three times. The parameters of the models in Section 2 were estimated with the methods for discretely sampled data described in Section 3. The number of transactions has increased on average by 5% each month from 1993 to 2000, see Figure 2. The outliers in Figure 2 are due to heavy trading in December, in particular on December 31st.

Year	$q_{0.01}$	$q_{0.1}$	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$q_{0.90}$	$q_{0.99}$	n
1993	20185	68629	100188	150415	252241	853120	6073847	1878
1994	24634	74605	117000	172000	335997	879500	5698800	2605
1995	130168	139100	200800	324000	757770	1815000	11400000	2701
1996	130020	150500	240000	400256	921000	1904000	10909000	5256
1997	130002	154724	244096	456000	971508	2190000	10500000	11821
1998	130074	176000	272337	555000	1010005	1944200	8080400	12350
1999	131078	197626	331500	773499	1320000	2860287	9900000	30153

Table 2: Quantiles of transactions in Icelandic kronas.

The activity in December is partly due to tax regulations, i.e., agents are buying tax deductions at the end of the year. The development of monthly turnover is described in Figure 3. Regression suggests that monthly turnover has increased by about 6% a month from 1993 to 2000.

There were several practical implications. The stochastic processes given by the definitions in Section 2 are of a continuous path type. The board of a company can make decisions that imply a jump in the value. Therefore, an extra parameter was added. This parameter is a constant that was used to multiply the variance matrix of the innovation if the transaction time was within 20 days from the annual meeting. Close to the annual meeting one might suspect that there are agents trading for a political position in the company. To allow for such a strange period of trading, the likelihood was then based on a modified variance matrix for the innovations. Generally, the innovation covariance matrix, $\boldsymbol{Q}(t, t_{i-1})$ from equation 15

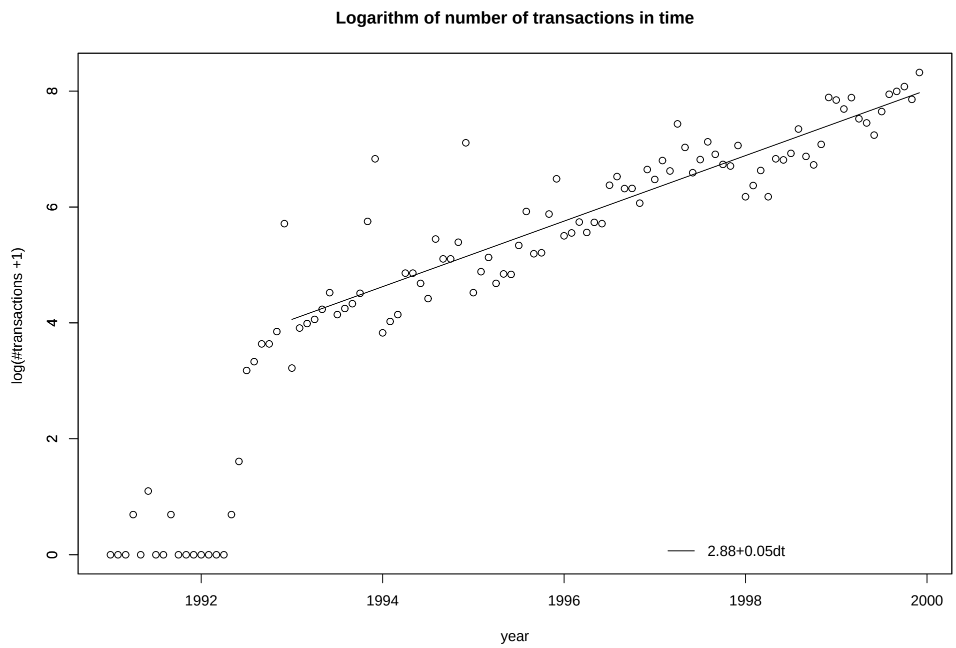

Figure 2: The logarithm of the number of monthly transactions.

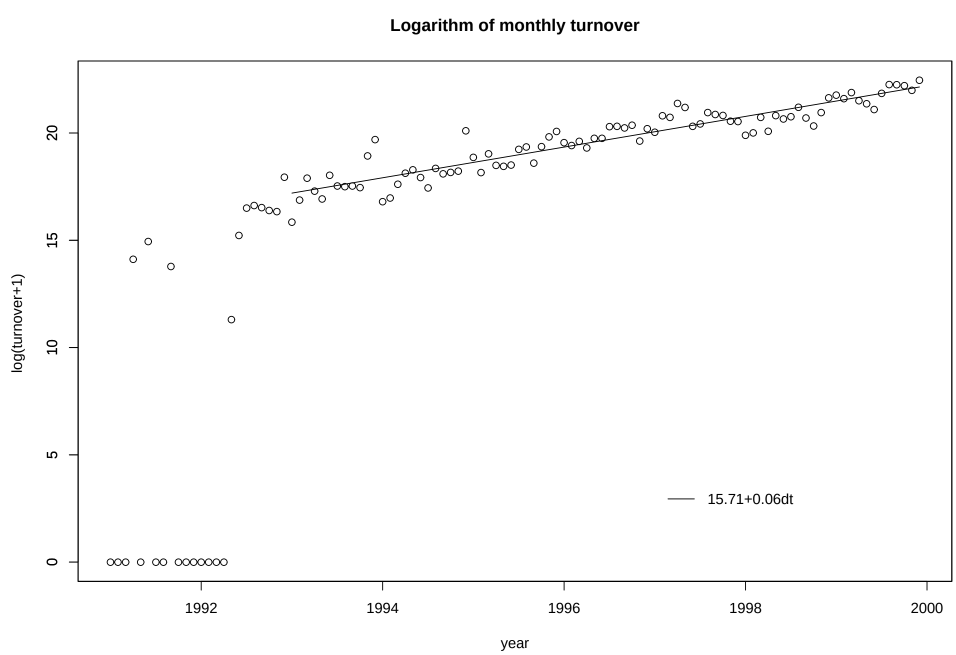

Figure 3: The logarithm of monthly turnover.

was used, but if t was closer to the annual meeting than 20 days, it was replaced by:

$$\phi \boldsymbol{Q}(t, t_{i-1}) \tag{23}$$

The idea with equation (23) is to allow for large innovations close to the annual meeting. The initial estimate of the state-vector $\boldsymbol{\alpha}(t_0)$ and the diagonal elements of its covariance matrix $\boldsymbol{P}(t|t_0)$ were also estimated. The estimate of the parameter σ_ε, the measurement noise, can be concentrated analytically simply by taking the derivative of the likelihood equation and solving the resulting equation. The remaining parameters have to be estimated numerically. The parameters estimated are shown in Table 3. The time variable, t was measured in days.

Model	Parameters estimated
M1	σ_ε, σ_1, $\alpha_1(t_0)$,$P_{11}(t_0)$,ϕ
M2	σ_ε, σ_1, $\alpha_1(t_0)$, $\alpha_2(t_0)$, $P_{11}(t_0)$,ϕ
M3	σ_ε, σ_1, σ_2, $\alpha_1(t_0)$, $\alpha_2(t_0)$, $P_{11}(t_0)$, $P_{22}(t_0)$, ϕ
M4	σ_ε, σ_1, σ_2, γ, r,$\alpha_1(t_0)$, $\alpha_2(t_0)$, $P_{11}(t_0)$, $P_{22}(t_0)$, ϕ

Table 3: Parameters estimated in the models analyzed

In the most complex case, M_4, there are 10 estimated parameters, of which five are of interest, and five are nuisance parameters. It is therefore clear that we cannot expect sensible results for small data series. The likelihood function was calculated with a Fortran subroutine, and the optimization was performed with the nlm routine of the R-statistical package(Ihaka and Gentleman, 1996). The numerical routines converged easily for over 70 stocks, and estimates of the parameters were obtained. Graphical inspection of most series suggests a long-term trend in most series, but the maximized log-likelihood values show that the trend is generally not statistically significant. The numerical estimates of the long-term drift suggest that the magnitude of the drift is mostly at a sensible level. A scatter plot of σ_ε versus σ_1 is shown in Figure 4. The figure suggests that the low noise stocks tend to have strong signals. The signal-to-noise ratio of standard deviations plotted against trading intensity is shown in Figure 5. There seems to be a strong relationship that more trading intensity leads to a higher signal-to-noise ratio. This result is by no means a natural one. It is is seen in Figures 6 and 7 that all of the rarely traded companies have high σ_ε, and the frequently traded ones have high σ_1. Figure 7 suggests that higher intensity leads to higher volatility. The explanation is that the companies with the highest intensity are traded in a short period of high volatility, see e.g., stock number 76 in Figure 80. From Figure 6 it is seen that the noise is smaller for stocks traded more often. This is a quite natural result because rarely traded stocks are prone to more bargaining due to higher uncertainty, and agents put a price on liquidity.

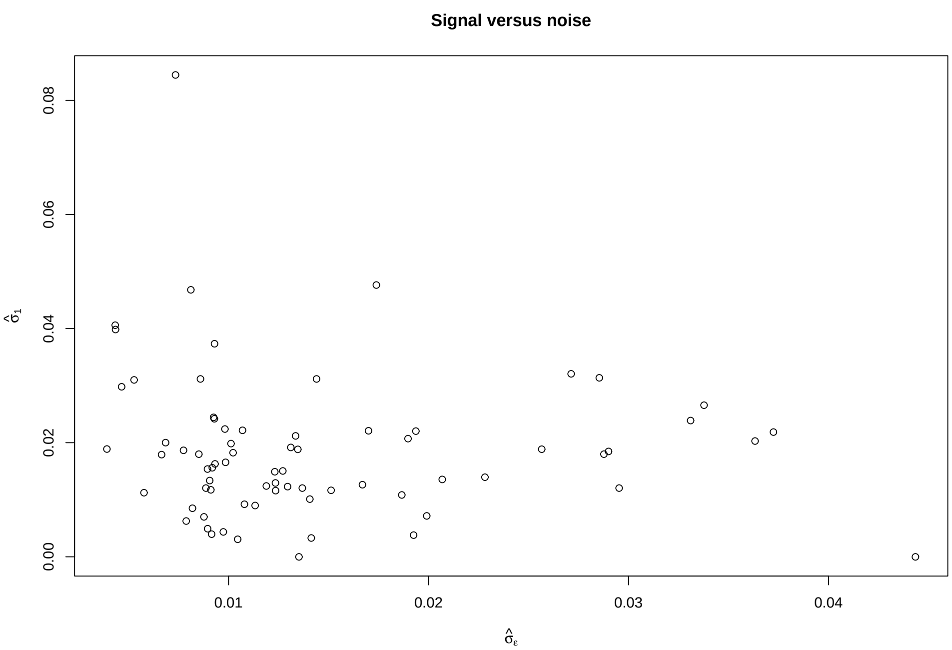

Figure 4: Signal σ_1 versus noise σ_ε

The numerical estimates of σ_ε vary from about half a percent up to a few percent. This is natural range keeping in mind what could be expected from guessing a reasonable bid-ask spread. The interpretation of the graphs is straightforward: lower liquidity means higher noise. This is analogous to theories on the relevance of the bid-ask spread (Campbell et al., 1997). The results suggest that the estimate of σ_ε is not much affected by the other parameters.

To get a grip on the reality behind the parameter estimates, it is useful to look at some specific stocks. Stock number 76 enters the market in late 1999. During four months, there are 1305 transactions, and the market price increases over 50%. The period is 136 days, of which the market was open for trading 97 days. Of these 1305 transaction there are 434, cases of zero time interval. For example, at a particular time point, all in the same second four transactions took place at the prices of 32.20, 32.40, 32.60 and 32.80. The standard deviation of the logarithm of these four numbers is 0.0079, which is close to the maximum-likelihood estimate of $\hat{\sigma}_{\varepsilon} = 0.0073$, so it could be argued that this is a representative sample! Each of the four transactions included 20,000 shares. The estimate of signal-noise ratio $\hat{\sigma}_1/\sigma_{\varepsilon} = 11.5$ is of course extreme. A few seconds before, there was a transaction of 20,000 shares at a price of 32.00; half a minute later there were a couple of transactions of 20,000 and 40,425 shares, respectively at a price of 33.00 and two minutes later there was a transaction of 100,000 shares at a price of 33.30. Simultaneous (at the same second) transactions at different prices can occur as the order book may contain many asking bids, say one for 1000 shares at 1 another for 20,000 shares at 1.2 and the third, for 100,000 shares at 1.5. Then a buyer comes and clears the entire order book and buys everything that is for sale. Similar circumstances may hold for the buying bids. There might be many offers on both sides, and there may be none. On the Iceland Stock Exchange some companies have made agreements with institutions, such as

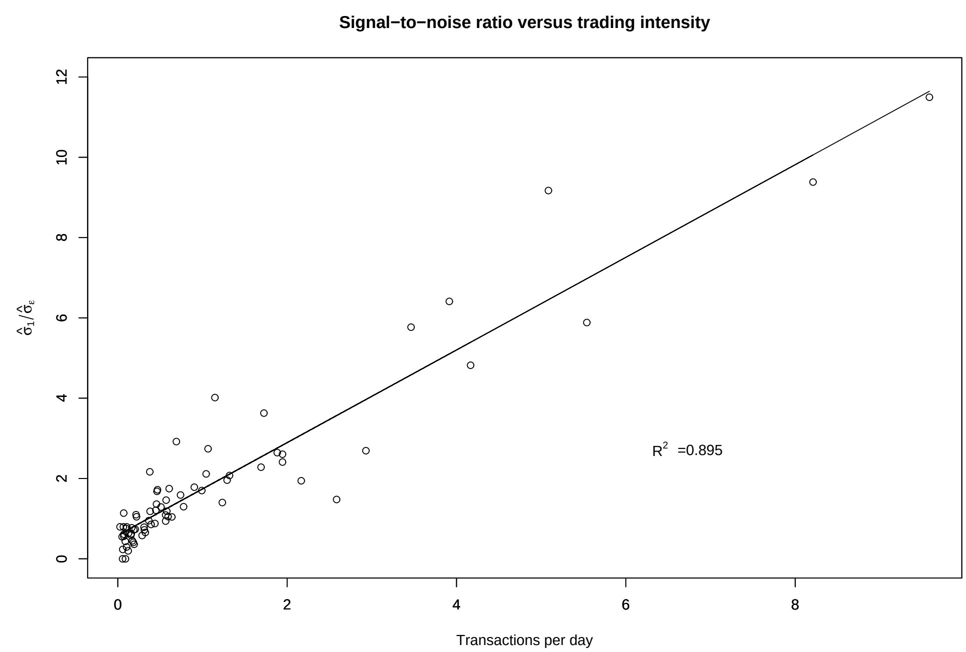

Figure 5: Signal-to-noise, $\sigma_1/\sigma_{\varepsilon}$, versus trading intensity.

banks, to act as market makers to ensure liquidity with their stock. These institutions commit themselves to maintaining permanent bids and asks. Other companies might not have any such agreement. In such an environment, it is hard to say what the bid-ask spread really is. The interpretation in this paper is that the parameter σ_ε picks up latent bid-ask spread and perhaps other market microstructure phenomena.

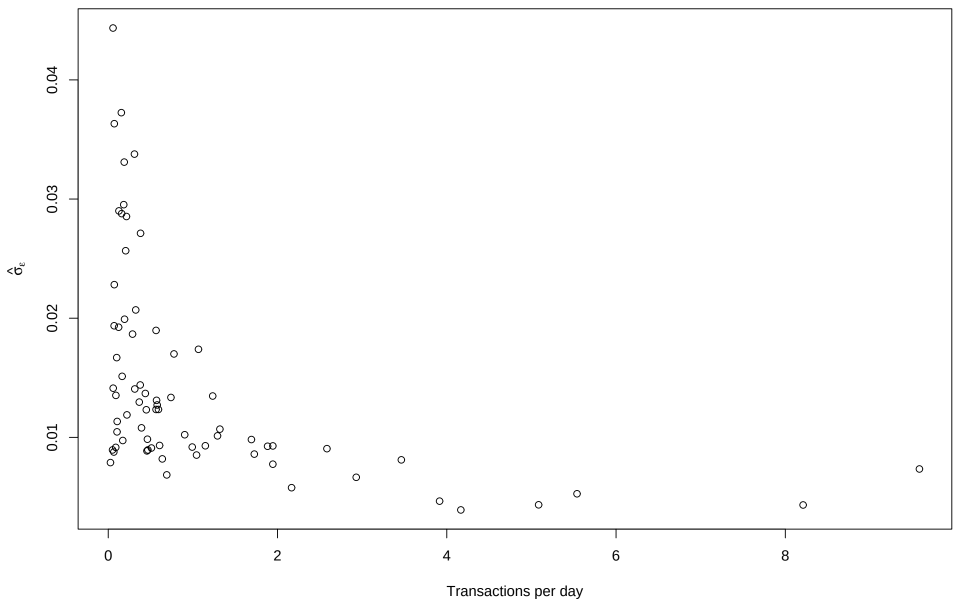

Figure 6: Noise versus trading intensity.

The data for each stock is shown in Figures 8 to 80. The scales in the figures are different in order to illustrate the behaviour of each individual stock. The estimation results of models $M_1, \ldots, M_4$ are given in Tables 4 to 76. The dates of the annual meeting are marked by M. The standard deviation of the noise ranges from half a percent up a few percent. The maximized log-likelihood does no in most cases suggest significant improvements in fit from model M_1 to M_4. The log-likelihood values are generally very similar for models M_2 and M_3. The eventual improvement in fit is usually due to the inclusion of drift, whereas the step from M_2 or M_3 to M_4 is marginal. The drift parameter, r, is drift per day. The estimated yearly rate is $exp(365 * \hat{r})$, so a 10% yearly rate is reflected by $r = log(1.1)/365 = 0.00026$. The natural magintude of r is deemed to be roughly 0.0001, and it is natural to assume that the standard deviation of changing drift is of similar magnitude. Skewness and kurtosis of the scaled prediction errors, $(y(t_i) - \hat{y}(t_i))/f(t_i)$, are shown. The distribution is generally not severely skewed, but kurtosis is generally high.

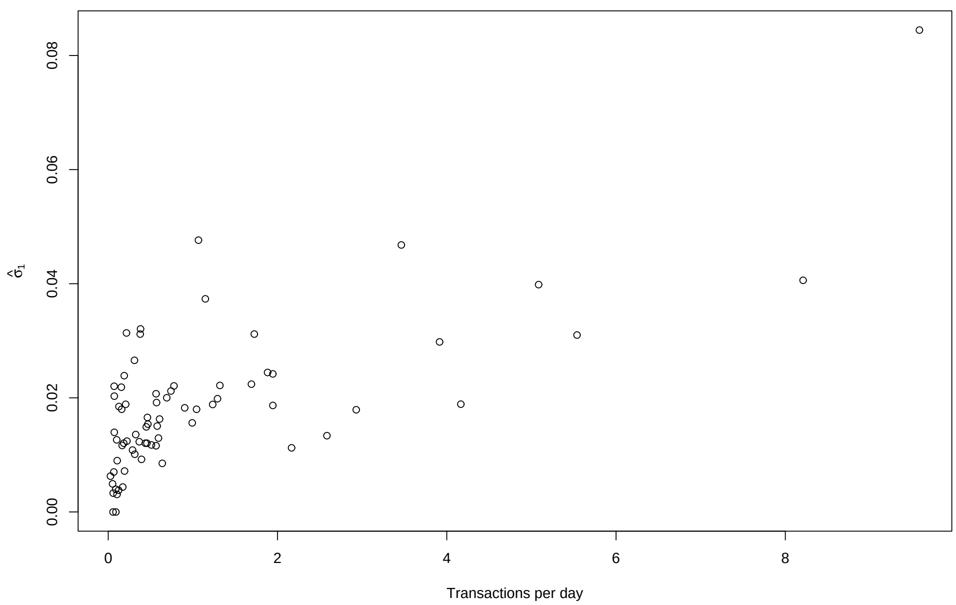

Figure 7: Signal versus trading intensity.

Stock number 2 is one of the most regularly traded stock, on the market with 6086 transactions in eight years. In Figure 9 it can be seen that there is a steady upward drift except for periods around the annual meeting. The inclusion of trend for this stock is statistically the most significant of all stocks, $log - likelihood(M_2/M_1) = 5$. The standard deviation of the noise, $\sigma_\varepsilon = 0.6\%$, is among the lowest in the study. The standard deviation of the daily innovations(σ_1) is around 1%, which is also among the lowest in the study, indicating that investing in stock is quite risky. The drift $r = 0.0007$, suggests a yearly increase in price of around 30%. Models M_3 and M_4 have $\hat{\sigma}_2 = 0$, which gives the unrealistic interpretation of a constant drift.

Data for stock number 18 are shown in Figure 25, and estimation results are given in Table 21. There are 929 transactions in six years. There is a period of rapid increase, another period of persistent decrease and then rapid increase again. Although not statistically significant, the case of stock number 18 illustrates what is meant to be accomplished with the stochastic trend component of model M_3. The low value of γ could be interpreted as there is a slow adaptation of the drift term to a permanent equilibrium drift.

5 Discussion

The standard deviation of the noise term, σ_ε, can be interpreted as microstructure deviance from the continuous-time textbook model. The standard deviation of the innovation σ_1 is a measure of how fast stock is moving from day to day. The signal-to-noise ratio relates the market innovation to the microstructure effects. Both σ_1 and the signal-to-noise ratio are useful parameters for risk management because an ideal investor would like something that will not move fast, and something that is easily converted into other assets.

In the analysis of the Icelandic stock market data, no significant sign of a stochastic trend was found. In view of the efficient market hypothesis, detecting a trend, deterministic or stochastic, should perhaps not be trivial. However, a kind of power game in the market is conceivable such that a strong investor consistently buys all offered stock in a particualar company over an period of time in order to build up a block that could be decisive in controlling the company. This is conceivable when trading is infrequent because, in such markets, there are by definition few agents; little is offered for sale, and buyers are rare. Then at a certain time point, this investor might threaten some parties wantint to control the company to sell a decisive block of shares to a competing group and then quit trading that stock. This kind of behavior would suggest that there would be an upward trend while this investor is buying and a gradual decline after the strong investor has left the scene and prices are converging to their "natural level". This behavior would suggest the existence of a stochastic trend, i.e., $\sigma_2 > 0$. The data do not suggest the existence of this phenomena although a nonsignficant illustration of it is hinted at for stock number 18.

In the application of this study, the variance of the state-innovation $\boldsymbol{Q}(t, t_i)$ was designed to increase linearly in $\delta_t = t - t_i$, regardless of what time of day or night it is, whether or not the stock is open for trade. One could easily add parameters to make this innovation variance more complex. That might be of interest in some situations. In the case of the Icelandic stock market, data are so sparse that this hardly leaves much scope for estimating a more complex structure of the innovation variances.

The same argument applies for the problem of updating the parameters σ_ε and σ_1. It is unrealistic to believe that they are constant in time. The limited data will not allow for sophisticated modeling of the updating scheme.

Agents using models of this kind might also want to use a parameter like ϕ in equation (23). There might be dates other than the annual meeting when the board is taking decisions on critical matters, mergers, closing plants, layoffs, etc. The annual meeting is typically a date when decisions on dividends and stock splits are taken. No effort is made here to explain the returns. It is expected that agents are risk averse and prefer a low value on σ_ε and σ_1, high dividends and big stock splits.

Acknowledgements

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Appendix

A Derivation of discretely observed dynamics

The key of deriving equation (14) and (15) lies in calculation of the matrix exponent, e^A. The continuous time state dynamics are given by:

$$d\boldsymbol{\alpha}(t) = A\boldsymbol{\alpha}(t)dt + \boldsymbol{b}dt + Cd\boldsymbol{W}(t)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\gamma \end{bmatrix} \quad C = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ \gamma r \end{bmatrix}$$

Write the matrix A as:

$$A = G\Lambda G^{-1}$$
$$G = \begin{bmatrix} 1 & -\frac{1}{\gamma} \\ 0 & 1 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 0 & 0 \\ 0 & -\gamma \end{bmatrix}$$

and define

$$\boldsymbol{\alpha}^*(t) = G^{-1}\boldsymbol{\alpha}(t)$$

then

$$\begin{aligned} d\boldsymbol{\alpha}^*(t) = G^{-1}d\boldsymbol{\alpha}(t) = \\ G^{-1}(A\boldsymbol{\alpha}(t)dt + \boldsymbol{b}dt + Cd\boldsymbol{W}(t) = \\ G^{-1}G\Lambda G^{-1}\boldsymbol{\alpha}(t)dt + G^{-1}\boldsymbol{b}dt + G^{-1}Cd\boldsymbol{W}(t) = \\ \Lambda\boldsymbol{\alpha}^*(t) + G^{-1}\boldsymbol{b}dt + G^{-1}Cd\boldsymbol{W}(t) \end{aligned}$$

Solving for initional condition at time t_{i-1} gives

$$\boldsymbol{\alpha}^*(t_i) = e^{\Lambda\delta_i}\boldsymbol{\alpha}^*(t_{i-1}) + \int_{t_{i-1}}^{t_i} e^{\Lambda(s-t_{i-1})}G^{-1}\boldsymbol{b}ds + \boldsymbol{\xi}^*(t_i, t_{i-1})$$

where

$$\boldsymbol{\xi}^*(t_i, t_{i-1}) = \int_{t_{i-1}}^{t_i} e^{\Lambda(s-t_{i-1})}G^{-1}Cd\boldsymbol{W}(s)$$

The variance of $\boldsymbol{\xi}^*(t_i, t_{i-1})$, is given by:

$$V(\boldsymbol{\xi}^*(t_i, t_{i-1})) = \int_{t_{i-1}}^{t_i} e^{\Lambda(s-t_{i-1})} G^{-1} CC'(G^{-1}) e^{\Lambda(s-t_{i-1})'} ds$$

Using the formula for matrix exponent:

$$e^A = I + A + A^2/2 + \dots$$

the special structure of the matrix Λ gives

$$e^{\Lambda(s-t_{i-1})} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\gamma(s-t_{i-1})} \end{bmatrix}$$

and transforming back gives the discretely observed state dynamics of $\boldsymbol{\alpha}(t)$,

$$\begin{aligned} \boldsymbol{\alpha}(t_i) &= G e^{\Lambda \delta_i} G^{-1} \boldsymbol{\alpha}(t_{i-1}) + G \int_{t_{i-1}}^{t_i} e^{\Lambda(s-t_{i-1})} G^{-1} \boldsymbol{b} ds + G \boldsymbol{\xi}^*(t_i) \\ &= T(t_i, t_{i-1}) \boldsymbol{\alpha}(t_{i-1}) + r \begin{bmatrix} \delta_i - \frac{1}{\gamma}(1 - e^{-\gamma \delta_i}) \\ 1 - e^{-\gamma \delta_i} \end{bmatrix} + \boldsymbol{\xi}(t_i, t_{i-1}) \end{aligned}$$

with

$$\begin{aligned} T(t_i, t_{i-1}) &= \begin{bmatrix} 1 & \frac{1}{\gamma}(1 - e^{-\gamma \delta_i}) \\ 0 & e^{-\gamma \delta_i} \end{bmatrix} \\ &= \begin{bmatrix} 1 & g_1(\delta_i, \gamma) \\ 0 & e^{-\gamma \delta_i} \end{bmatrix} \end{aligned}$$

and the variance, $\boldsymbol{Q}(t_i, t_{i-1})$ of $\boldsymbol{\xi}(t_i, t_{i-1}) = G\boldsymbol{\xi}^*(t_i, t_{i-1})$ is:

$$\boldsymbol{Q}(t_i, t_{i-1}) = V(\boldsymbol{\xi}(t_i, t_{i-1})) = \int_{t_{i-1}}^{t_i} G e^{\Lambda(s-t_{i-1})} G^{-1} CC'(G^{-1})' e^{\Lambda(s-t_{i-1})'} G' ds =$$

$$\int_{t_{i-1}}^{t_i} \begin{bmatrix} \sigma_1^2 + \frac{\sigma_2^2}{\gamma^2}(1 - e^{-\gamma(s-t_{i-1})})^2 & \frac{\sigma_2^2}{\gamma}(1 - e^{-\gamma(s-t_{i-1})}) \\ \frac{\sigma_2^2}{\gamma}(1 - e^{-\gamma(s-t_{i-1})}) & \sigma_2^2 \end{bmatrix} ds =$$

$$\begin{bmatrix} \sigma_1^2 \delta_i + \frac{\sigma_2^2}{\gamma^2}(\delta_i - 2\frac{1}{\gamma}(1 - e^{-\gamma \delta_i}) + \frac{1}{2\gamma}(1 - e^{-2\gamma \delta_i})) & \frac{\sigma_2^2}{\gamma}((1 - e^{-\gamma \delta_i}) - \frac{1}{2\gamma}(1 - e^{-2\gamma \delta_i})) \\ \frac{\sigma_2^2}{\gamma}(\frac{1}{\gamma}(1 - e^{-\gamma \delta_i}) - \frac{1}{2\gamma}(1 - e^{-2\gamma \delta_i})) & \frac{\sigma_2^2}{2\gamma}(1 - e^{-2\gamma \delta_i}) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 \delta_i + \sigma_2^2 g_3(\delta_i, \gamma) & \sigma_2^2 g_2(\delta_i, \gamma) \\ \sigma_2^2 g_2(\delta_i, \gamma) & \sigma_2^2 g_1(\delta_i, 2\gamma) \end{bmatrix}$$

The functions g_i's are defined by:

$$
\begin{aligned}
g_1(\delta,\gamma) &= \frac{1}{\gamma}(1-e^{-\gamma\delta}) \\
g_2(\delta,\gamma) &= \frac{1}{\gamma}(g_1(\delta,\gamma) - g(\delta,2\gamma)) \\
g_3(\delta,\gamma) &= \frac{1}{\gamma^2}(\delta - 2g_1(\delta,\gamma) + g_1(\delta,2\gamma))
\end{aligned}
$$

Standard calculus gives:

$$
\begin{aligned}
\lim_{\gamma\to 0} g_1(\delta,\gamma) &= \delta \\
\lim_{\gamma\to 0} g_2(\delta,\gamma) &= \delta^2/2 \\
\lim_{\gamma\to 0} g_3(\delta,\gamma) &= \delta^3/3
\end{aligned}
$$

B Data and estimation results for individual stocks

The figures plots of logged transaction prices for each stock during the period of trade on the Icelandic exchange. The dates of the annual meeting are marked with, M. The tables show estimation result for the models $M_1, \ldots, M_4$. The stocks are numerated in the order of registration on the Icelandic stock exchange. Stocks numer 54, 57 and 74 had 35, 3 and 7 transaction respectively, and was not enough data to estimate models and are therefore excluded from the analysis. The result for stock number 37 are based on 12 transactions do not make much sense either. Note that the parameter r is not identifiable in model M_4 when $\gamma = 0$. Therefore some strange estimates on r are to be expected when γ is close to zero in model M_4.

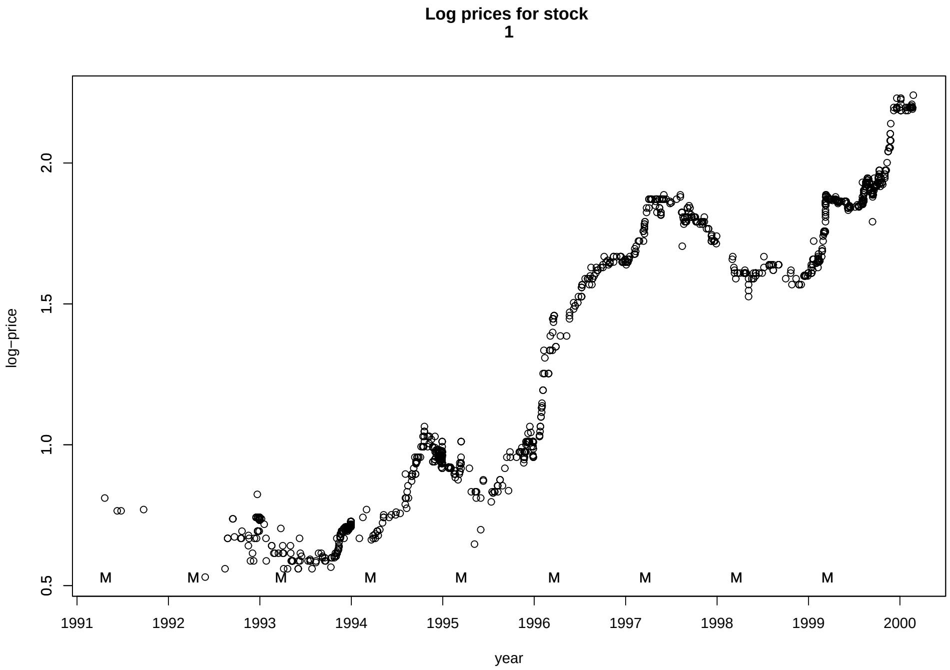

Figure 8: Log-prices for stock number 1

Stock number 1: A total of 1181 transactions from 04/22/91 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0129	0.0129	0.0129	0.0130
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.9683	0.9582	0.9582	0.9493
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0020
$\hat{r}$	0	0.0004	0	0.0007
log-likelihood	2896.2620	2897.9210	2897.9210	2899.3860
skewness	-0.1355	-0.0503	-0.0503	-0.0274
kurtosis	11.3232	11.1006	11.1004	11.0754

Table 4: Comparison of models for stock number 1

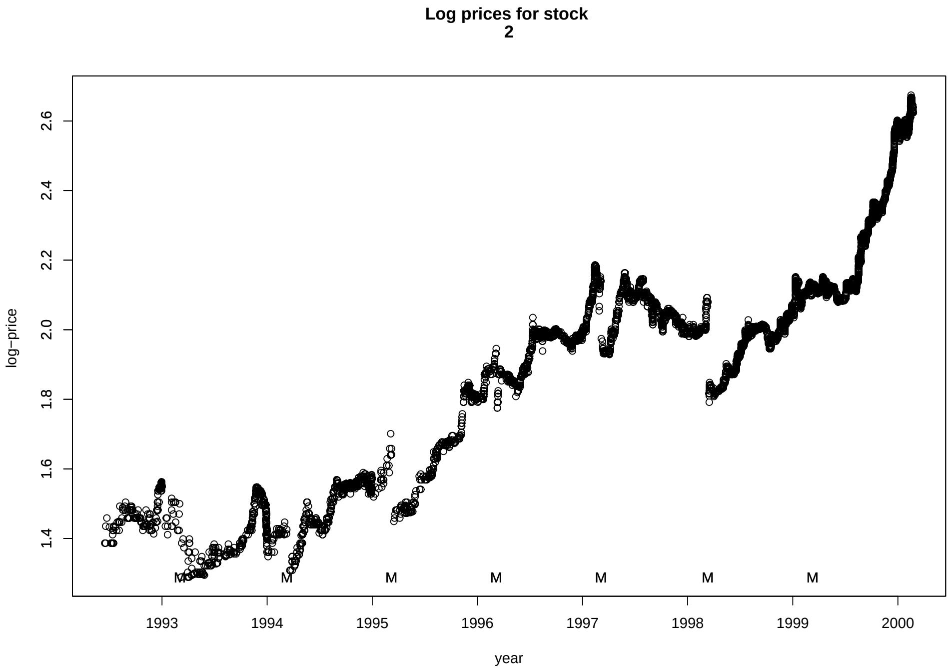

Figure 9: Log-prices for stock number 2

Stock number 2: A total of 6086 transactions from 06/16/92 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0058	0.0058	0.0058	0.0058
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.9659	1.9474	1.9475	1.9432
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.2657
$\hat{r}$	0	0.0007	0	0.0007
log-likelihood	20291.1500	20296.1500	20296.1500	20297.4700
skewness	-0.5375	-0.4711	-0.4711	-0.4735
kurtosis	13.9771	13.7542	13.7543	13.7212

Table 5: Comparison of models for stock number 2

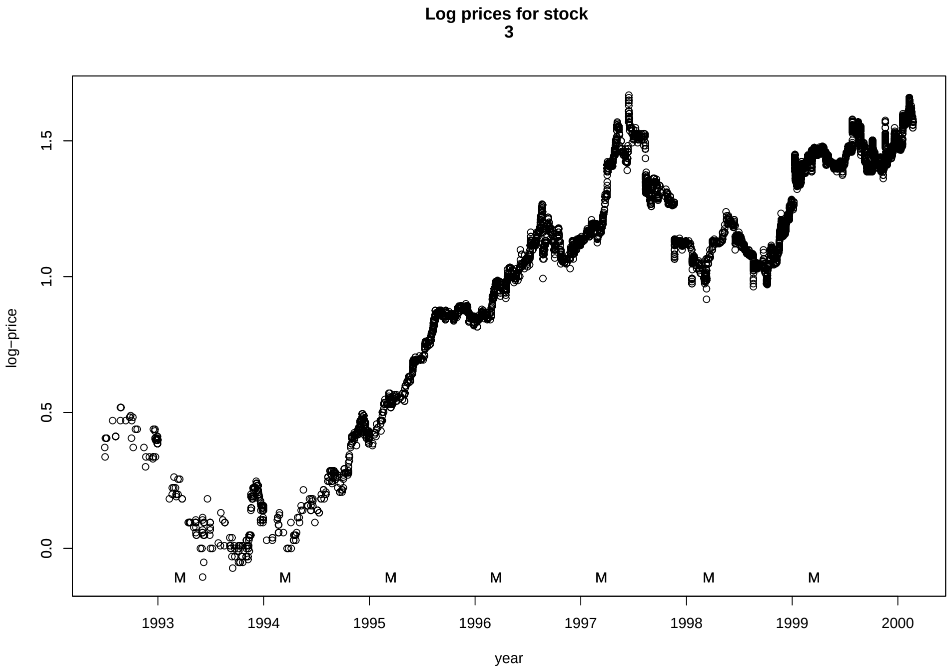

Figure 10: Log-prices for stock number 3

Stock number 3: A total of 5435 transactions from 07/01/92 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0093	0.0093	0.0093	0.0093
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.6070	2.6050	2.6050	2.6039
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0025
$\hat{r}$	0	0.0004	0	0.0007
log-likelihood	15269.6300	15270.0700	15270.0700	15270.3800
skewness	-0.5304	-0.5103	-0.5103	-0.5053
kurtosis	30.8869	30.7976	30.7976	30.7851

Table 6: Comparison of models for stock number 3

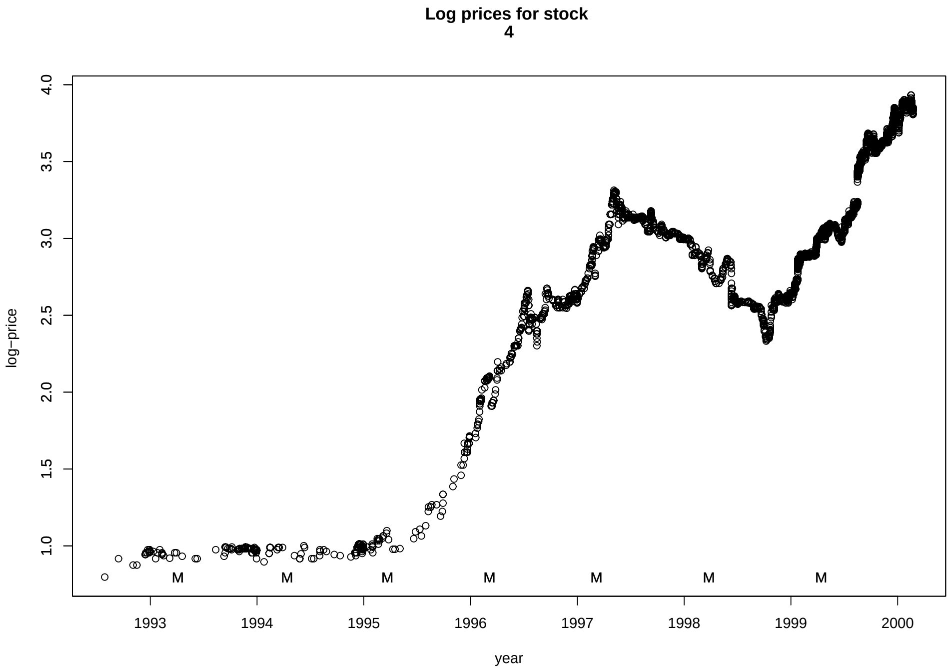

Figure 11: Log-prices for stock number 4

Stock number 4: A total of 3171 transactions from 07/29/92 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0093	0.0093	0.0093	0.0093
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	4.0351	4.0175	4.0176	4.0170
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.2113
$\hat{r}$	0	0.0010	0	0.0010
log-likelihood	8245.8580	8246.9230	8246.9230	8246.9620
$skewness$	-0.3579	-0.3192	-0.3191	-0.3199
$kurtosis$	18.9759	18.8752	18.8749	18.8787

Table 7: Comparison of models for stock number 4

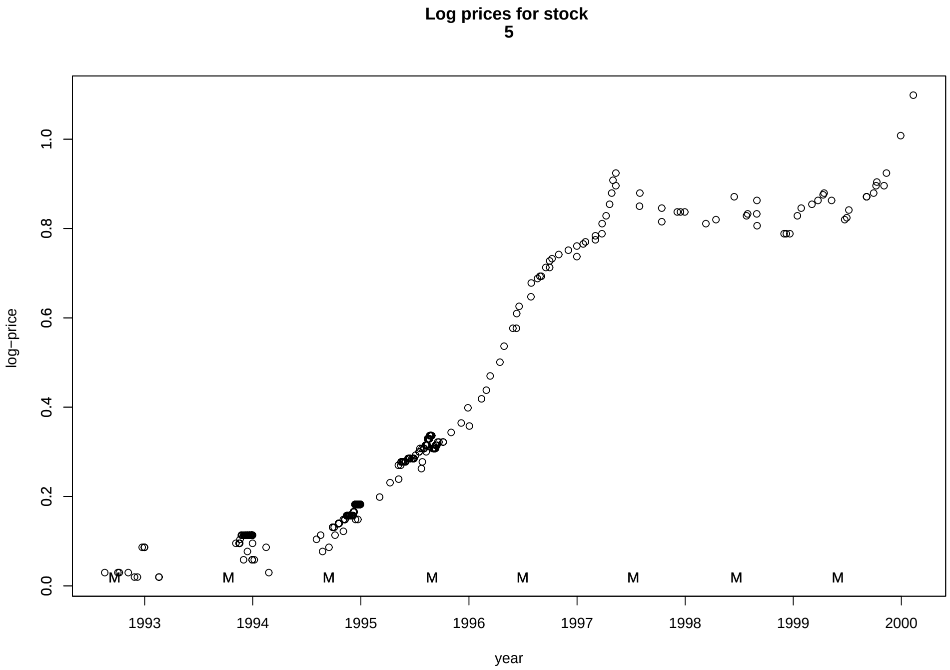

Figure 12: Log-prices for stock number 5

Stock number 5: A total of 284 transactions from 08/19/92 to 02/11/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0099	0.0100	0.0102	0.0105
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.5368	0.4827	0.3941	0.2947
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.3171	1.4213
$\hat{\gamma}$	0	∞	0	0.0216
$\hat{r}$	0	0.0004	0	0.0004
log-likelihood	764.2870	772.3572	774.7022	777.2343
$skewness$	-1.2349	-0.8207	-0.8861	-0.9401
$kurtosis$	8.4445	7.6026	7.8330	8.0391

Table 8: Comparison of models for stock number 5

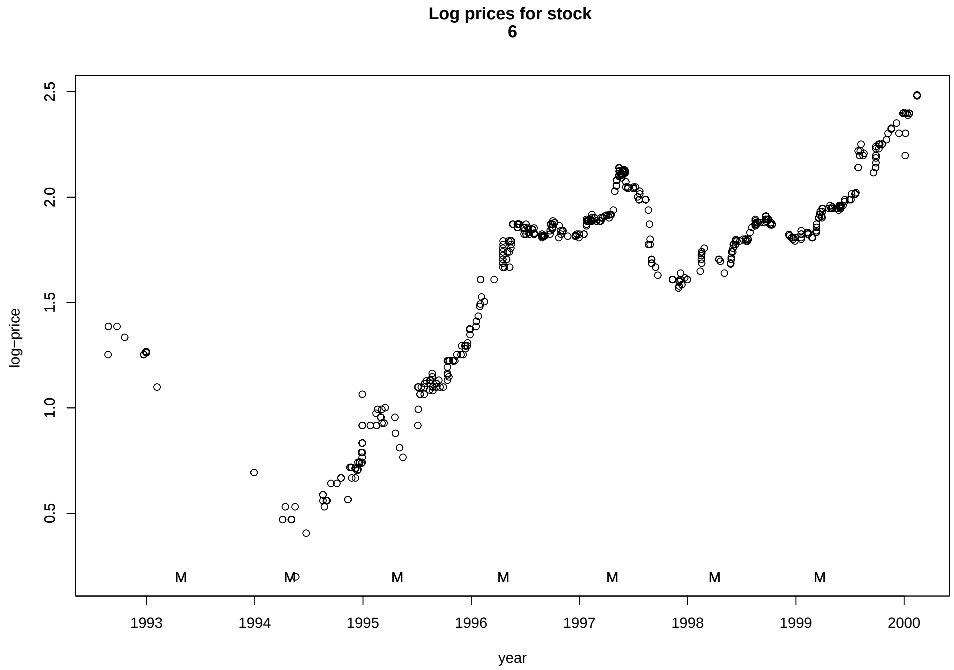

Figure 13: Log-prices for stock number 6

Stock number 6: A total of 498 transactions from 08/24/92 to 02/14/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0291	0.0291	0.0295	0.0295
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.4793	0.4740	0.4067	0.4078
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.2632	0.3030
$\hat{\gamma}$	0	∞	0	0.0026
$\hat{r}$	0	0.0004	0	0.0008
log-likelihood	854.5007	855.7273	860.3294	861.9577
$skewness$	0.1934	0.3028	0.3385	0.3456
$kurtosis$	22.2319	21.8463	20.9055	21.2240

Table 9: Comparison of models for stock number 6

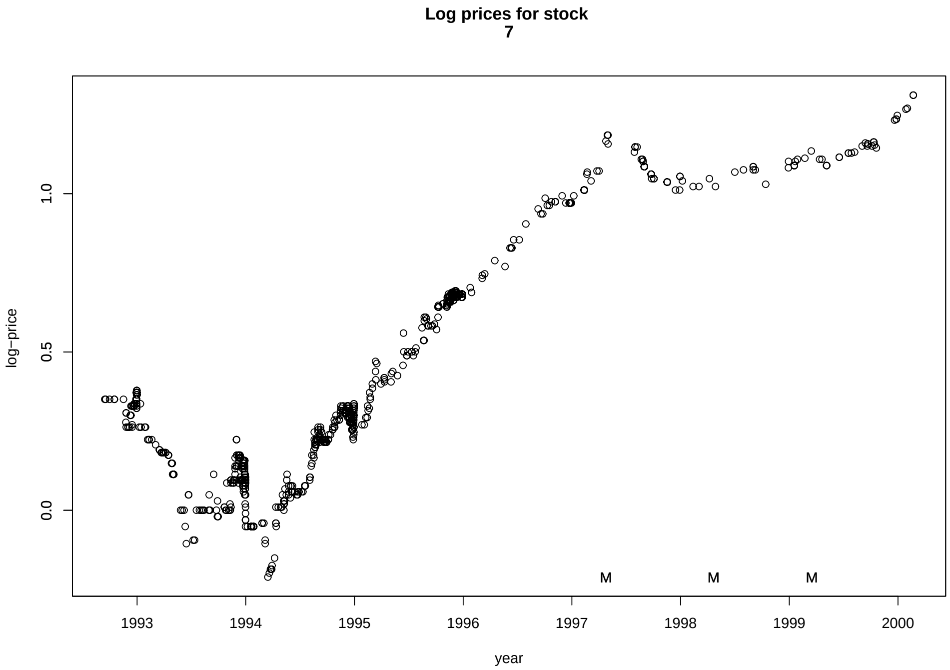

Figure 14: Log-prices for stock number 7

Stock number 7: A total of 777 transactions from 09/14/92 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0186	0.0186	0.0186	0.0187
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.5977	0.5919	0.5919	0.5804
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0035
$\hat{r}$	0	0.0004	0	0.0006
log-likelihood	1718.4300	1719.8660	1719.8660	1722.2480
$skewness$	-0.9521	-0.8486	-0.8486	-0.8549
$kurtosis$	9.5710	9.3295	9.3294	9.3601

Table 10: Comparison of models for stock number 7

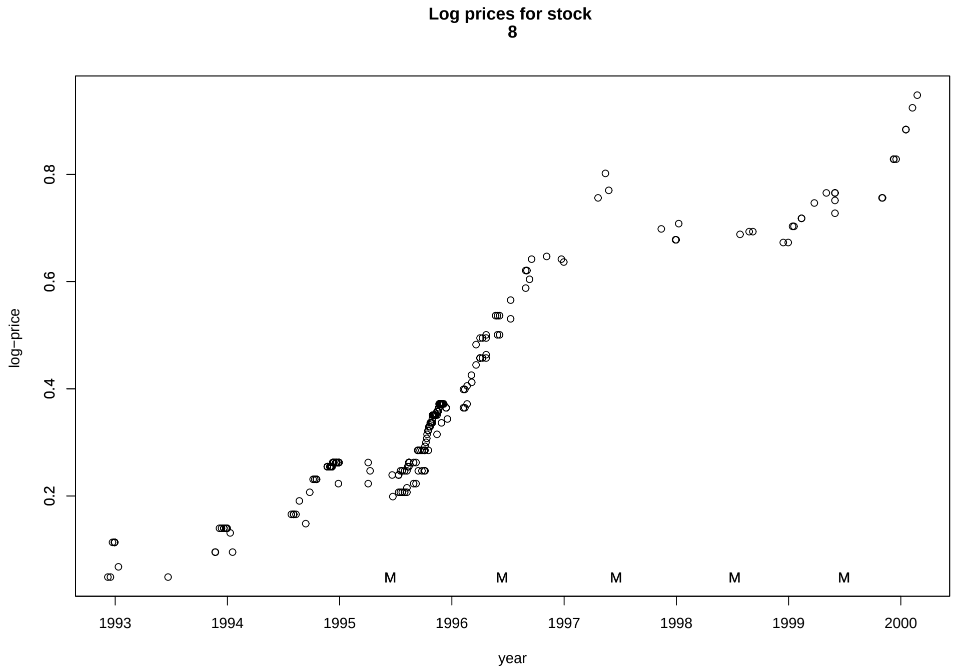

Figure 15: Log-prices for stock number 8

Stock number 8: A total of 240 transactions from 12/08/92 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0131	0.0131	0.0132	0.0135
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.4084	0.3688	0.3340	0.0000
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.1542	5.0799
$\hat{\gamma}$	0	∞	0	0.1178
$\hat{r}$	0	0.0003	0	0.0003
log-likelihood	604.9081	610.2188	609.6230	615.3386
$skewness$	-0.7556	-0.4001	-0.3912	-0.4034
$kurtosis$	3.7812	3.1990	3.2969	3.3575

Table 11: Comparison of models for stock number 8

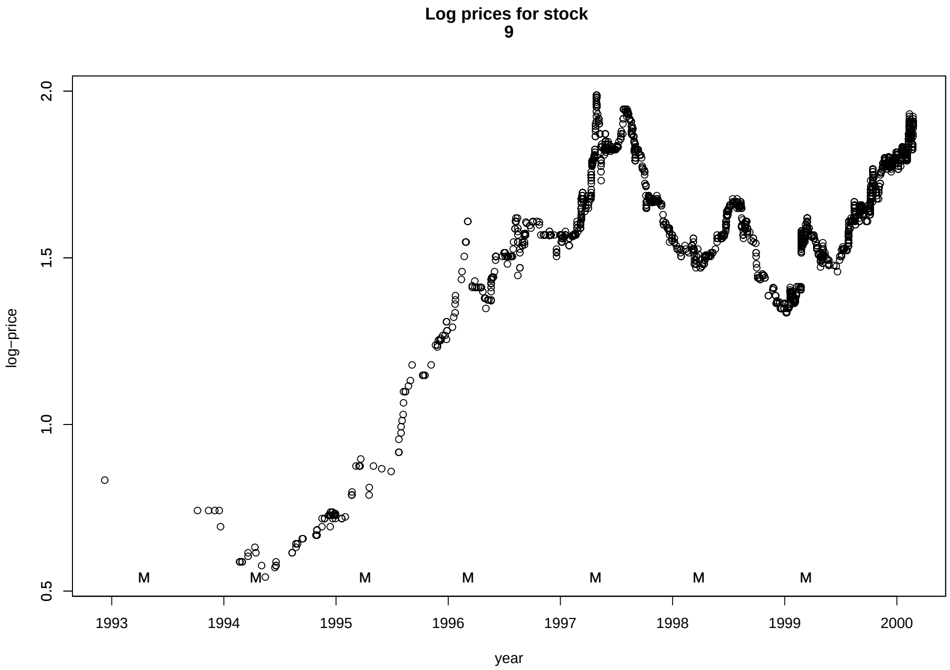

Figure 16: Log-prices for stock number 9

Stock number 9: A total of 1599 transactions from 12/09/92 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0093	0.0093	0.0093	0.0093
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.7550	1.7500	1.7500	1.7460
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0037
$\hat{r}$	0	0.0004	0	0.0005
log-likelihood	4325.0550	4325.6890	4325.6890	4326.1600
$skewness$	-0.4595	-0.4116	-0.4117	-0.3961
$kurtosis$	13.2182	13.0589	13.0589	13.0157

Table 12: Comparison of models for stock number 9

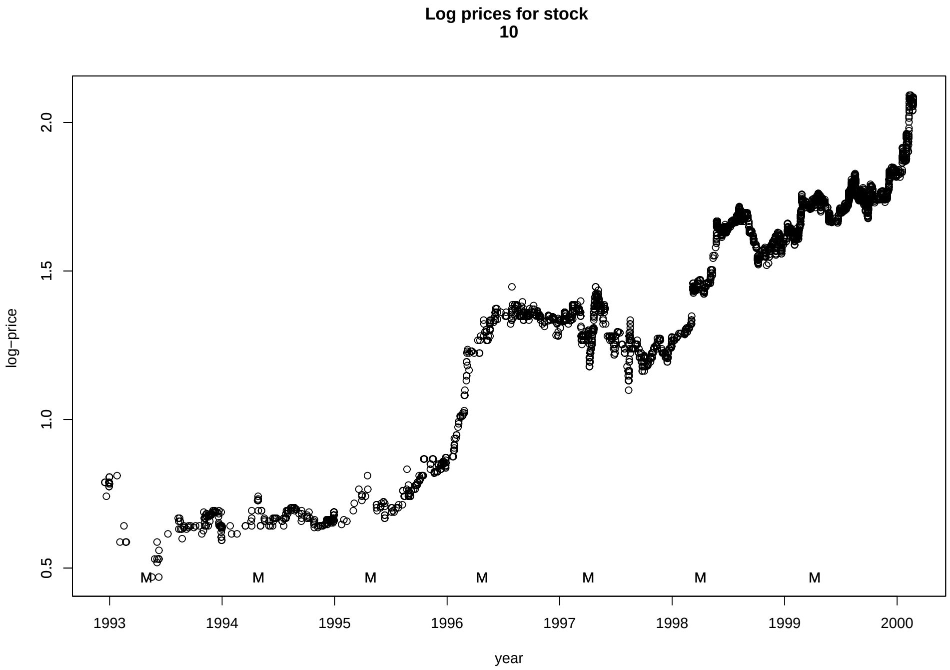

Figure 17: Log-prices for stock number 10

Stock number 10: A total of 2606 transactions from 12/16/92 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0092	0.0092	0.0092	0.0092
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.7153	1.7080	1.7080	1.6992
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0081
$\hat{r}$	0	0.0005	0	0.0007
log-likelihood	7286.8920	7288.1740	7288.1750	7289.7270
$skewness$	0.6546	0.7003	0.7003	0.7116
$kurtosis$	11.5022	11.4756	11.4756	11.4183

Table 13: Comparison of models for stock number 10

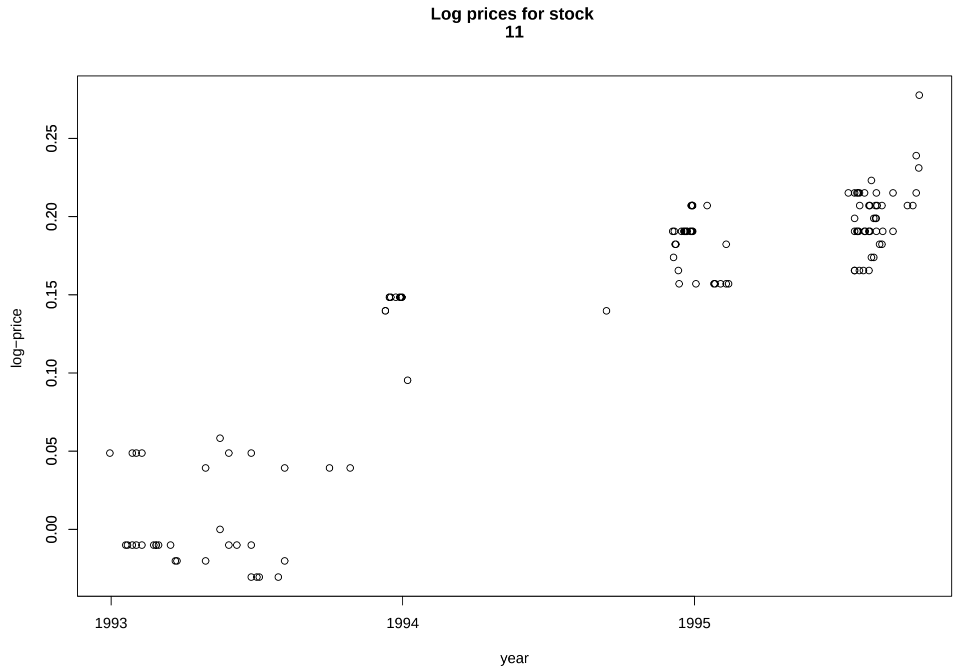

Figure 18: Log-prices for stock number 11

Stock number 11: A total of 124 transactions from 12/30/92 to 10/10/95

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0192	0.0193	0.0193	0.0193
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.2286	0.2099	0.2099	0.1975
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0300
$\hat{r}$	0	0.0002	0	0.0003
log-likelihood	286.5907	287.8055	287.8055	289.3843
skewness	-0.0125	0.1465	0.1464	0.1755
kurtosis	3.2249	3.0823	3.0823	3.3655

Table 14: Comparison of models for stock number 11

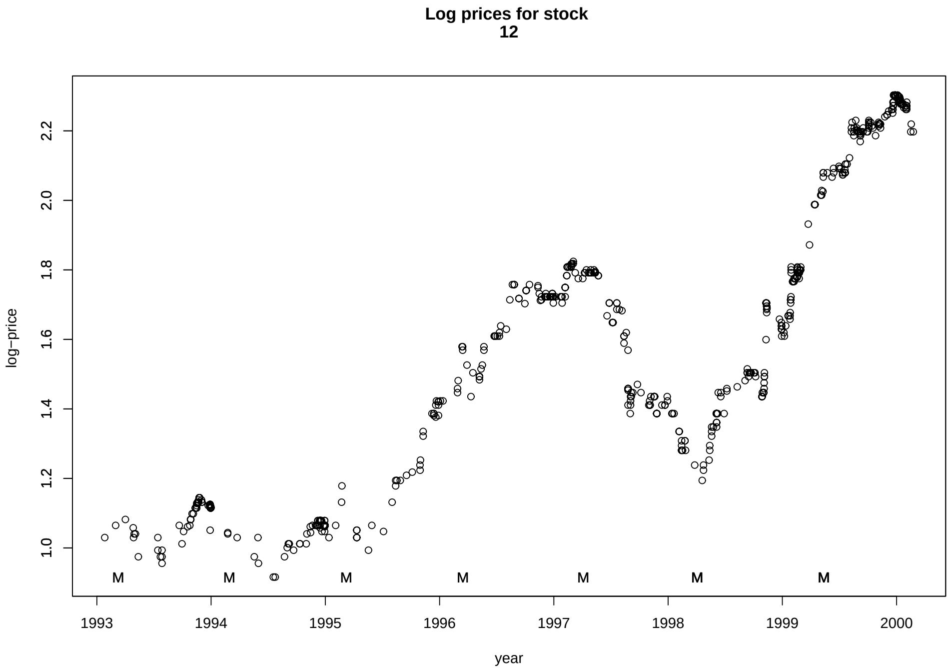

Figure 19: Log-prices for stock number 12

Stock number 12: A total of 571 transactions from 01/26/93 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0119	0.0119	0.0119	0.0119
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.0580	1.0473	1.0473	1.0447
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0001	0.0001
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0005	0	0.4568
log-likelihood	1342.4560	1344.1290	1344.1290	1344.5790
skewness	-0.5775	-0.4178	-0.4180	-0.4025
kurtosis	20.0526	19.4079	19.4086	19.3018

Table 15: Comparison of models for stock number 12

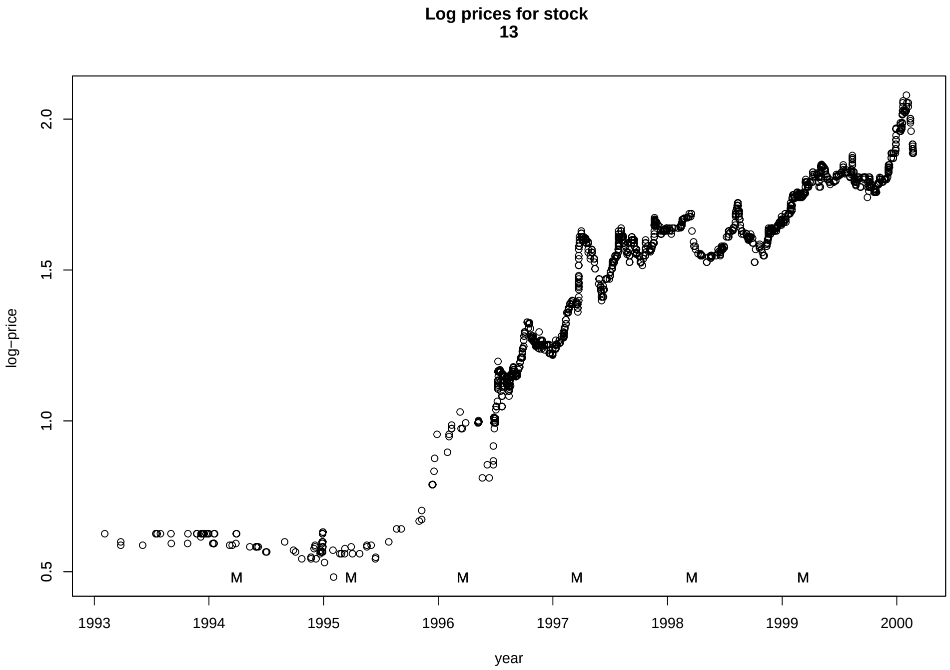

Figure 20: Log-prices for stock number 13

Stock number 13: A total of 1317 transactions from 02/03/93 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0091	0.0091	0.0091	0.0091
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.2986	1.2911	1.2911	1.2879
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0023
$\hat{r}$	0	0.0004	0	0.0007
log-likelihood	3590.4500	3592.1860	3592.1860	3592.9320
skewness	0.3416	0.4175	0.4175	0.4346
kurtosis	11.3520	11.1753	11.1753	11.1456

Table 16: Comparison of models for stock number 13

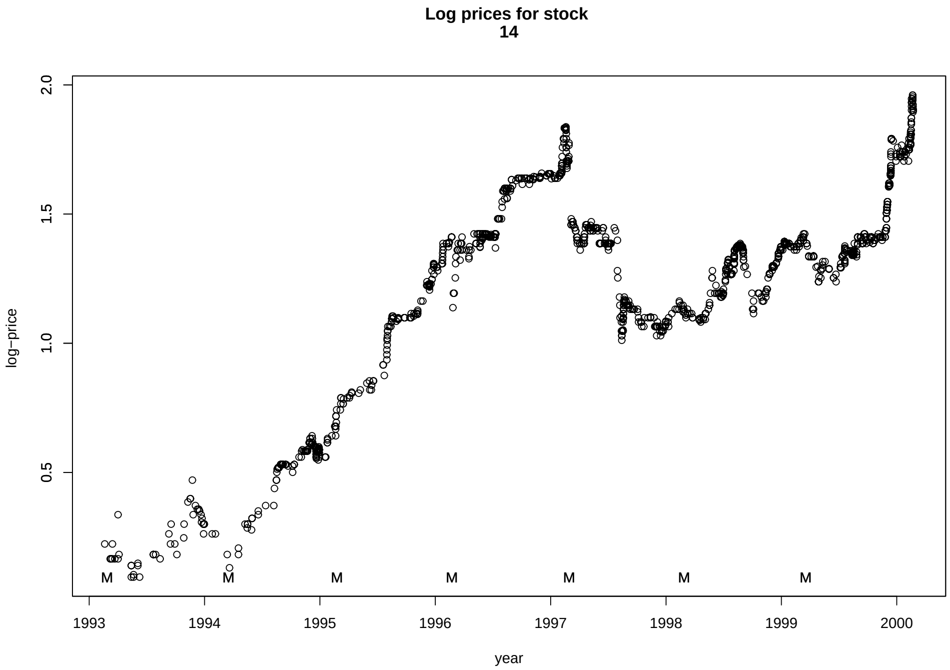

Figure 21: Log-prices for stock number 14

Stock number 14: A total of 1153 transactions from 02/19/93 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0123	0.0123	0.0123	0.0123
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.2364	1.2119	1.2120	1.2105
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0092
$\hat{r}$	0	0.0010	0	0.0010
log-likelihood	2727.4730	2732.4720	2732.4720	2732.8100
$skewness$	-0.1032	0.0717	0.0718	0.0825
$kurtosis$	16.0654	15.3378	15.3379	15.5027

Table 17: Comparison of models for stock number 14

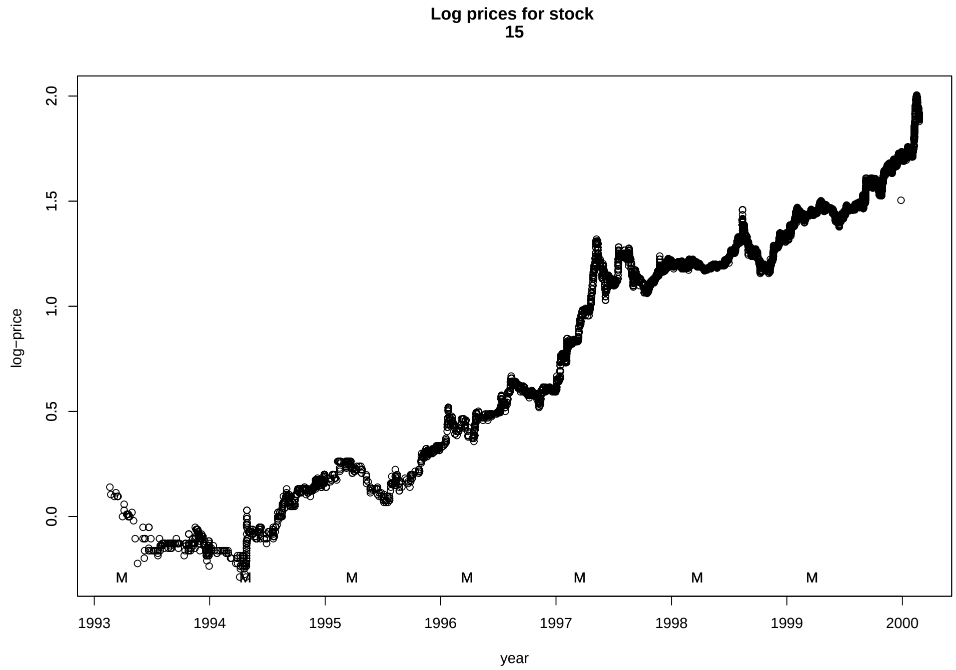

Figure 22: Log-prices for stock number 15

Stock number 15: A total of 7503 transactions from 02/19/93 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0066	0.0066	0.0066	0.0067
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.7064	2.6972	2.6972	2.6936
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.4333
$\hat{r}$	0	0.0007	0	0.0007
log-likelihood	23968.7800	23970.5400	23970.5400	23971.3000
skewness	-0.7324	-0.7058	-0.7058	-0.7073
kurtosis	53.1092	53.0061	53.0056	53.0290

Table 18: Comparison of models for stock number 15

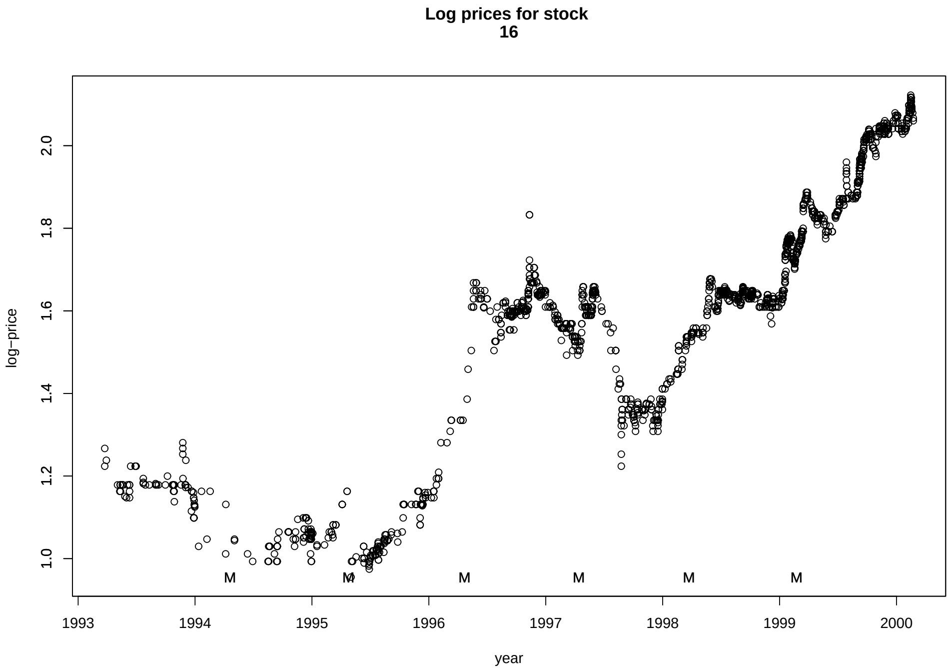

Figure 23: Log-prices for stock number 16

Stock number 16: A total of 1429 transactions from 03/25/93 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0123	0.0123	0.0123	0.0124
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.9476	0.9433	0.9433	0.9386
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0022
$\hat{r}$	0	0.0003	0	0.0006
log-likelihood	3650.3070	3651.2730	3651.2730	3652.4520
$skewness$	-0.1045	-0.0400	-0.0400	-0.0203
$kurtosis$	16.7040	16.4748	16.4747	16.4947

Table 19: Comparison of models for stock number 16

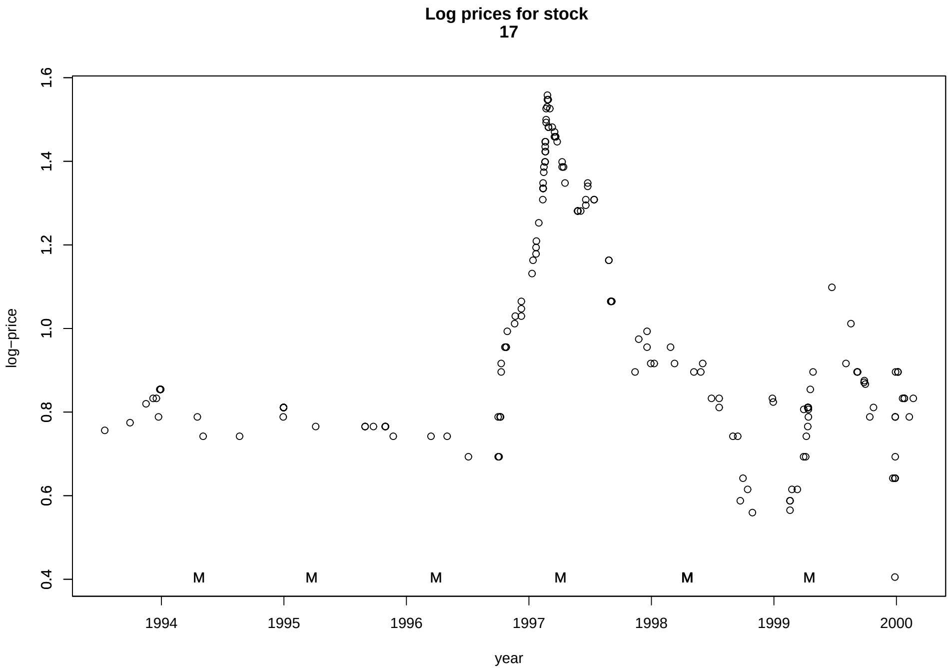

Figure 24: Log-prices for stock number 17

Stock number 17: A total of 171 transactions from 07/16/93 to 02/21/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0363	0.0363	0.0363	0.0363
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.5588	0.5588	0.5588	0.5584
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0151
$\hat{r}$	0	0.0000	0	0.0000
log-likelihood	217.6940	217.6958	217.6958	217.7114
skewness	0.3494	0.3569	0.3569	0.3558
kurtosis	7.7693	7.7592	7.7593	7.7636

Table 20: Comparison of models for stock number 17

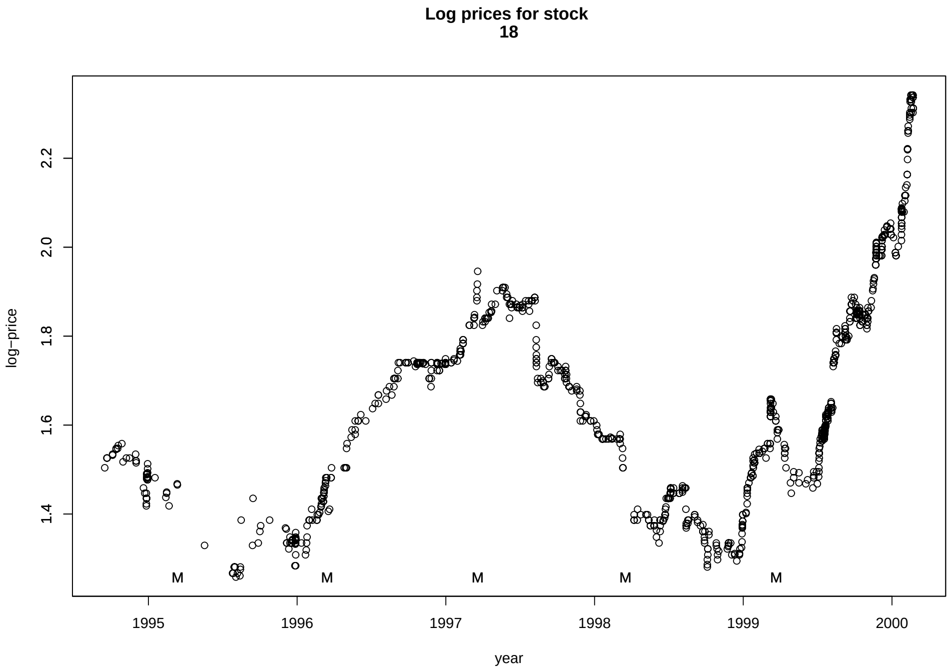

Figure 25: Log-prices for stock number 18

Stock number 18: A total of 929 transactions from 09/16/94 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0089	0.0089	0.0090	0.0089
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.7664	1.7593	1.7169	1.7178
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.5855	0.6570
$\hat{\gamma}$	0	∞	0	0.0019
$\hat{r}$	0	0.0004	0	0.0011
log-likelihood	2435.4420	2436.0930	2436.2740	2437.0740
skewness	-0.5598	-0.4839	-0.5202	-0.4608
kurtosis	9.4651	9.2185	9.4241	9.2373

Table 21: Comparison of models for stock number 18

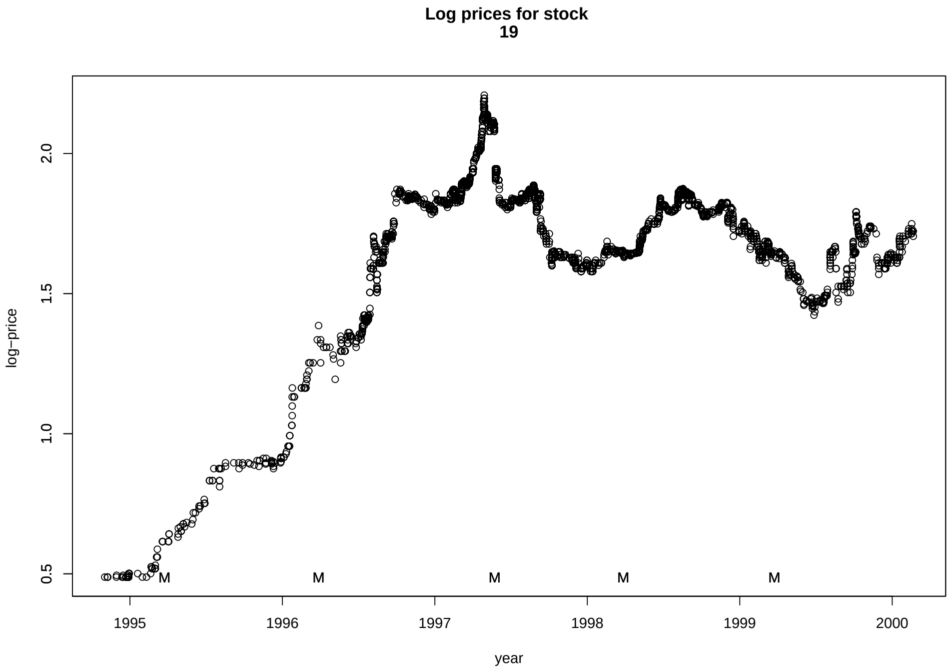

Figure 26: Log-prices for stock number 19

Stock number 19: A total of 2019 transactions from 11/02/94 to 02/21/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0085	0.0085	0.0085	0.0085
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.1304	2.1201	2.1201	2.1128
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0003	0.0003
$\hat{\gamma}$	0	∞	0	0.0002
$\hat{r}$	0	0.0007	0	-0.0040
log-likelihood	5579.4070	5580.7320	5580.7320	5581.8330
$skewness$	-0.4290	-0.3618	-0.3618	-0.3723
$kurtosis$	17.1625	16.9440	16.9441	16.9924

Table 22: Comparison of models for stock number 19

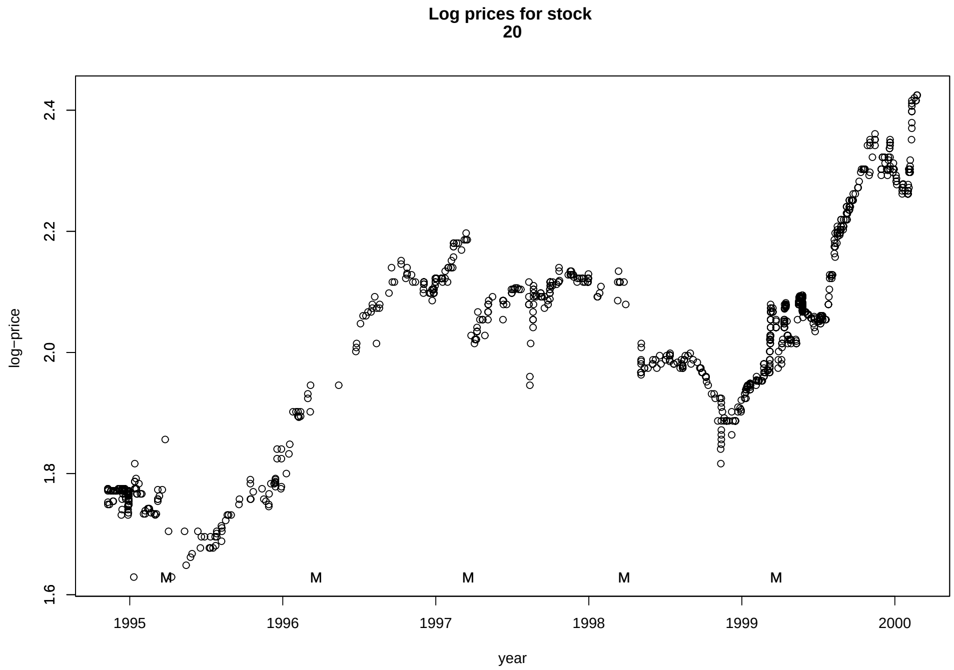

Figure 27: Log-prices for stock number 20

Stock number 20: A total of 885 transactions from 11/10/94 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0088	0.0089	0.0089	0.0089
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.3784	1.3619	1.3619	1.3597
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0086
$\hat{r}$	0	0.0005	0	0.0005
log-likelihood	2397.5060	2398.9360	2398.9360	2399.1950
$skewness$	-0.0166	0.0836	0.0837	0.0817
$kurtosis$	11.5571	11.4210	11.4209	11.5346

Table 23: Comparison of models for stock number 20

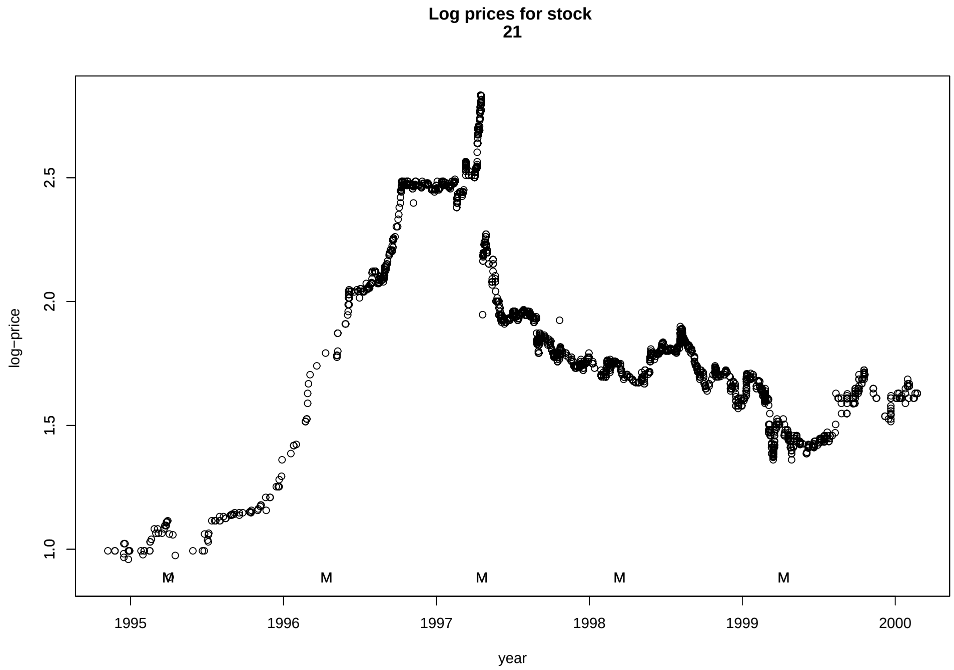

Figure 28: Log-prices for stock number 21

Stock number 21: A total of 1745 transactions from 11/08/94 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0102	0.0102	0.0102	0.0102
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.8101	1.7994	1.7759	1.7851
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.3446	0.0000
$\hat{\gamma}$	0	∞	0	0.0004
$\hat{r}$	0	0.0007	0	-0.0034
log-likelihood	4532.4580	4533.6610	4534.1760	4535.3840
$skewness$	-0.3486	-0.2824	-0.2600	-0.2898
$kurtosis$	14.8300	14.6361	14.5921	14.6642

Table 24: Comparison of models for stock number 21

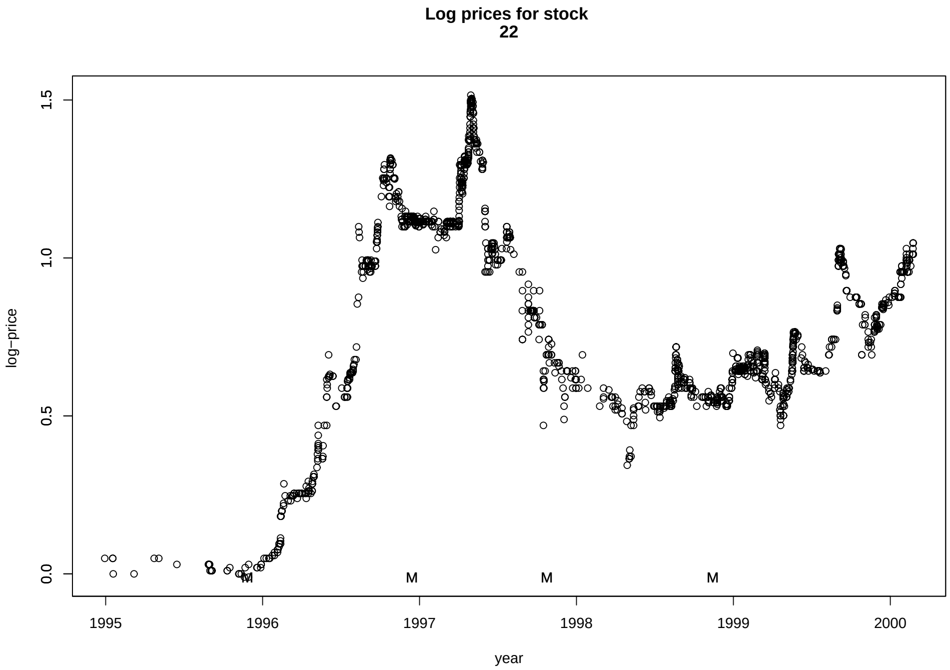

Figure 29: Log-prices for stock number 22

Stock number 22: A total of 1463 transactions from 12/30/94 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0170	0.0170	0.0170	0.0170
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.3030	1.2999	1.2999	1.2993
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0129
$\hat{r}$	0	0.0005	0	0.0006
log-likelihood	3222.3880	3222.9370	3222.9370	3223.0560
$skewness$	0.3238	0.3669	0.3671	0.3709
$kurtosis$	20.0670	19.9254	19.9251	19.9141

Table 25: Comparison of models for stock number 22

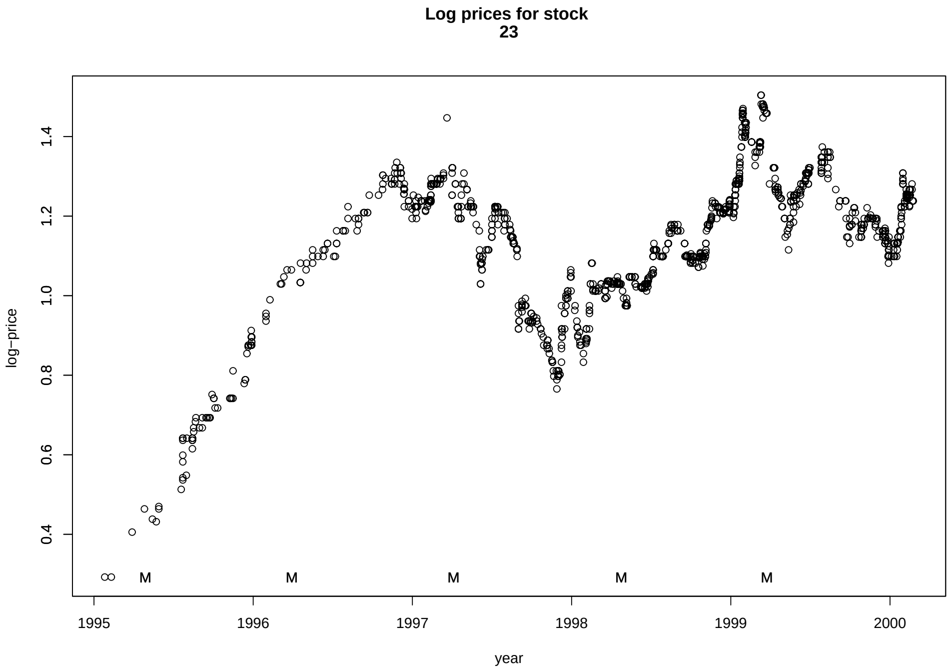

Figure 30: Log-prices for stock number 23

Stock number 23: A total of 1071 transactions from 01/26/95 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0127	0.0127	0.0127	0.0127
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.1979	1.1882	1.1882	1.1834
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0001	0.0007
$\hat{\gamma}$	0	∞	0	0.0021
$\hat{r}$	0	0.0007	0	0.0001
log-likelihood	2609.7890	2611.4720	2611.4720	2612.5920
$skewness$	-0.0637	0.0352	0.0352	0.0117
$kurtosis$	9.4045	9.1901	9.1901	9.2482

Table 26: Comparison of models for stock number 23

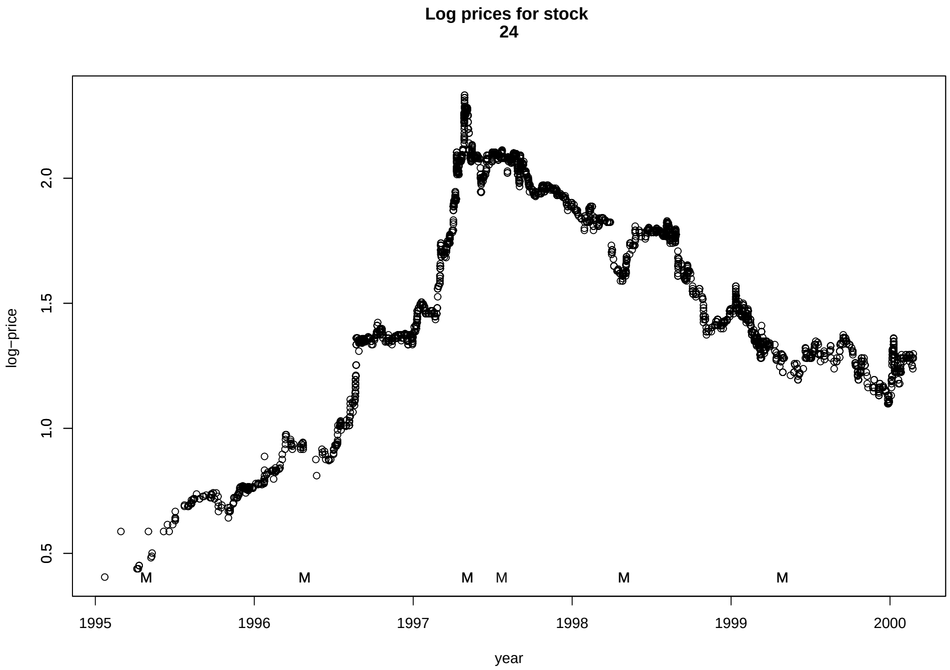

Figure 31: Log-prices for stock number 24

Stock number 24: A total of 2398 transactions from 01/23/95 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0101	0.0101	0.0101	0.0101
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.9718	1.9695	1.9535	1.9602
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.4570	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0004	0	-0.1154
log-likelihood	6392.3400	6392.7570	6393.2430	6394.3900
$skewness$	-0.1955	-0.1638	-0.1375	-0.1675
$kurtosis$	9.8506	9.7874	9.7174	9.7938

Table 27: Comparison of models for stock number 24

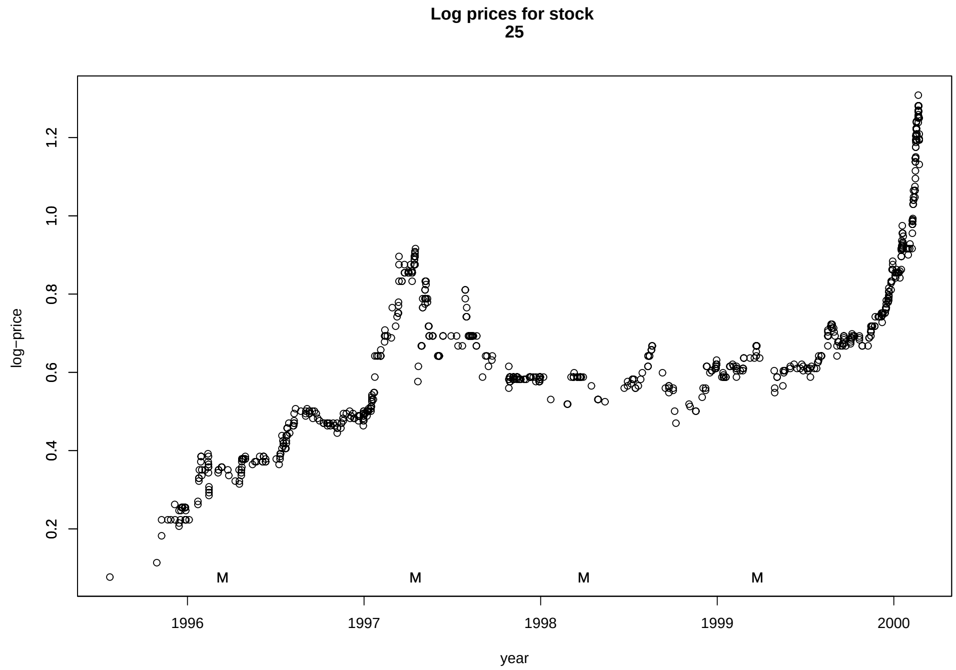

Figure 32: Log-prices for stock number 25

Stock number 25: A total of 774 transactions from 07/25/95 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0098	0.0099	0.0099	0.0099
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.6982	1.6800	1.6799	1.6800
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0008	0	0.0009
log-likelihood	1943.7840	1945.5680	1945.5680	1945.5680
$skewness$	-0.4471	-0.3153	-0.3153	-0.3152
$kurtosis$	10.1530	9.8128	9.8127	9.8126

Table 28: Comparison of models for stock number 25

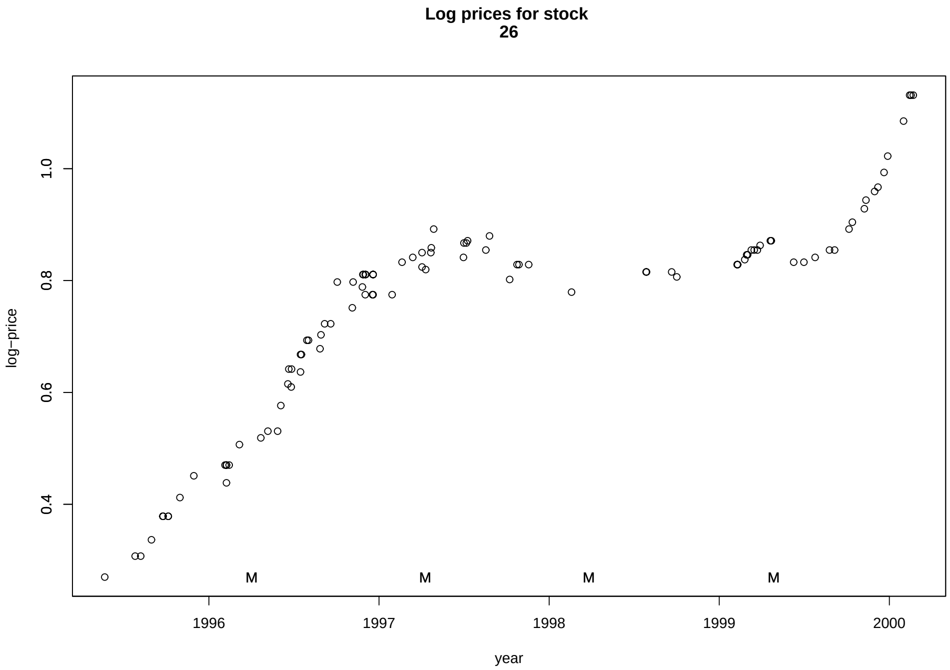

Figure 33: Log-prices for stock number 26

Stock number 26: A total of 102 transactions from 05/23/95 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0126	0.0131	0.0141	0.0141
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.4958	0.4083	0.2411	0.2333
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.3306	0.4182
$\hat{\gamma}$	0	∞	0	0.0038
$\hat{r}$	0	0.0004	0	0.0006
log-likelihood	230.4309	236.1116	239.9949	240.7744
$skewness$	-1.4765	-0.5727	-0.4679	-0.3940
$kurtosis$	4.7233	3.0812	3.0669	3.0503

Table 29: Comparison of models for stock number 26

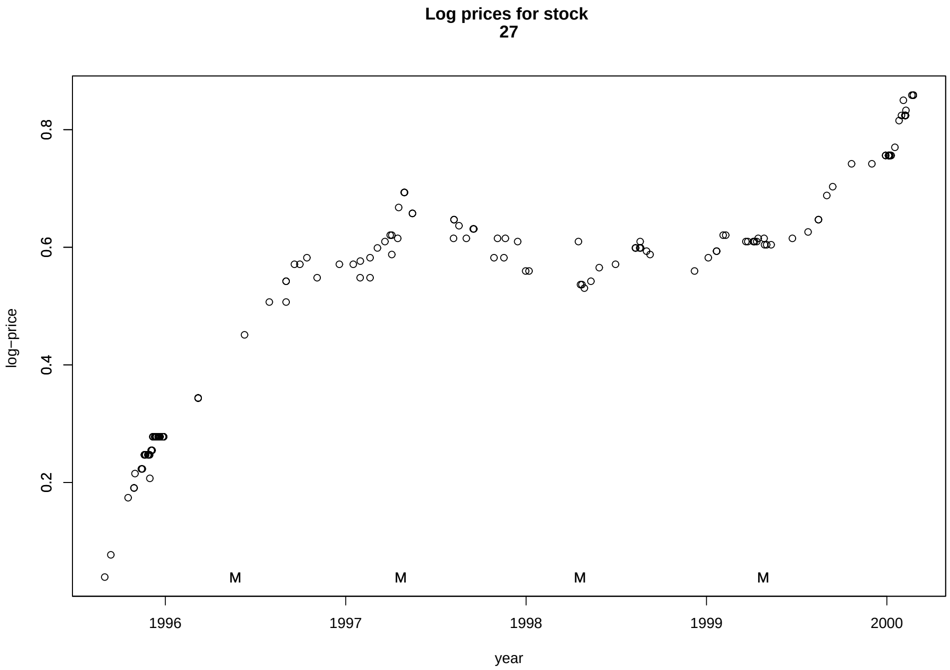

Figure 34: Log-prices for stock number 27

Stock number 27: A total of 145 transactions from 09/01/95 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0086	0.0088	0.0092	0.0092
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6868	0.6038	0.4364	0.4320
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.3748	0.3576
$\hat{\gamma}$	0	∞	0	0.0066
$\hat{r}$	0	0.0005	0	0.0003
log-likelihood	370.8616	377.2277	383.0530	387.0535
skewness	-0.9351	-0.2988	-0.1825	-0.2494
kurtosis	5.9547	4.8617	5.0283	5.0778

Table 30: Comparison of models for stock number 27

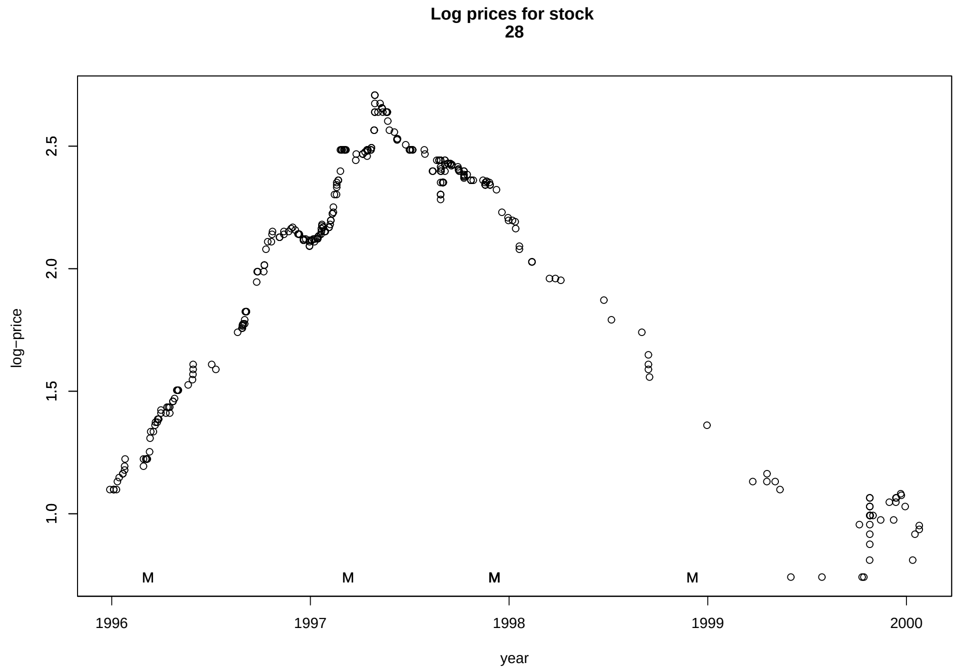

Figure 35: Log-prices for stock number 28

Stock number 28: A total of 307 transactions from 12/29/95 to 01/25/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0253	0.0253	0.0253	0.0257
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.8046	0.8045	0.8045	0.7353
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0020
$\hat{r}$	0	-0.0000	0	-0.0023
log-likelihood	527.7458	527.7464	527.7464	534.7267
skewness	0.0972	0.0936	0.0935	0.1934
kurtosis	8.6673	8.6694	8.6694	8.5186

Table 31: Comparison of models for stock number 28

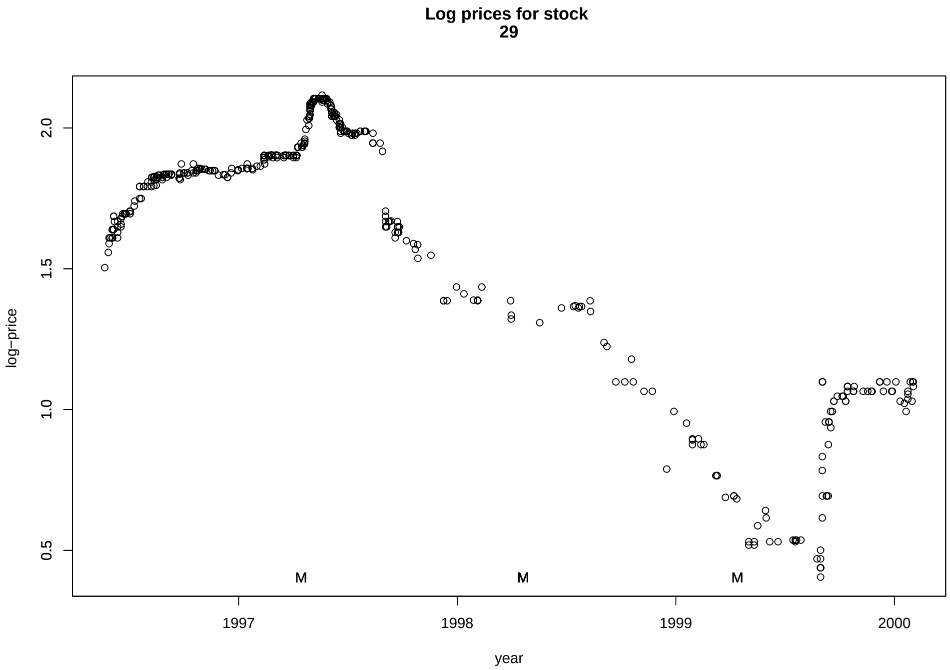

Figure 36: Log-prices for stock number 29

Stock number 29: A total of 418 transactions from 05/22/96 to 02/02/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0337	0.0337	0.0337	0.0338
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.7995	0.7995	0.8012	0.7863
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0231
$\hat{r}$	0	-0.0000	0	-0.0003
log-likelihood	618.8483	618.8496	618.8495	619.9664
skewness	3.4748	3.4723	3.4739	3.5182
kurtosis	46.7575	46.7640	46.8176	47.1106

Table 32: Comparison of models for stock number 29

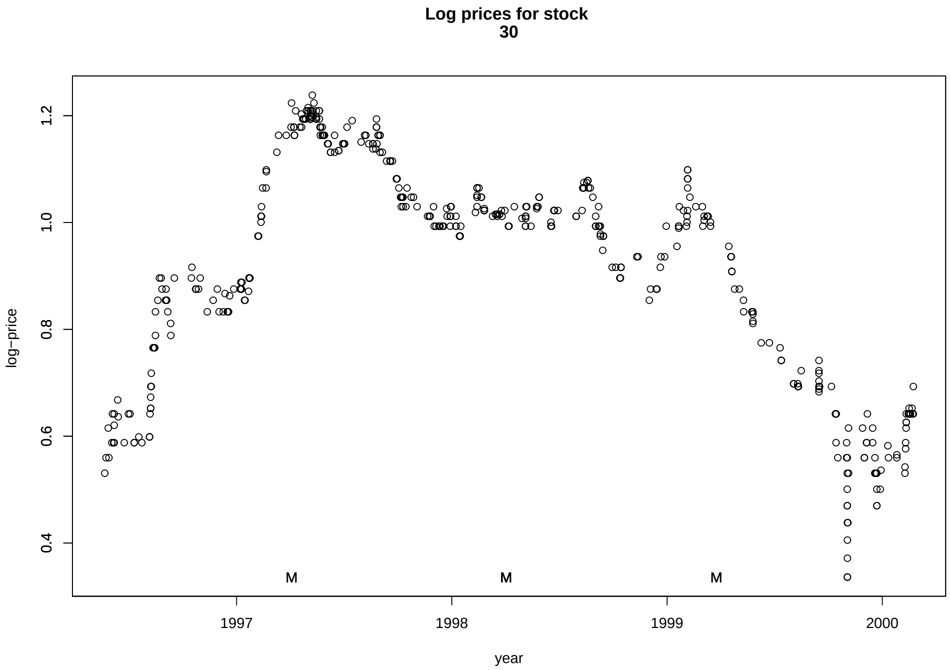

Figure 37: Log-prices for stock number 30

Stock number 30: A total of 445 transactions from 05/22/96 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0205	0.0206	0.0207	0.0207
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6847	0.6842	0.6639	0.6547
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.2032	0.0000
$\hat{\gamma}$	0	∞	0	0.0028
$\hat{r}$	0	0.0002	0	-0.0009
log-likelihood	903.6611	903.7677	904.4830	907.9821
skewness	-1.2363	-1.1916	-1.0854	-1.1668
kurtosis	13.9684	13.8032	13.4772	13.7880

Table 33: Comparison of models for stock number 30

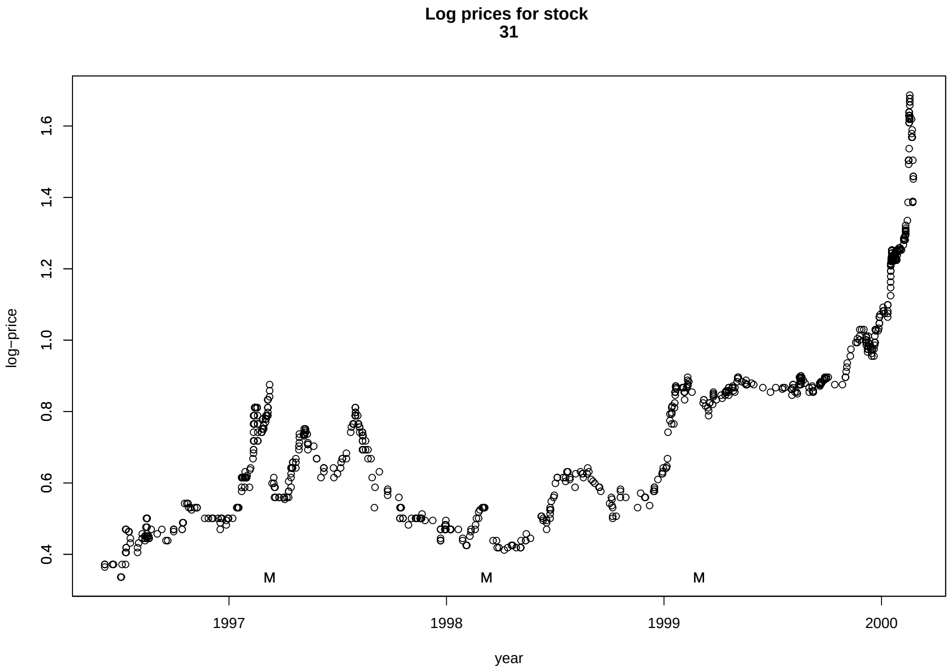

Figure 38: Log-prices for stock number 31

Stock number 31: A total of 774 transactions from 06/06/96 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0131	0.0131	0.0131	0.0131
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.4752	1.4611	1.4611	1.4612
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0002	0.1317
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0009	0	0.0016
log-likelihood	1787.8910	1789.4750	1789.4750	1793.3770
$skewness$	-0.4434	-0.3182	-0.3182	-0.3306
$kurtosis$	13.1623	12.8544	12.8544	12.8958

Table 34: Comparison of models for stock number 31

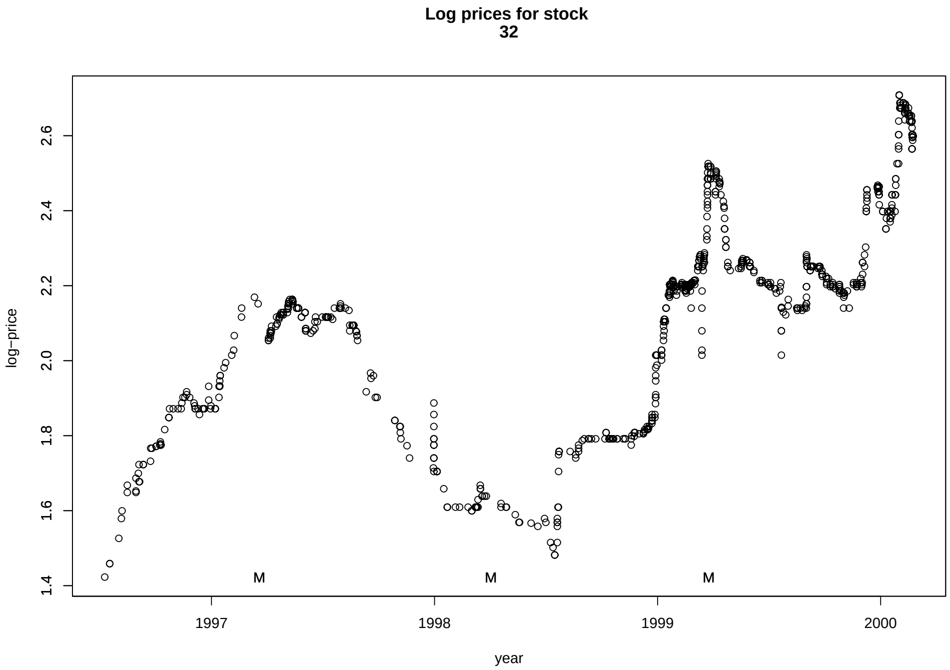

Figure 39: Log-prices for stock number 32

Stock number 32: A total of 748 transactions from 07/10/96 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0189	0.0189	0.0189	0.0190
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.1096	1.1047	1.1046	1.0910
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0114
$\hat{r}$	0	0.0007	0	0.0003
log-likelihood	1538.5370	1539.2890	1539.2890	1540.9670
$skewness$	0.8257	0.8818	0.8817	0.8788
$kurtosis$	14.2290	14.1022	14.1021	14.1836

Table 35: Comparison of models for stock number 32

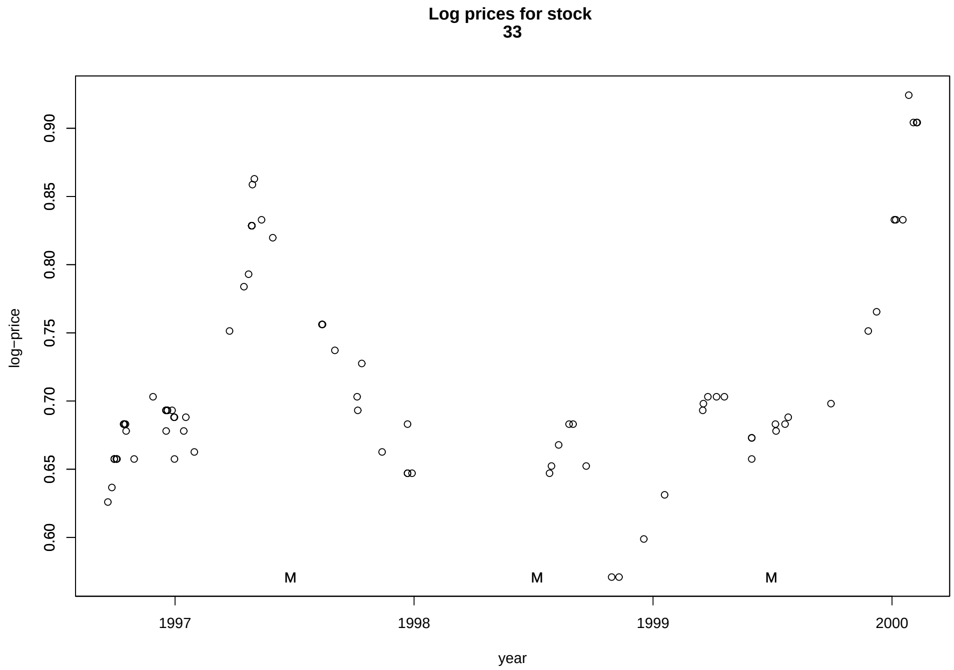

Figure 40: Log-prices for stock number 33

Stock number 33: A total of 81 transactions from 09/20/96 to 02/09/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0089	0.0087	0.0087	0.0088
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.8135	0.8163	0.8163	0.7963
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0786
$\hat{r}$	0	0.0003	0	0.0003
log-likelihood	195.5161	196.9725	196.9725	197.6980
skewness	-0.3585	-0.0019	-0.0019	0.0692
kurtosis	6.3028	5.9365	5.9365	6.0564

Table 36: Comparison of models for stock number 33

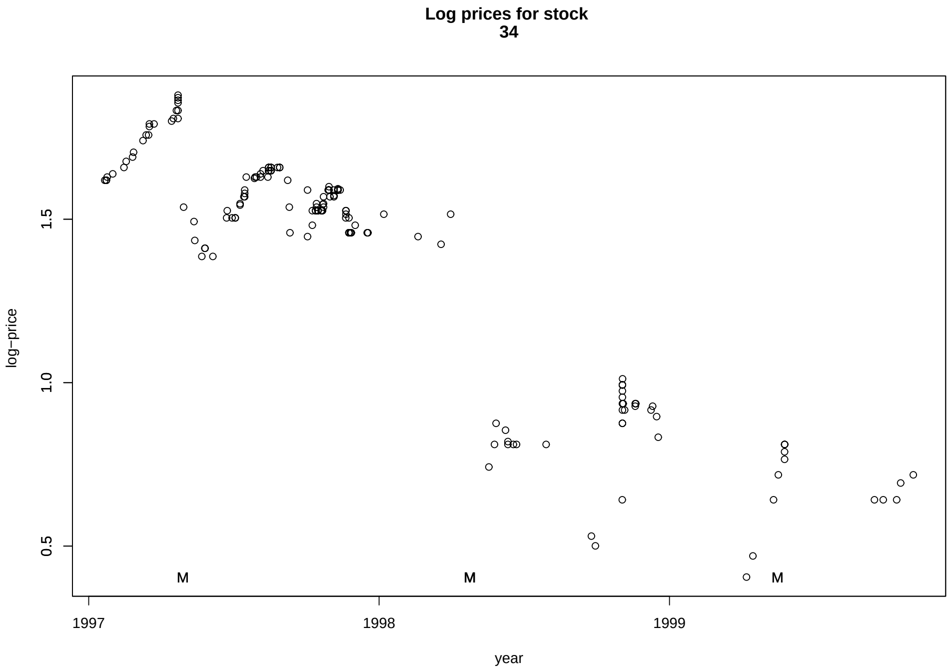

Figure 41: Log-prices for stock number 34

Stock number 34: A total of 157 transactions from 01/21/97 to 11/04/99

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0372	0.0372	0.0372	0.0372
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6022	0.5871	0.5871	0.5871
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	452.5997
$\hat{r}$	0	-0.0010	0	-0.0010
log-likelihood	212.3939	213.3849	213.3850	213.3850
skewness	0.4101	0.2364	0.2364	0.2364
kurtosis	9.3610	9.1917	9.1917	9.1917

Table 37: Comparison of models for stock number 34

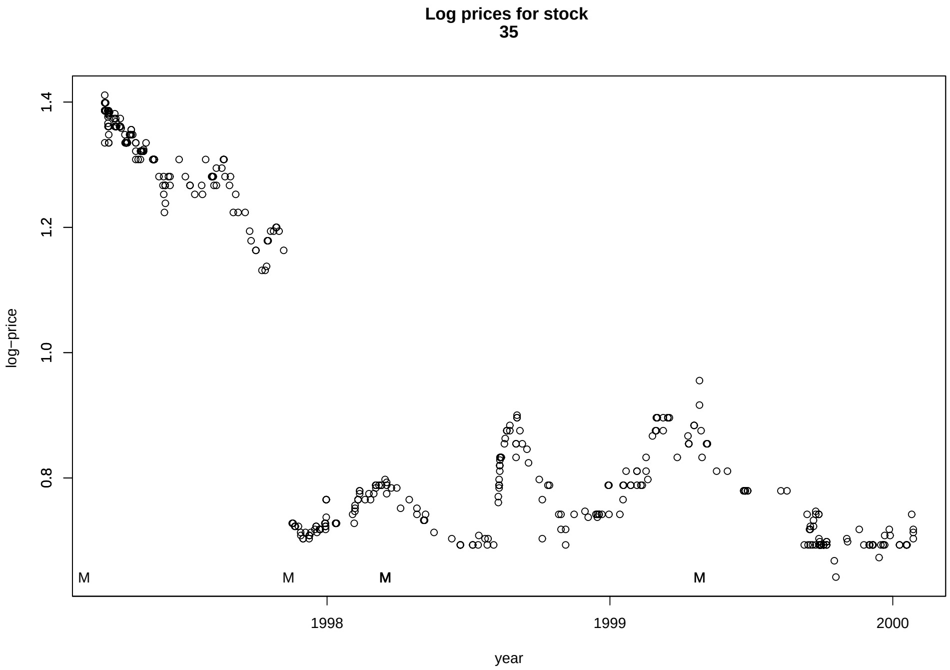

Figure 42: Log-prices for stock number 35

Stock number 35: A total of 410 transactions from 03/20/97 to 01/28/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0108	0.0108	0.0108	0.0108
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.8640	0.8614	0.8618	0.8532
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0080
$\hat{r}$	0	-0.0002	0	-0.0000
log-likelihood	1059.9980	1060.2920	1060.2920	1061.0380
skewness	-0.1271	-0.1999	-0.1996	-0.2241
kurtosis	4.9672	5.0012	5.0026	5.0454

Table 38: Comparison of models for stock number 35

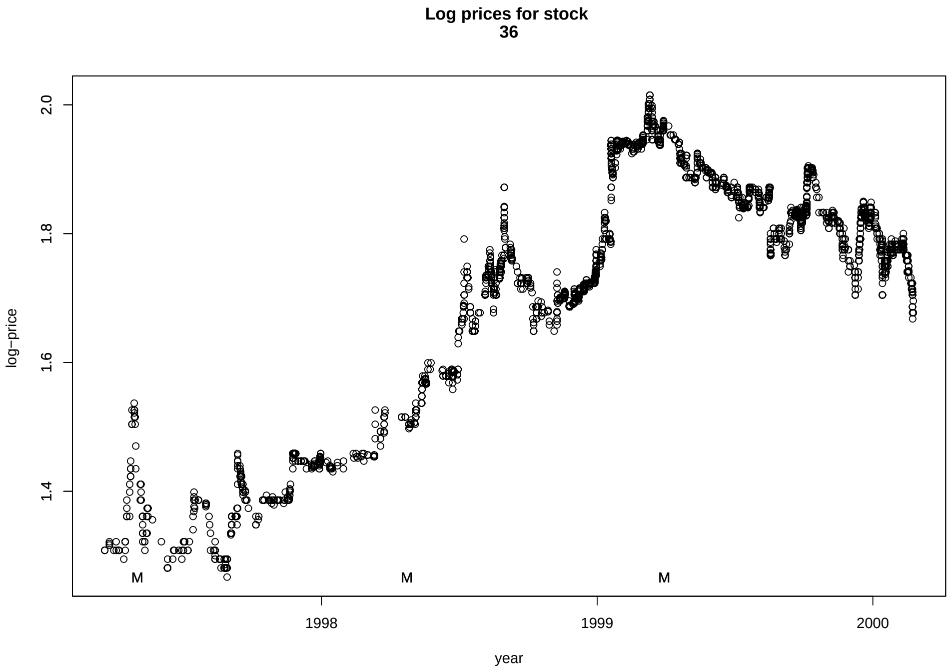

Figure 43: Log-prices for stock number 36

Stock number 36: A total of 2083 transactions from 03/20/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0077	0.0077	0.0077	0.0077
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.4202	2.4163	2.4164	2.4096
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0004	0	-0.1660
log-likelihood	6226.9370	6227.1620	6227.1620	6227.5530
skewness	0.8534	0.8753	0.8752	0.8703
kurtosis	18.7676	18.7600	18.7599	18.7493

Table 39: Comparison of models for stock number 36

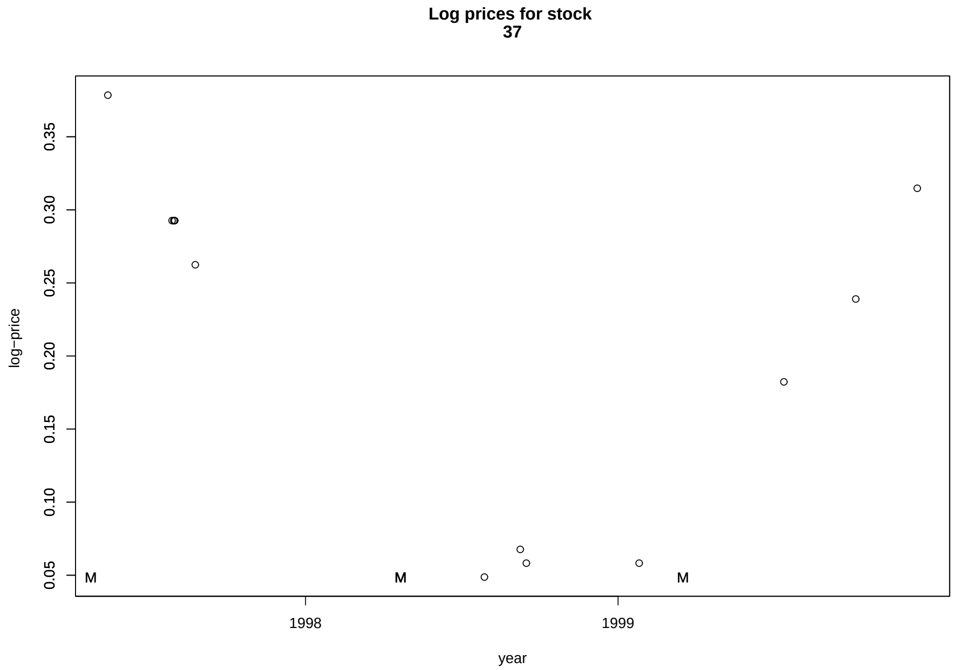

Figure 44: Log-prices for stock number 37

Stock number 37: A total of 12 transactions from 05/15/97 to 12/17/99

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0000	0.0000	0.0000	0.0000
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1009.0605	1009.0605	1307.0265	24414402708968266 1376.0000
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	2457.4836	46.1715
$\hat{\gamma}$	0	∞	0	0.0001
$\hat{r}$	0	-0.0001	0	0.0207
log-likelihood	24.2264	24.2778	32.3272	39.0920
skewness	0.1412	-0.0823	-1.1778	-0.0966
kurtosis	2.1974	2.0779	3.8137	2.4104

Table 40: Comparison of models for stock number 37

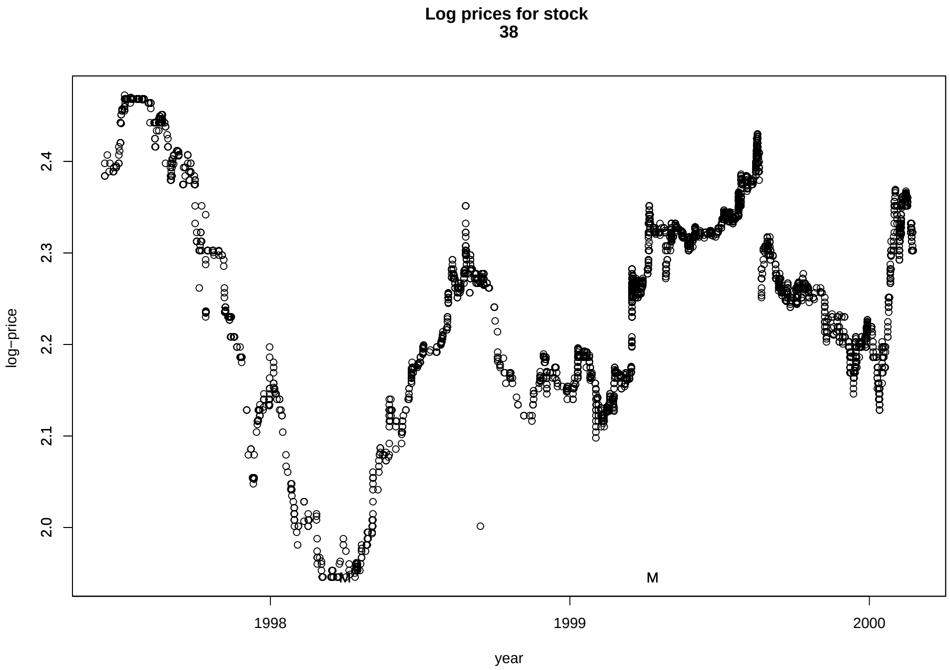

Figure 45: Log-prices for stock number 38

Stock number 38: A total of 2549 transactions from 06/13/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0091	0.0091	0.0091	0.0091
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.4798	1.4798	1.4794	1.4761
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0022
$\hat{r}$	0	-0.0001	0	0.0006
log-likelihood	7650.5760	7650.6150	7650.6150	7651.2000
$skewness$	-1.6040	-1.6141	-1.6145	-1.6125
$kurtosis$	118.8603	119.0568	119.0578	118.9875

Table 41: Comparison of models for stock number 38

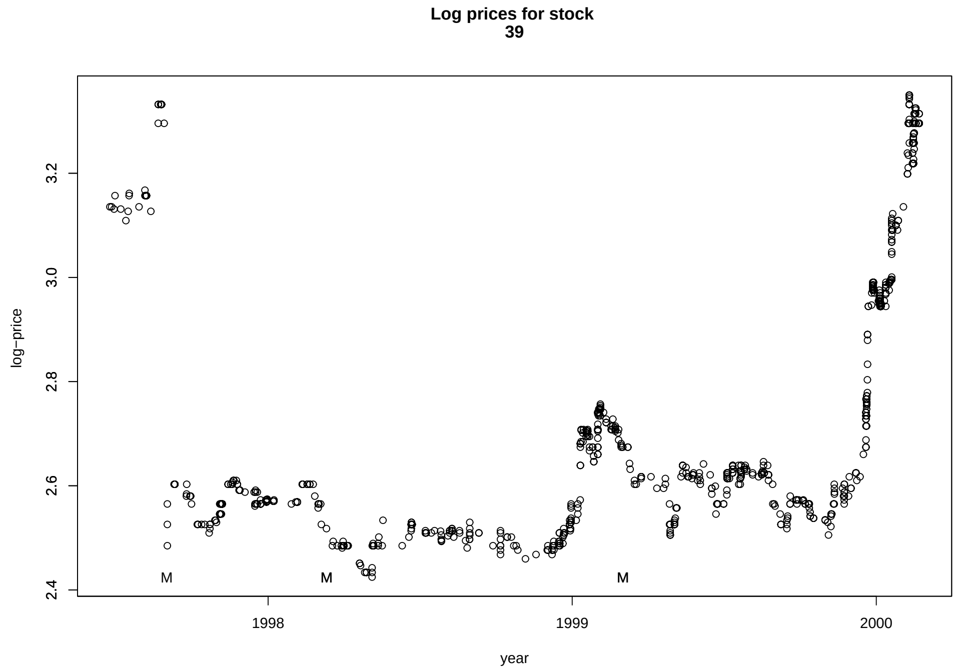

Figure 46: Log-prices for stock number 39

Stock number 39: A total of 720 transactions from 06/25/97 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0133	0.0133	0.0133	0.0134
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.6287	1.6049	1.6049	1.5870
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0011	0	0.4340
log-likelihood	1673.2250	1674.3710	1674.3710	1675.4860
skewness	0.0398	0.1394	0.1394	0.1470
kurtosis	8.3214	8.0525	8.0525	7.9962

Table 42: Comparison of models for stock number 39

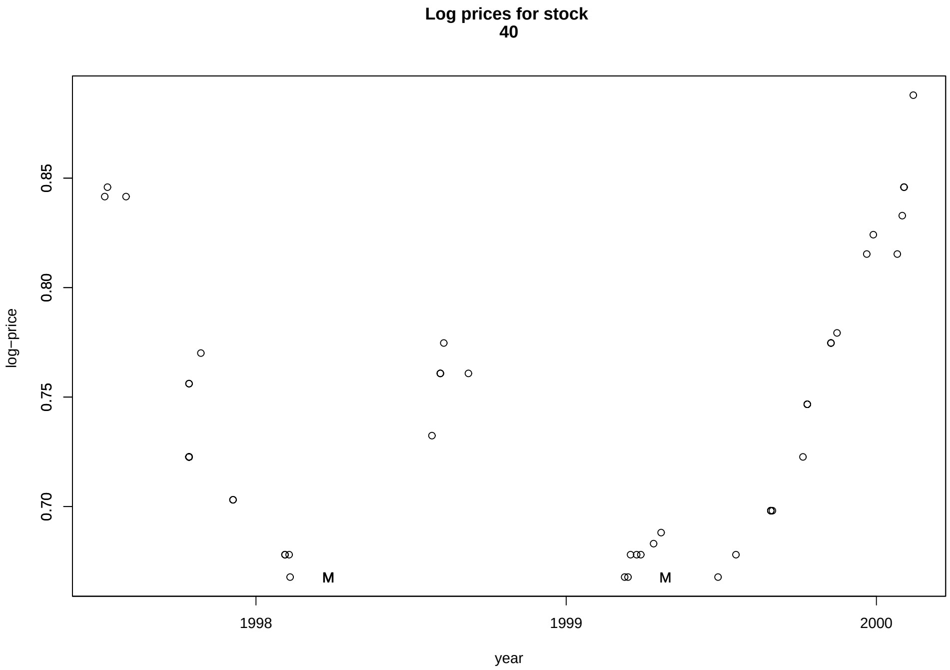

Figure 47: Log-prices for stock number 40

Stock number 40: A total of 49 transactions from 07/07/97 to 02/14/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0089	0.0089	0.0091	0.0089
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6616	0.6616	0.5122	0.5492
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.4870	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0000	0	0.3245
log-likelihood	123.6167	123.6516	124.9076	127.5755
$skewness$	-0.1304	-0.0549	-0.3752	0.0549
$kurtosis$	4.2130	4.2247	4.4270	4.0914

Table 43: Comparison of models for stock number 40

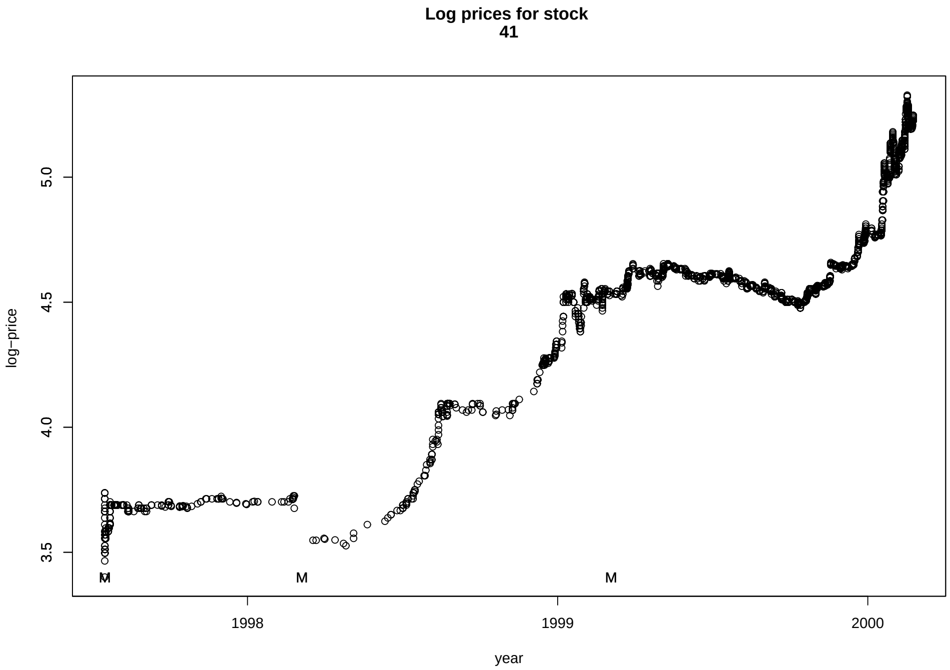

Figure 48: Log-prices for stock number 41

Stock number 41: A total of 1643 transactions from 07/17/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0086	0.0086	0.0086	0.0086
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	3.6665	3.6398	3.6398	3.6273
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0017	0	0.2642
log-likelihood	4445.6660	4446.9600	4446.9600	4447.5620
skewness	0.0150	0.0721	0.0721	0.0815
kurtosis	9.8064	9.6759	9.6759	9.6428

Table 44: Comparison of models for stock number 41

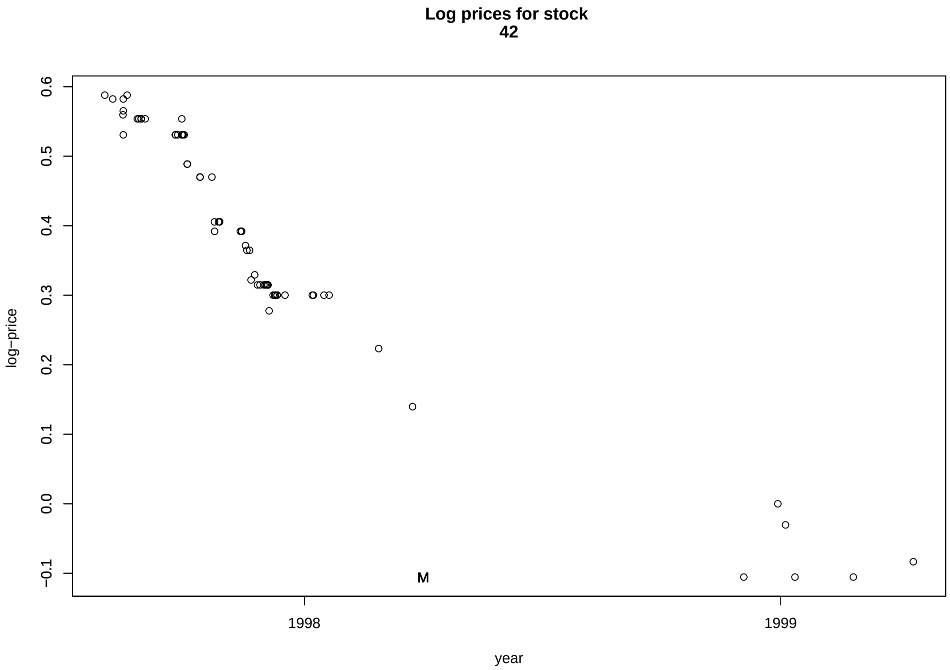

Figure 49: Log-prices for stock number 42

Stock number 42: A total of 65 transactions from 08/01/97 to 04/13/99

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0105	0.0110	0.0110	0.0113
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.0444	0.8868	0.8868	0.7935
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	-0.0011	0	0.7941
log-likelihood	152.3486	155.4317	155.4317	157.0650
skewness	1.0057	0.3442	0.3442	0.0915
kurtosis	6.2303	5.1096	5.1096	4.7822

Table 45: Comparison of models for stock number 42

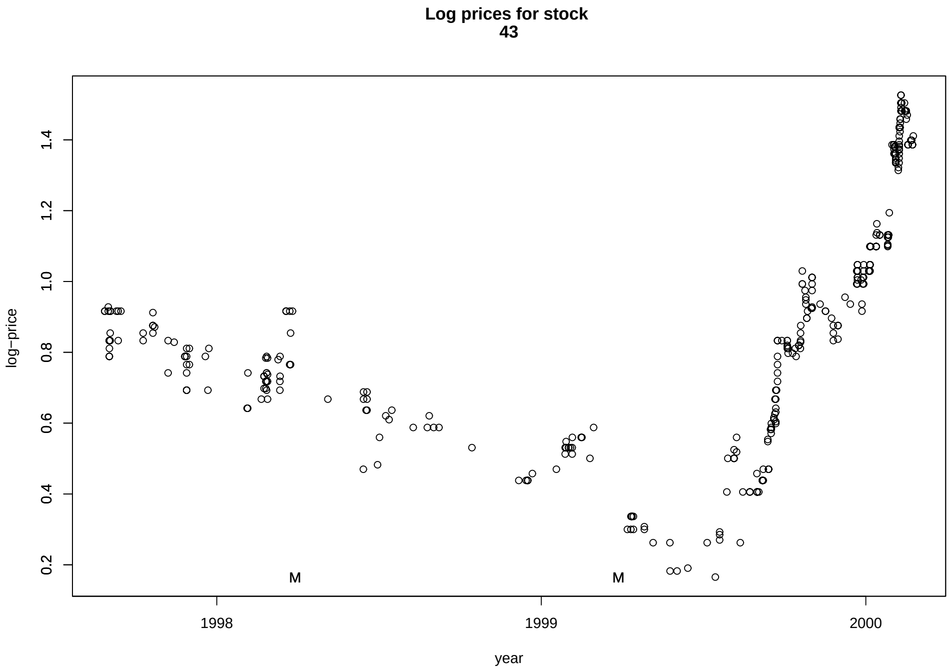

Figure 50: Log-prices for stock number 43

Stock number 43: A total of 348 transactions from 08/28/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0269	0.0270	0.0270	0.0271
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.2281	1.2219	1.2219	1.1818
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0009	0	1.3986
log-likelihood	571.7848	572.0879	572.0879	573.9782
skewness	-0.2679	-0.1975	-0.1974	-0.1338
kurtosis	5.7089	5.5773	5.5772	5.6000

Table 46: Comparison of models for stock number 43

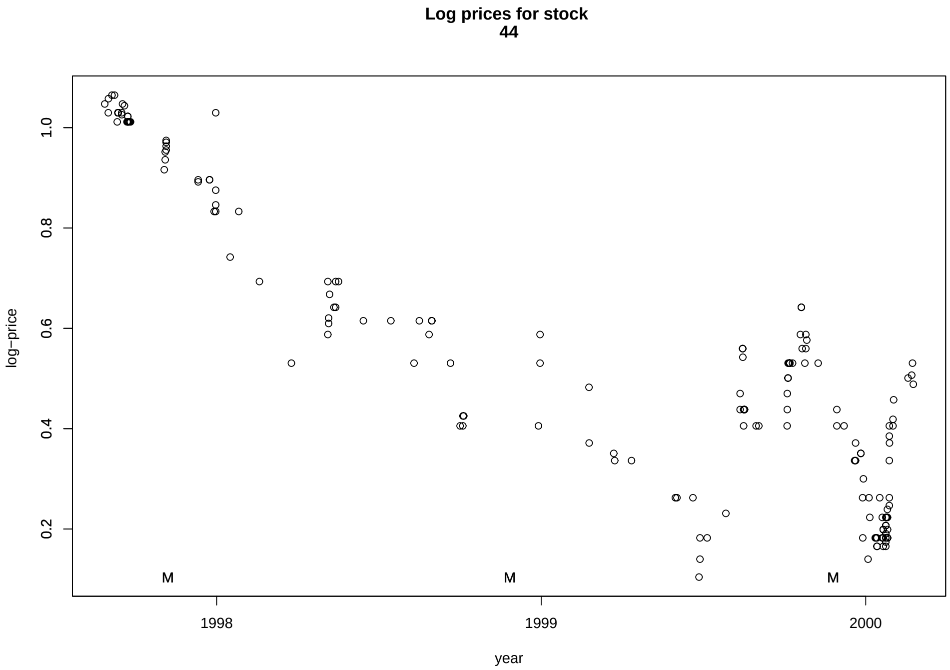

Figure 51: Log-prices for stock number 44

Stock number 44: A total of 172 transactions from 08/28/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0329	0.0330	0.0330	0.0331
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.7439	0.7415	0.7416	0.7215
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	-0.0009	0	0.1143
log-likelihood	257.7642	258.3022	258.3022	258.6399
skewness	-0.0358	-0.1777	-0.1777	-0.1392
kurtosis	4.8939	4.9784	4.9786	5.0118

Table 47: Comparison of models for stock number 44

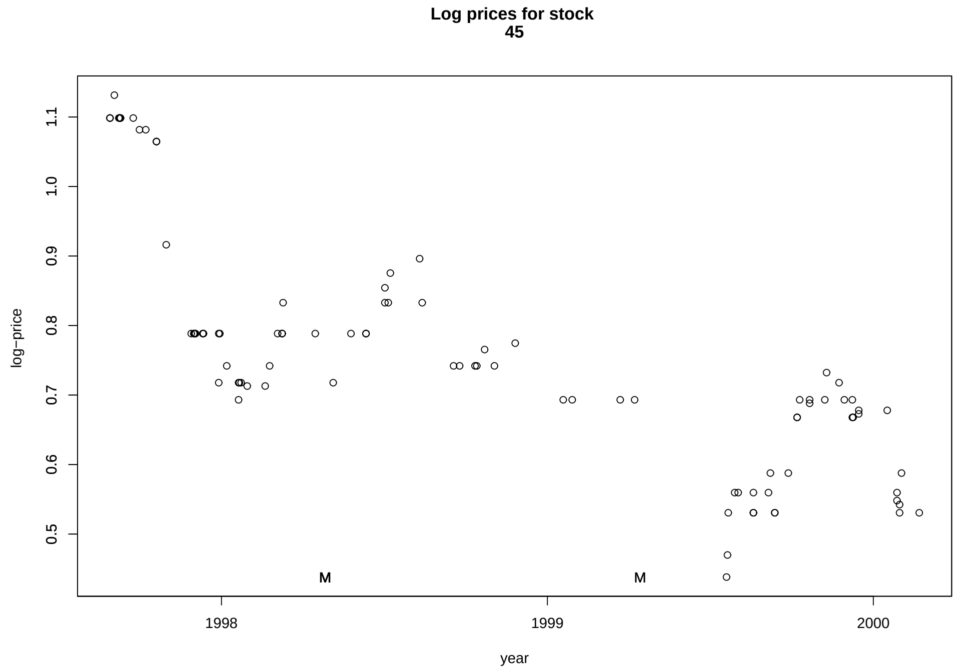

Figure 52: Log-prices for stock number 45

Stock number 45: A total of 90 transactions from 08/29/97 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0162	0.0164	0.0164	0.0167
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.8298	0.7947	0.7947	0.7561
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0075
$\hat{r}$	0	-0.0007	0	-0.0003
log-likelihood	167.1926	168.4652	168.4652	169.5388
skewness	0.1068	-0.2665	-0.2665	-0.3254
kurtosis	4.5760	4.4192	4.4192	4.1308

Table 48: Comparison of models for stock number 45

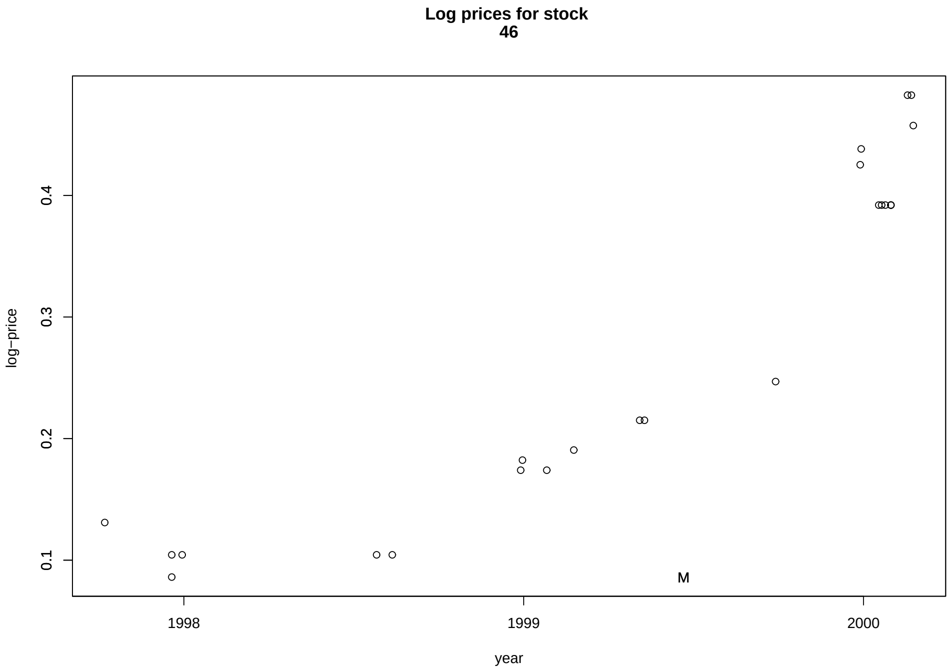

Figure 53: Log-prices for stock number 46

Stock number 46: A total of 23 transactions from 10/08/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0071	0.0074	0.0073	0.0079
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.0881	0.9804	0.9805	0.7966
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0004	0	0.0780
log-likelihood	48.6837	49.8202	49.8202	51.6056
skewness	-0.9333	-0.1982	-0.1982	-0.0054
kurtosis	5.7324	3.9890	3.9889	3.6793

Table 49: Comparison of models for stock number 46

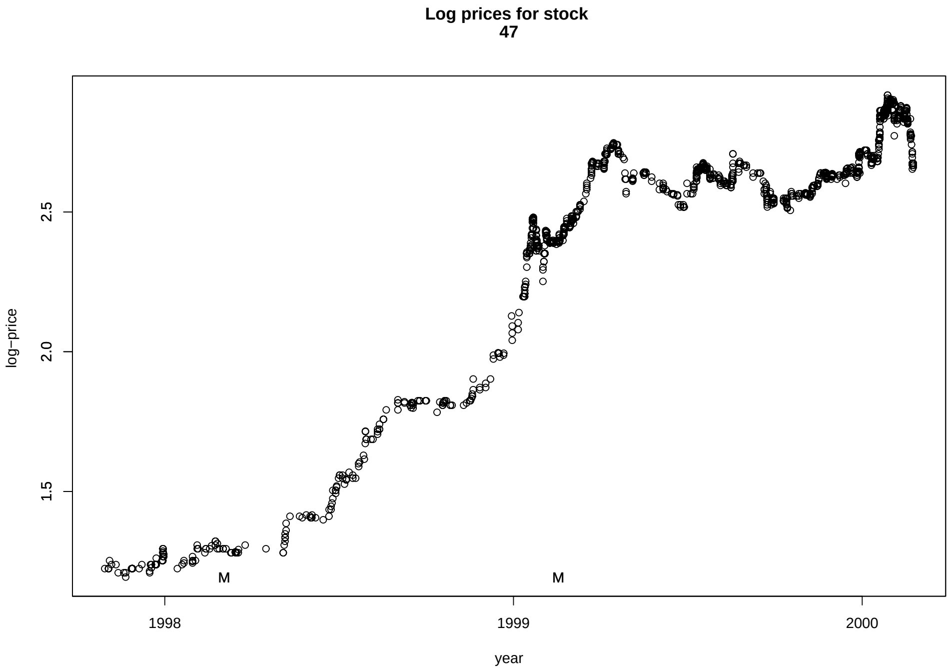

Figure 54: Log-prices for stock number 47

Stock number 47: A total of 1118 transactions from 10/30/97 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0107	0.0107	0.0107	0.0107
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.1106	2.0741	2.0741	2.0742
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0007	0.2673
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0015	0	0.0002
log-likelihood	2954.8670	2957.2600	2957.2600	2958.5000
$skewness$	-0.2440	-0.1314	-0.1314	-0.1228
$kurtosis$	10.7317	10.5542	10.5542	10.5457

Table 50: Comparison of models for stock number 47

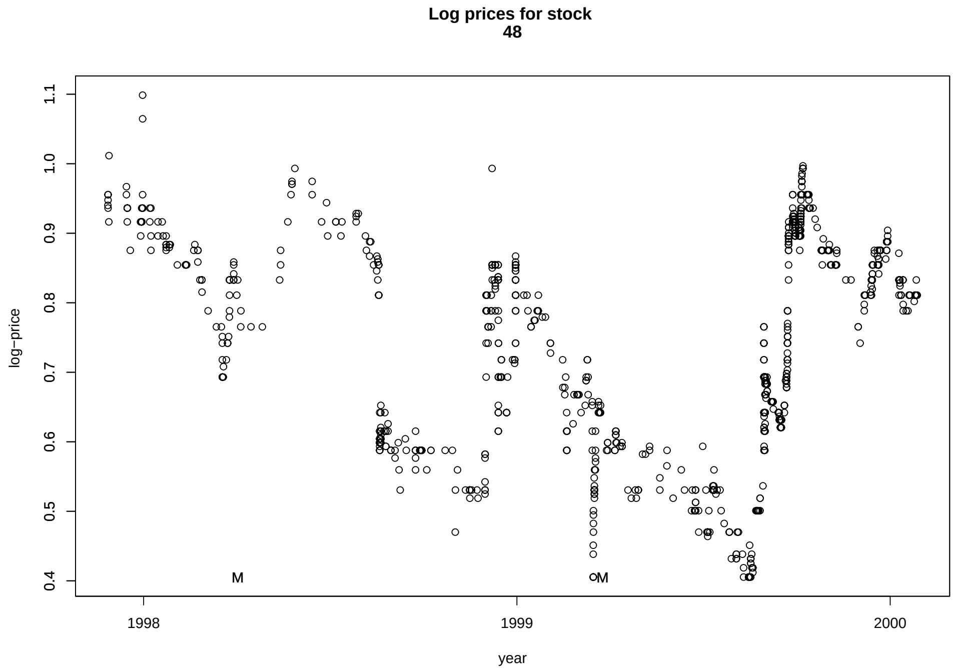

Figure 55: Log-prices for stock number 48

Stock number 48: A total of 845 transactions from 11/27/97 to 01/28/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0174	0.0174	0.0174	0.0174
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.7402	2.7402	2.7401	2.7389
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0044
$\hat{r}$	0	-0.0002	0	0.0000
log-likelihood	1701.8610	1701.8670	1701.8670	1701.9070
$skewness$	-0.6066	-0.6130	-0.6130	-0.6171
$kurtosis$	12.9399	12.9544	12.9542	12.9687

Table 51: Comparison of models for stock number 48

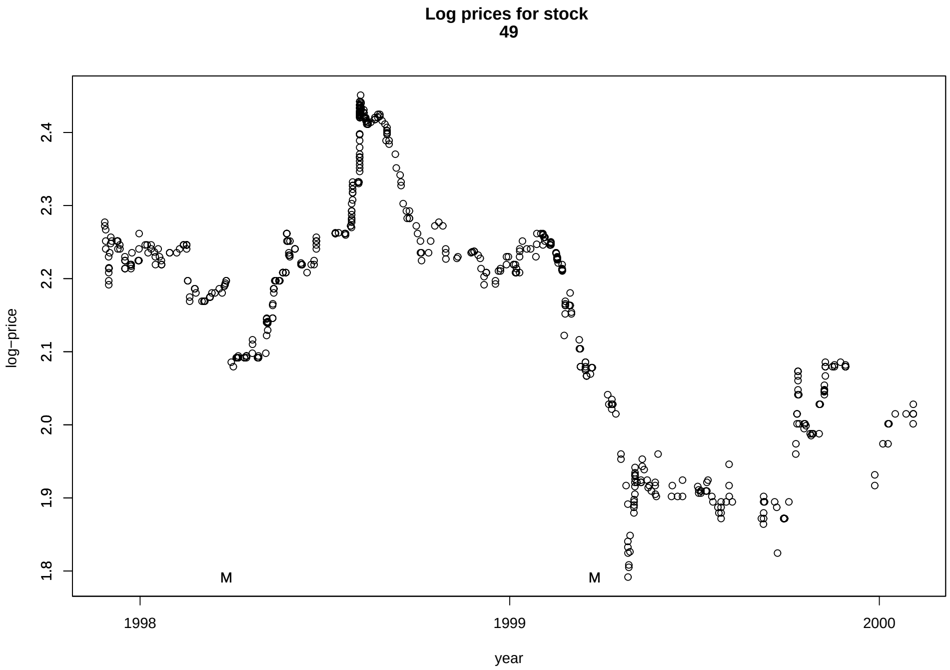

Figure 56: Log-prices for stock number 49

Stock number 49: A total of 553 transactions from 11/27/97 to 02/04/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0068	0.0068	0.0068	0.0069
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.9548	2.9546	2.9546	2.9198
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.7330
$\hat{r}$	0	-0.0004	0	-0.0003
log-likelihood	1514.6220	1514.7920	1514.7920	1515.7810
$skewness$	-0.1414	-0.1893	-0.1893	-0.1899
$kurtosis$	8.8006	8.8758	8.8758	8.9159

Table 52: Comparison of models for stock number 49

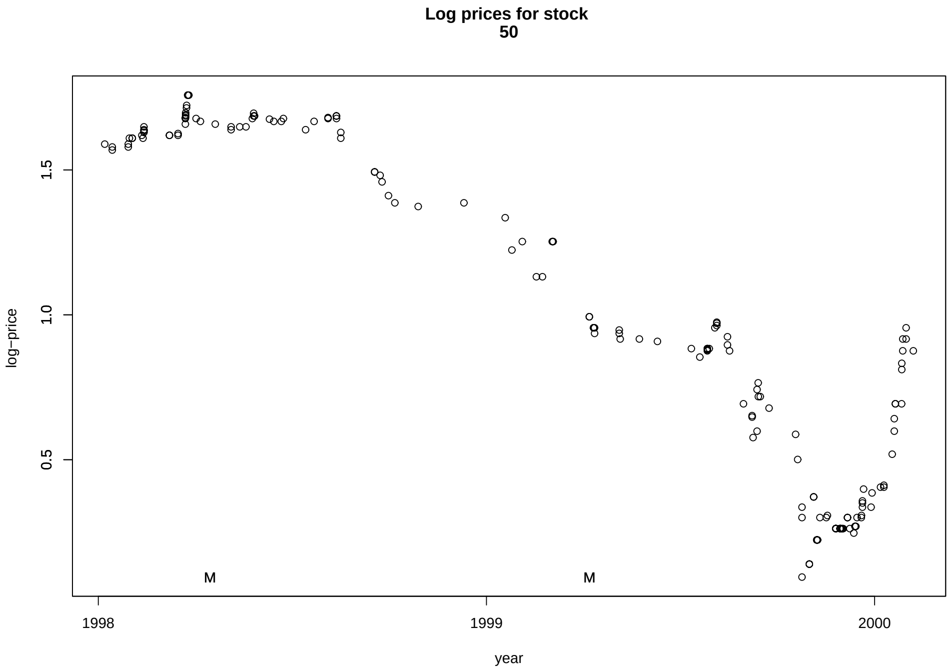

Figure 57: Log-prices for stock number 50

Stock number 50: A total of 163 transactions from 01/07/98 to 02/07/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0286	0.0285	0.0285	0.0285
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.0955	1.1000	1.1000	1.0994
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0002
$\hat{r}$	0	-0.0013	0	0.0001
log-likelihood	230.9019	231.5230	231.5230	231.5290
$skewness$	-0.9054	-1.1086	-1.1086	-1.1082
$kurtosis$	17.1322	17.9645	17.9644	17.9587

Table 53: Comparison of models for stock number 50

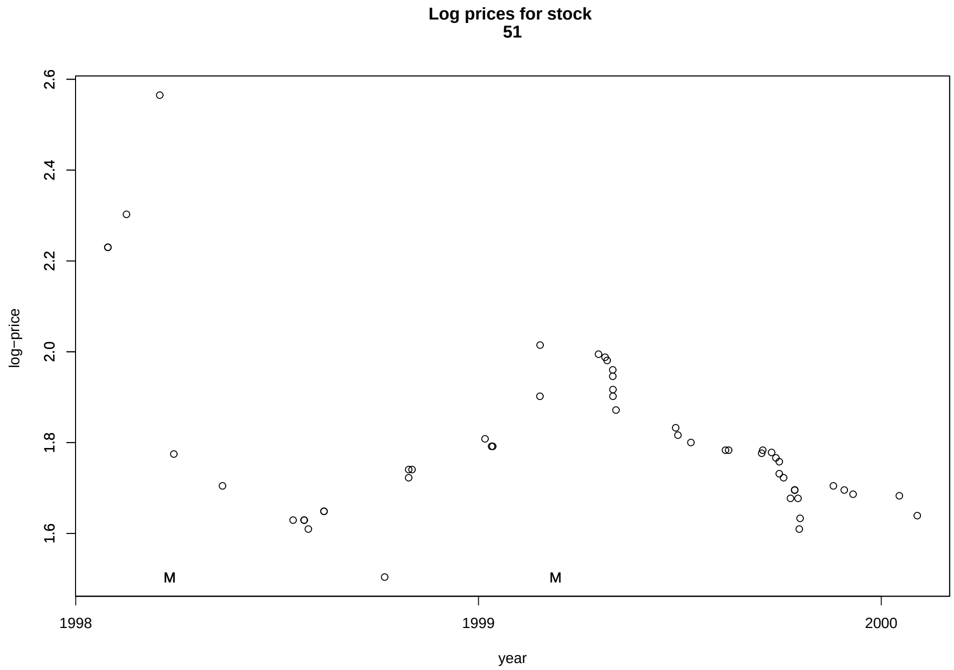

Figure 58: Log-prices for stock number 51

Stock number 51: A total of 52 transactions from 01/30/98 to 02/03/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0228	0.0228	0.0228	0.0228
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6191	0.6184	0.6184	0.6112
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	-0.0001	0	-1.1357
log-likelihood	78.7432	78.7628	78.7628	79.0241
$skewness$	0.6035	0.5420	0.5420	0.5124
$kurtosis$	6.6508	6.6222	6.6222	6.4655

Table 54: Comparison of models for stock number 51

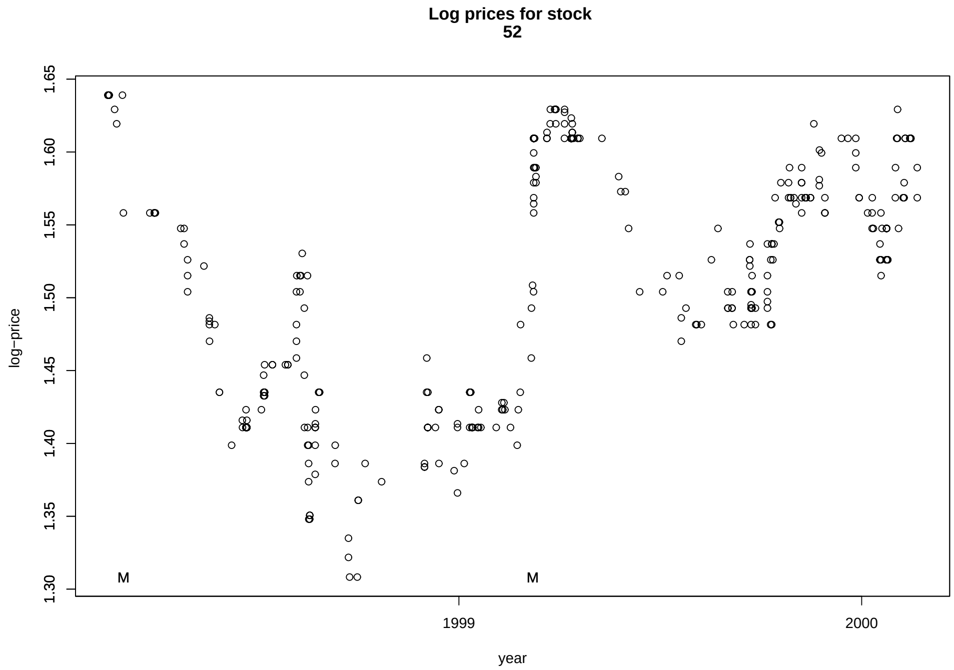

Figure 59: Log-prices for stock number 52

Stock number 52: A total of 321 transactions from 02/17/98 to 02/21/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0137	0.0137	0.0137	0.0137
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.8905	0.8889	0.8889	0.8801
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0046
$\hat{r}$	0	-0.0002	0	0.0003
log-likelihood	762.4140	762.4712	762.4712	762.8603
$skewness$	-0.4484	-0.4815	-0.4815	-0.4710
$kurtosis$	6.3665	6.3947	6.3949	6.3228

Table 55: Comparison of models for stock number 52

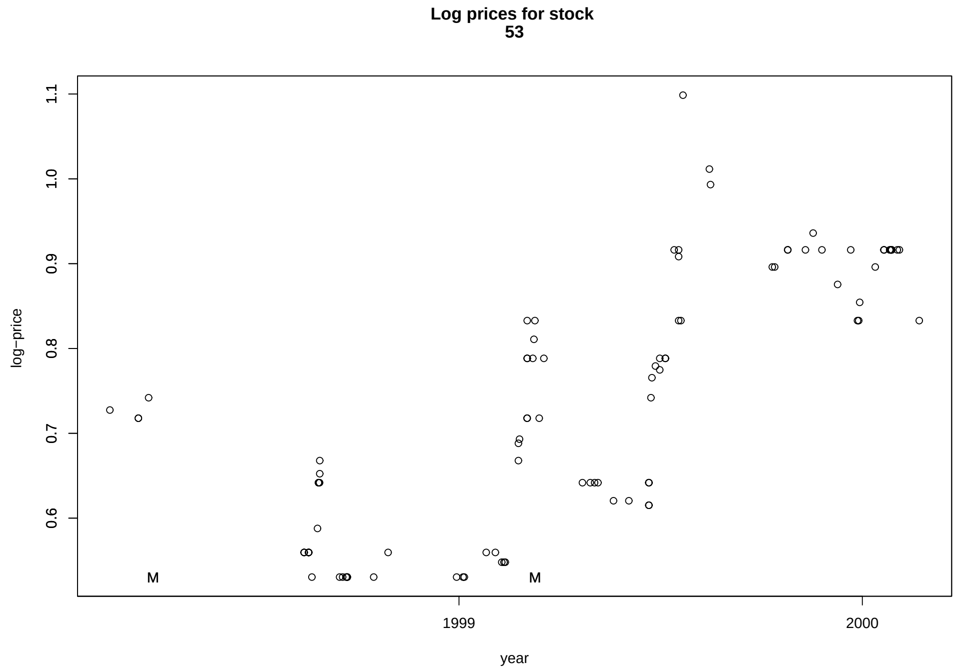

Figure 60: Log-prices for stock number 53

Stock number 53: A total of 93 transactions from 02/19/98 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0288	0.0288	0.0288	0.0290
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6512	0.6511	0.6511	0.6366
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0060
$\hat{r}$	0	0.0001	0	0.0006
log-likelihood	141.8471	141.8638	141.8638	142.1513
skewness	0.0859	0.1179	0.1179	0.1612
kurtosis	12.8210	12.5670	12.5670	12.2462

Table 56: Comparison of models for stock number 53

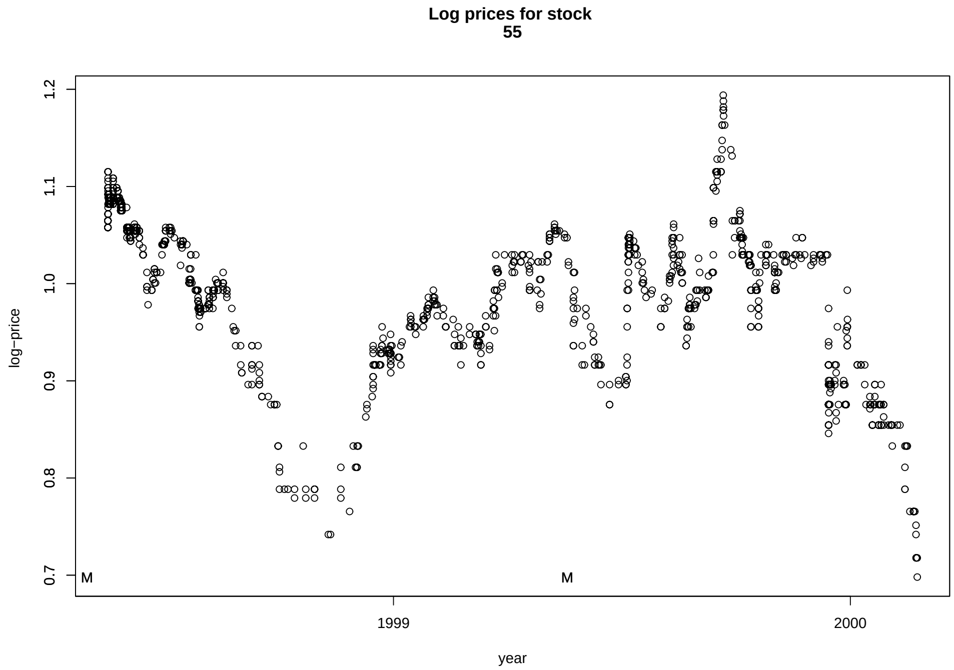

Figure 61: Log-prices for stock number 55

Stock number 55: A total of 1095 transactions from 05/18/98 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0098	0.0098	0.0098	0.0098
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.2865	2.2807	2.2807	2.2799
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0126
$\hat{r}$	0	-0.0006	0	-0.0005
log-likelihood	2985.9470	2986.2010	2986.2010	2986.2250
skewness	-0.1876	-0.2253	-0.2252	-0.2276
kurtosis	8.5240	8.5542	8.5542	8.5584

Table 57: Comparison of models for stock number 55

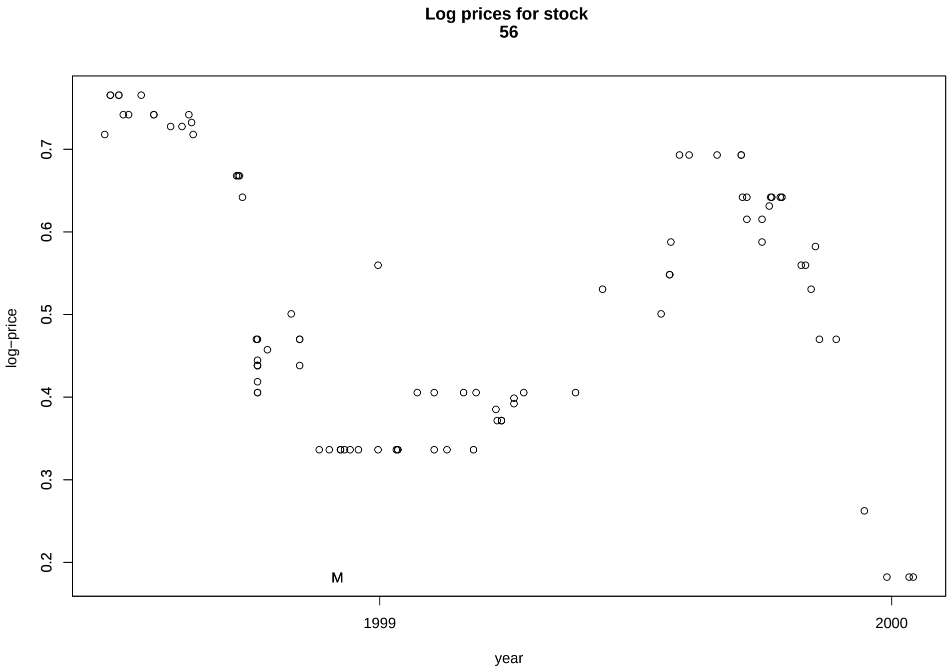

Figure 62: Log-prices for stock number 56

Stock number 56: A total of 92 transactions from 06/19/98 to 01/17/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0286	0.0288	0.0288	0.0288
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.6457	0.6251	0.6251	0.6251
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	-0.0009	0	-0.0005
log-likelihood	143.2473	144.0949	144.0949	144.0949
$skewness$	0.0446	-0.2151	-0.2150	-0.2149
$kurtosis$	9.4974	8.7073	8.7077	8.7077

Table 58: Comparison of models for stock number 56

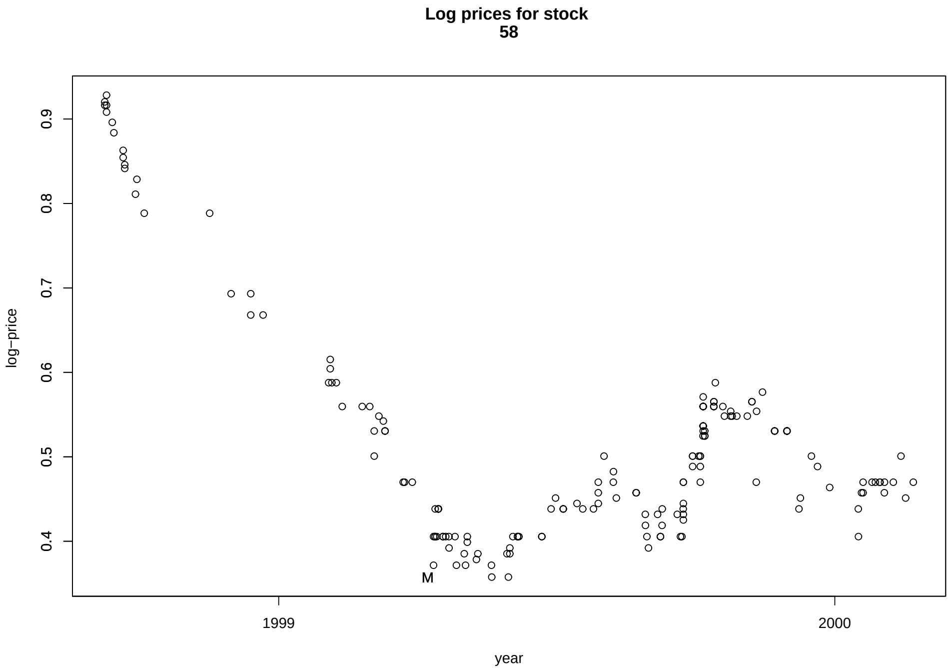

Figure 63: Log-prices for stock number 58

Stock number 58: A total of 166 transactions from 09/09/98 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0136	0.0138	0.0140	0.0141
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.8327	0.7989	0.7401	0.7193
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.5335	0.0000
$\hat{\gamma}$	0	∞	0	0.0059
$\hat{r}$	0	-0.0008	0	0.0005
log-likelihood	387.6714	388.9833	389.3403	391.7243
skewness	0.1505	-0.1012	-0.2013	-0.0774
kurtosis	3.8580	3.7828	4.0534	4.0953

Table 59: Comparison of models for stock number 58

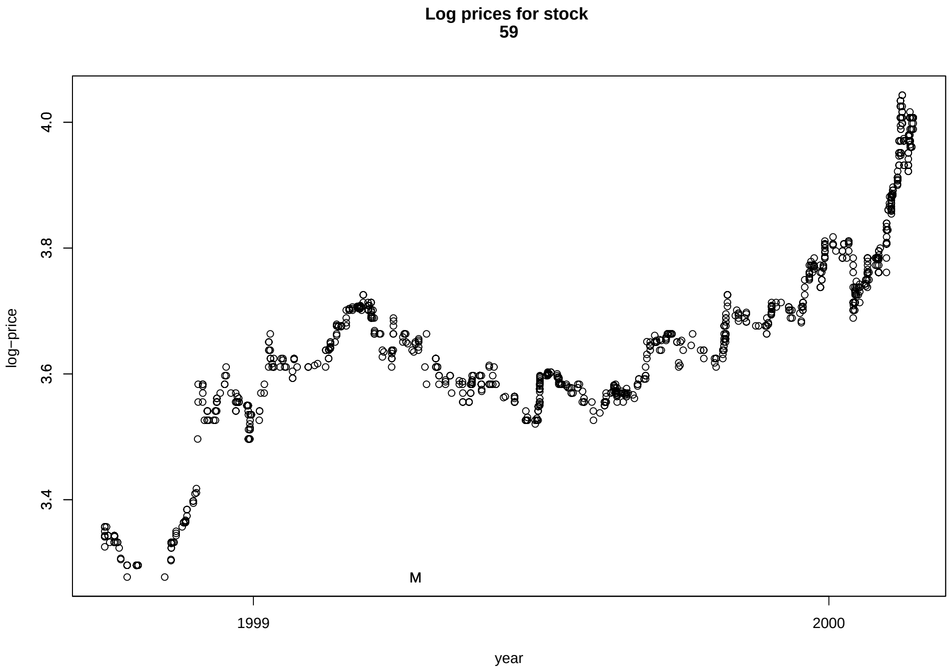

Figure 64: Log-prices for stock number 59

Stock number 59: A total of 966 transactions from 09/29/98 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0092	0.0092	0.0093	0.0092
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.6478	2.6478	2.6414	2.6433
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0152
$\hat{r}$	0	0.0002	0	0.0000
log-likelihood	2679.7280	2679.7430	2679.7510	2679.8120
$skewness$	-0.7293	-0.7209	-0.7145	-0.7169
$kurtosis$	9.1955	9.1606	9.1693	9.1576

Table 60: Comparison of models for stock number 59

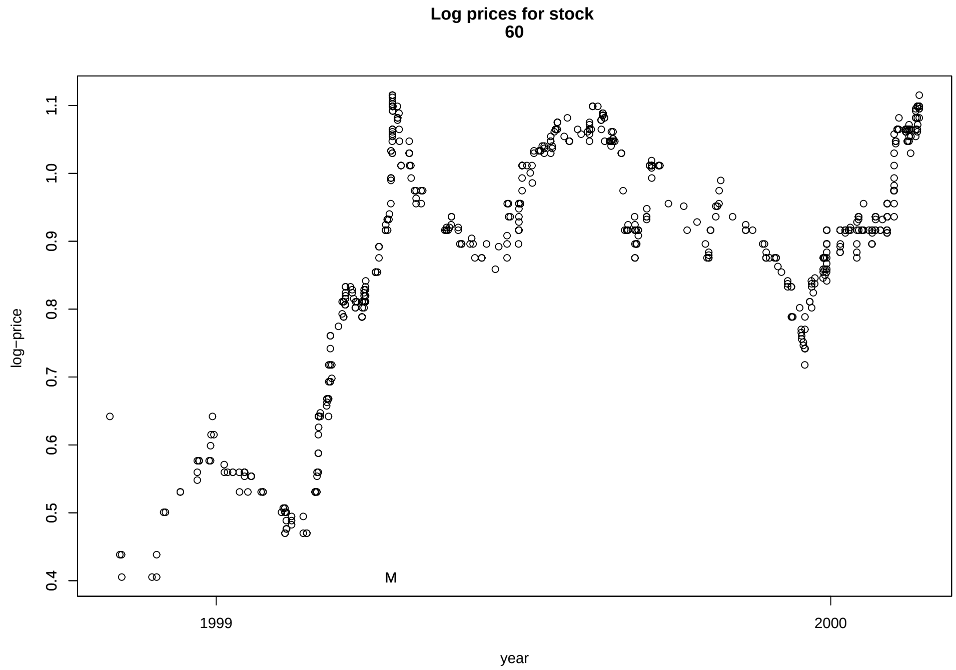

Figure 65: Log-prices for stock number 60

Stock number 60: A total of 594 transactions from 10/30/98 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0133	0.0133	0.0133	0.0135
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.5390	1.5350	1.5349	1.3992
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.2702
$\hat{r}$	0	0.0006	0	0.0012
log-likelihood	1446.1950	1446.4240	1446.4240	1455.5730
skewness	0.1661	0.2062	0.2062	0.2363
kurtosis	5.6853	5.6652	5.6652	5.2720

Table 61: Comparison of models for stock number 60

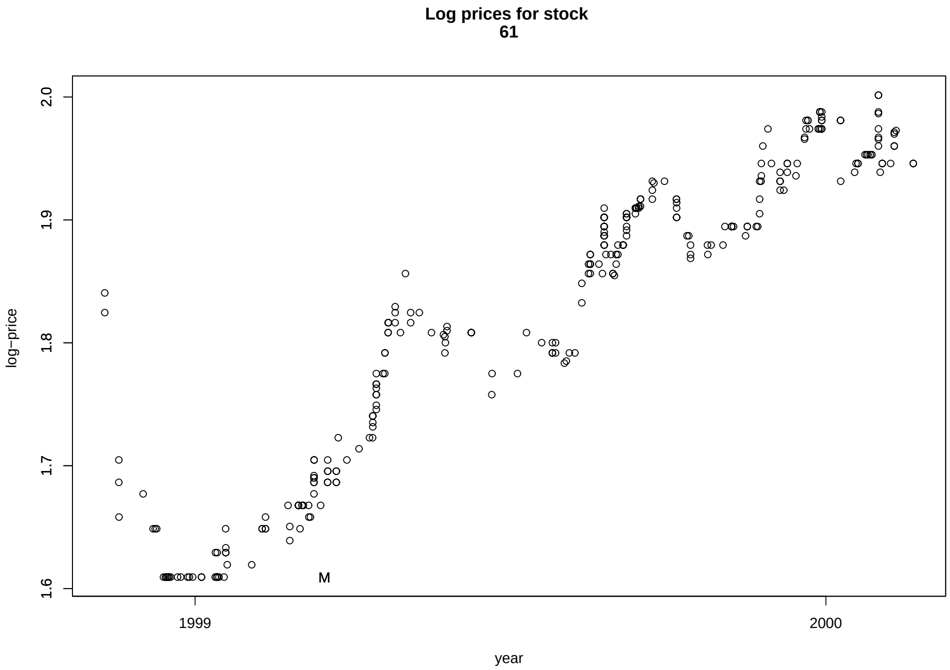

Figure 66: Log-prices for stock number 61

Stock number 61: A total of 299 transactions from 11/10/98 to 02/21/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0081	0.0081	0.0081	0.0082
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.2837	1.2837	1.1997	1.0401
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	2.7368	0.0000
$\hat{\gamma}$	0	∞	0	0.2287
$\hat{r}$	0	0.0002	0	0.0006
log-likelihood	862.3123	862.4025	862.5558	877.4117
skewness	-0.7200	-0.6677	-0.6911	-0.5709
kurtosis	7.5142	7.4394	6.7979	5.9771

Table 62: Comparison of models for stock number 61

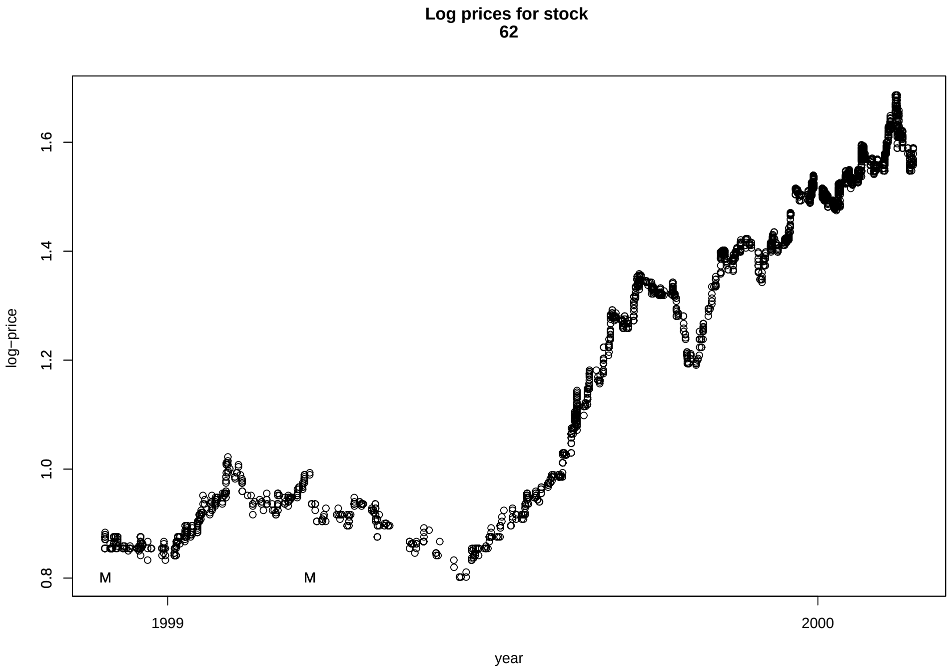

Figure 67: Log-prices for stock number 62

Stock number 62: A total of 2516 transactions from 11/27/98 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0053	0.0053	0.0053	0.0053
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	5.8970	5.8827	5.8826	5.8827
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0013	0	0.0010
log-likelihood	8485.0240	8485.4850	8485.4850	8485.4850
skewness	-0.5102	-0.4823	-0.4822	-0.4823
kurtosis	9.4002	9.3065	9.3064	9.3065

Table 63: Comparison of models for stock number 62

Figure 68: Log-prices for stock number 63

Stock number 63: A total of 3729 transactions from 11/27/98 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0043	0.0043	0.0043	0.0043
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	9.3914	9.3823	9.3824	9.3823
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0018	0	0.0002
log-likelihood	12927.3300	12927.7600	12927.7700	12927.7700
skewness	-0.9390	-0.9170	-0.9169	-0.9170
kurtosis	18.4659	18.3709	18.3701	18.3708

Table 64: Comparison of models for stock number 63

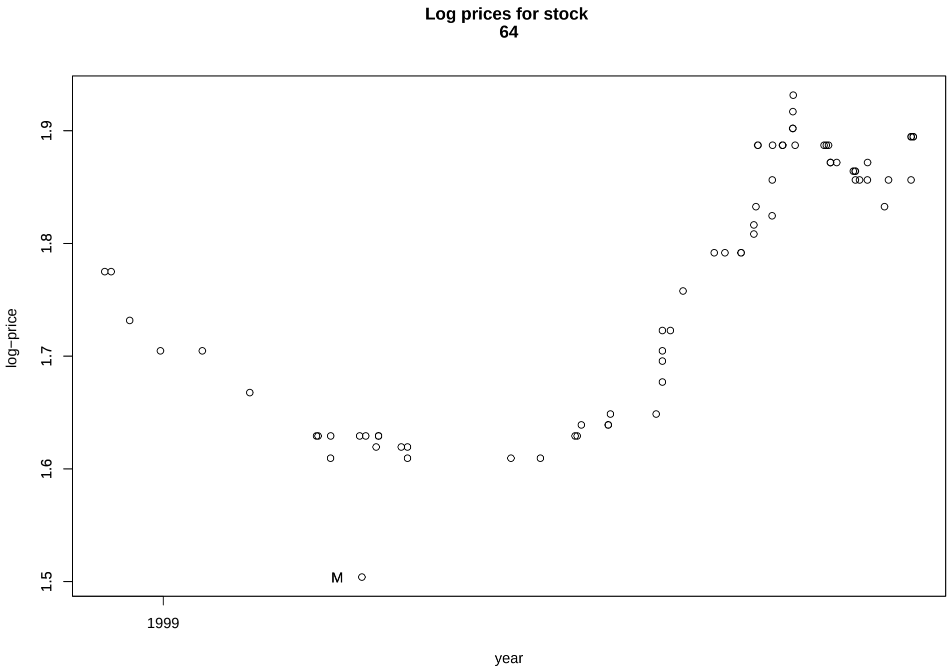

Figure 69: Log-prices for stock number 64

Stock number 64: A total of 75 transactions from 12/04/98 to 12/29/99

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0191	0.0191	0.0200	0.0199
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.4975	0.4975	0.3457	0.3593
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.6774	0.0001
$\hat{\gamma}$	0	∞	0	0.0063
$\hat{r}$	0	0.0002	0	0.0021
log-likelihood	158.8465	158.9612	160.3113	163.1084
$skewness$	-0.7838	-0.6703	-0.5935	-0.5058
$kurtosis$	9.3968	9.1515	7.3496	8.3500

Table 65: Comparison of models for stock number 64

Figure 70: Log-prices for stock number 65

Stock number 65: A total of 1510 transactions from 12/15/98 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0081	0.0081	0.0081	0.0081
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	5.7818	5.7683	5.7684	5.7683
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0025	0	0.0001
log-likelihood	4241.6400	4242.3540	4242.3540	4242.3540
skewness	0.6783	0.7194	0.7194	0.7194
kurtosis	13.7495	13.7198	13.7200	13.7198

Table 66: Comparison of models for stock number 65

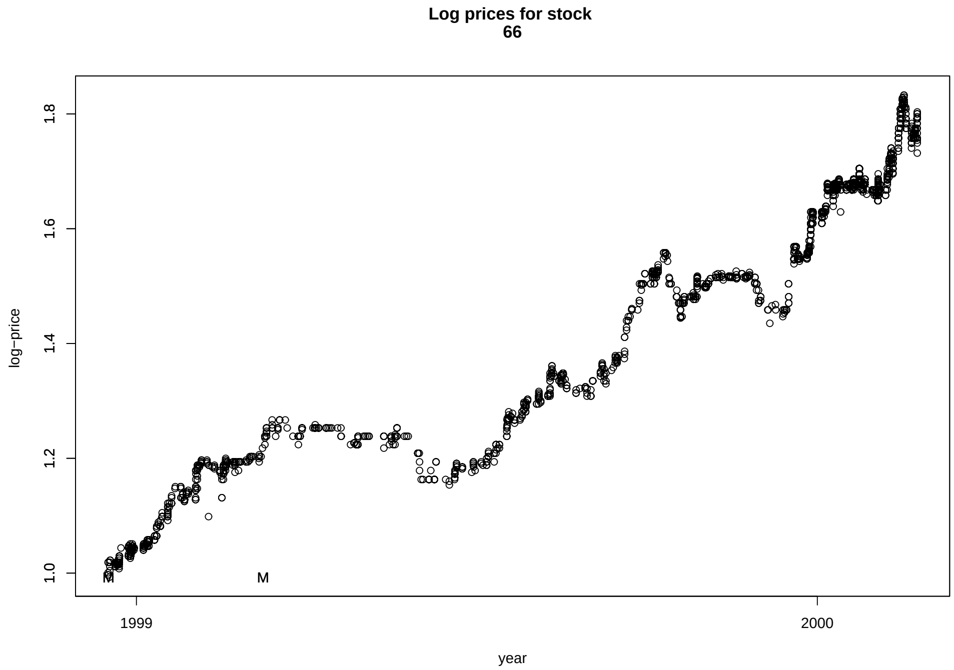

Figure 71: Log-prices for stock number 66

Stock number 66: A total of 1700 transactions from 12/17/98 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0046	0.0046	0.0046	0.0046
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	6.4524	6.4092	6.4093	6.4111
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0024
$\hat{r}$	0	0.0024	0	0.0001
log-likelihood	5748.5030	5750.2420	5750.2420	5750.3220
$skewness$	-0.8260	-0.7503	-0.7502	-0.7565
$kurtosis$	13.9958	13.5586	13.5579	13.5759

Table 67: Comparison of models for stock number 66

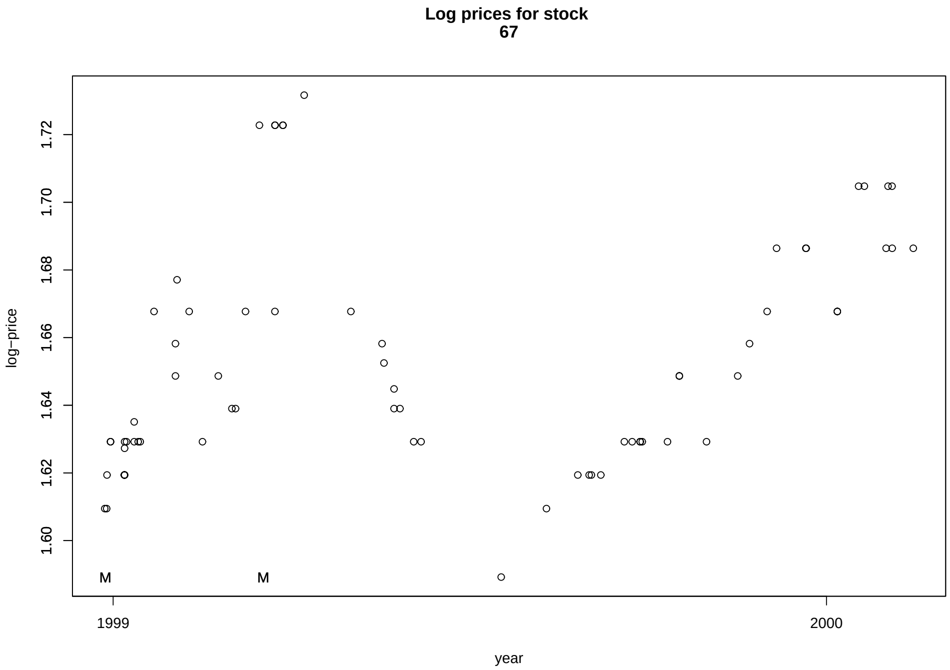

Figure 72: Log-prices for stock number 67

Stock number 67: A total of 71 transactions from 12/28/98 to 02/15/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0097	0.0097	0.0097	0.0097
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.4702	0.4702	0.4704	0.4480
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0000	0	0.2078
log-likelihood	191.8190	191.8209	191.8210	192.3043
skewness	-0.5736	-0.5569	-0.5576	-0.5523
kurtosis	6.4602	6.4387	6.4380	6.3476

Table 68: Comparison of models for stock number 67

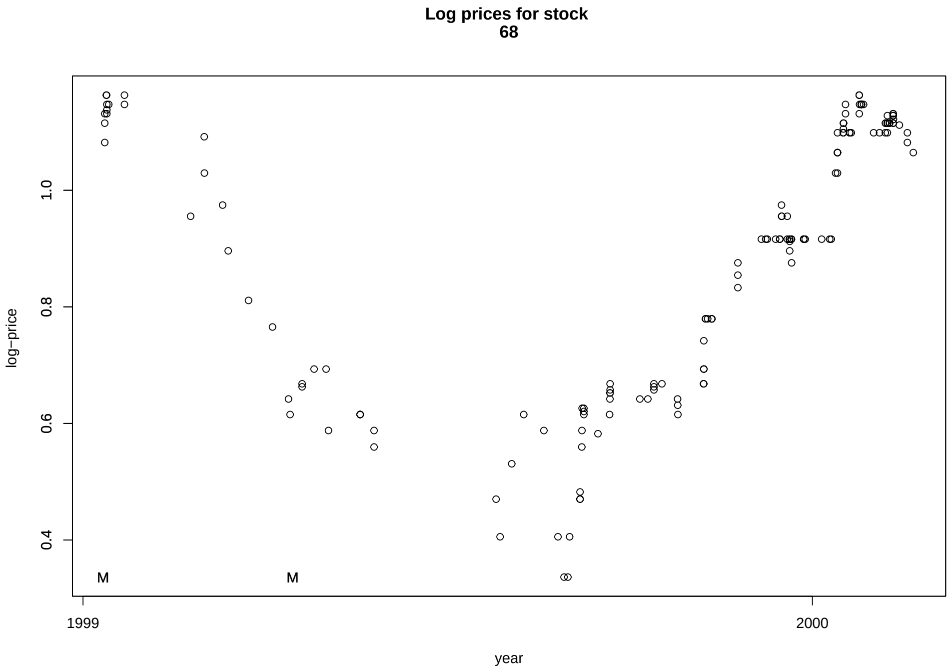

Figure 73: Log-prices for stock number 68

Stock number 68: A total of 153 transactions from 01/12/99 to 02/21/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0143	0.0143	0.0143	0.0144
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	2.2536	2.2536	2.2538	2.1647
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0017	0.0000
$\hat{\gamma}$	0	∞	0	0.0017
$\hat{r}$	0	-0.0001	0	0.0143
log-likelihood	304.1943	304.1987	304.1988	305.7892
skewness	-0.2747	-0.2894	-0.2895	-0.2072
kurtosis	4.5159	4.5342	4.5334	4.4676

Table 69: Comparison of models for stock number 68

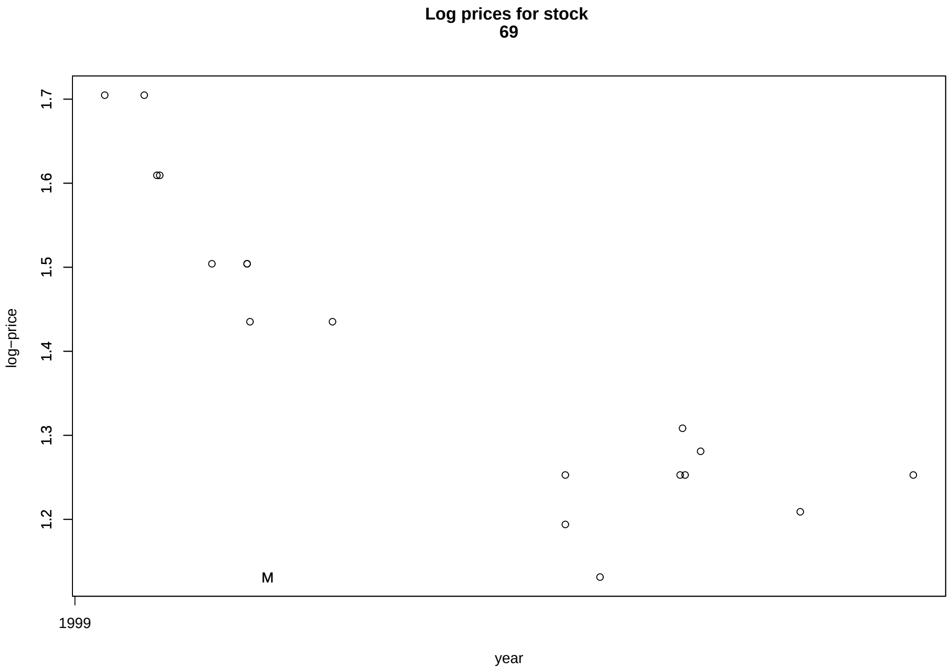

Figure 74: Log-prices for stock number 69

Stock number 69: A total of 18 transactions from 01/13/99 to 12/02/99

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0306	0.0311	0.0311	0.0443
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.5615	0.4656	0.4656	0.0000
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0056
$\hat{r}$	0	-0.0016	0	0.0019
log-likelihood	23.6386	25.3676	25.3676	30.5408
skewness	1.2965	0.1789	0.1789	-0.4214
kurtosis	3.3928	2.2294	2.2294	3.0548

Table 70: Comparison of models for stock number 69

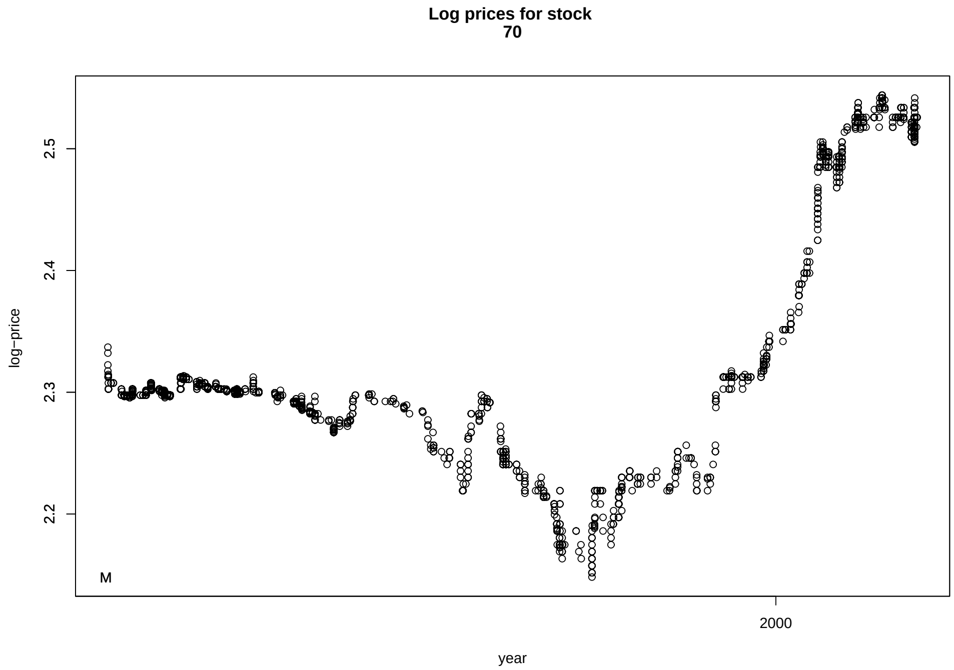

Figure 75: Log-prices for stock number 70

Stock number 70: A total of 1255 transactions from 04/28/99 to 02/23/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0039	0.0039	0.0039	0.0039
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	4.8358	4.8259	4.8260	4.8198
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	4.1080
$\hat{r}$	0	0.0006	0	0.0008
log-likelihood	4562.4970	4562.6360	4562.6360	4574.8270
$skewness$	0.6393	0.6595	0.6594	0.6620
$kurtosis$	9.1141	9.1119	9.1119	9.1084

Table 71: Comparison of models for stock number 70

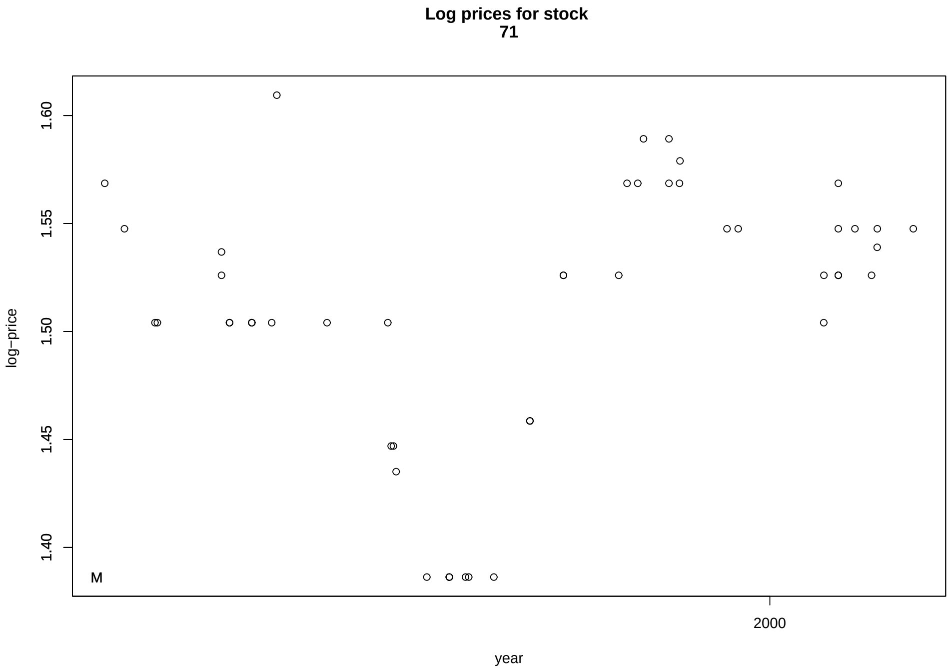

Figure 76: Log-prices for stock number 71

Stock number 71: A total of 48 transactions from 05/07/99 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0154	0.0153	0.0153	0.0151
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	0.7784	0.7895	0.7895	0.7707
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0786
$\hat{r}$	0	-0.0002	0	0.0002
log-likelihood	100.0445	100.0977	100.0977	101.1196
$skewness$	-0.0189	-0.1232	-0.1232	-0.0065
$kurtosis$	6.8873	7.2037	7.2037	7.3861

Table 72: Comparison of models for stock number 71

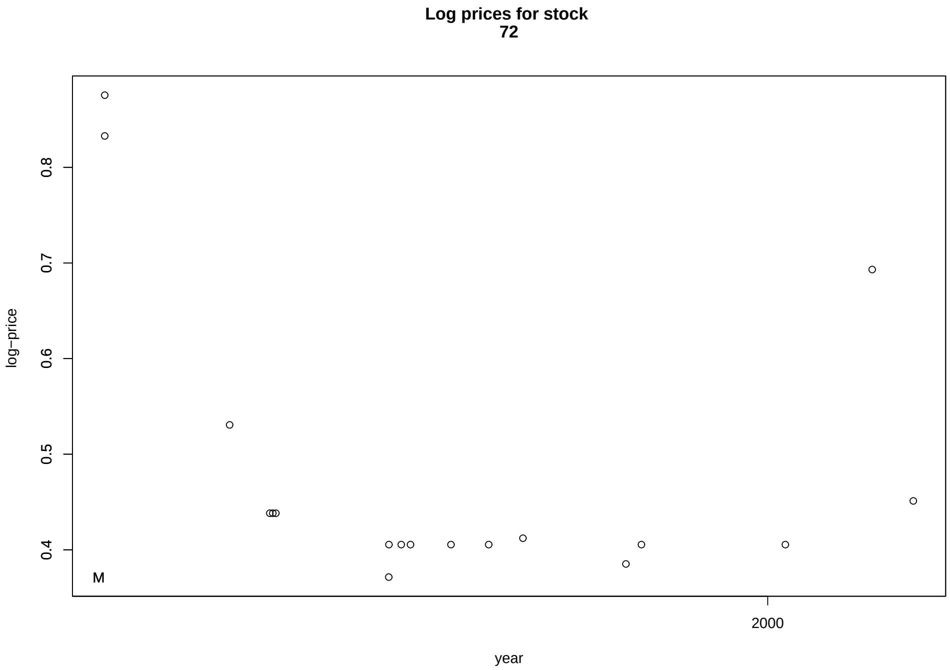

Figure 77: Log-prices for stock number 72

Stock number 72: A total of 18 transactions from 06/03/99 to 02/17/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0187	0.0186	0.0186	0.0194
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.4647	1.4240	1.4240	1.1370
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0241
$\hat{r}$	0	-0.0015	0	0.0008
log-likelihood	21.5909	22.0036	22.0036	24.1787
skewness	0.3644	-0.1570	-0.1570	-0.0983
kurtosis	3.5807	3.5639	3.5639	5.6486

Table 73: Comparison of models for stock number 72

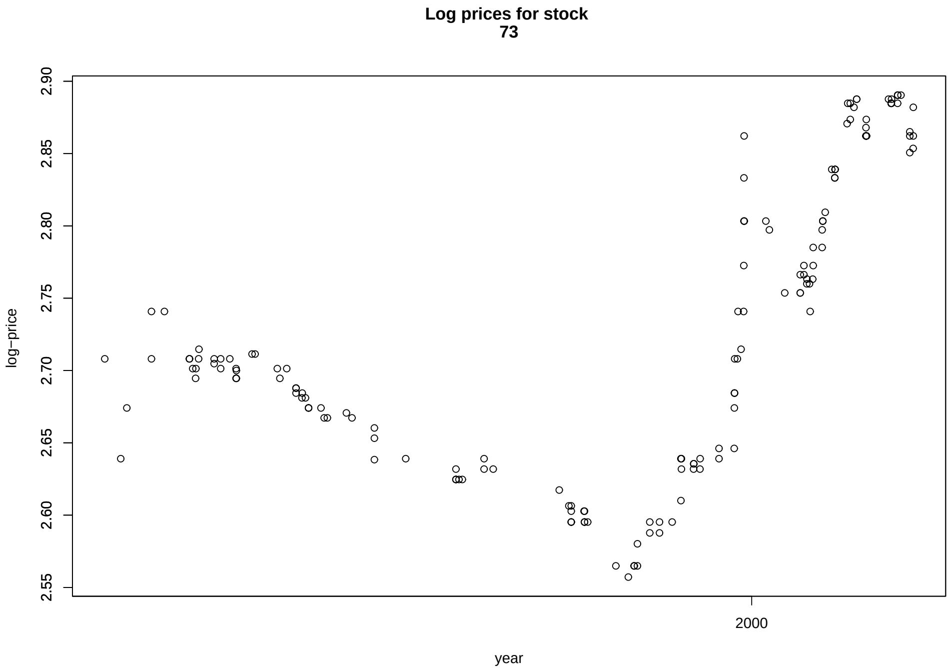

Figure 78: Log-prices for stock number 73

Stock number 73: A total of 153 transactions from 06/09/99 to 02/22/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0123	0.0123	0.0123	0.0123
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	1.0919	1.0824	1.0824	1.0499
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x 10^2$	0	0	0.0002	0.0002
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0006	0	0.7000
log-likelihood	379.4526	379.7496	379.7496	380.9638
$skewness$	1.1309	1.2004	1.2004	1.2332
$kurtosis$	13.1938	12.8699	12.8702	12.8317

Table 74: Comparison of models for stock number 73

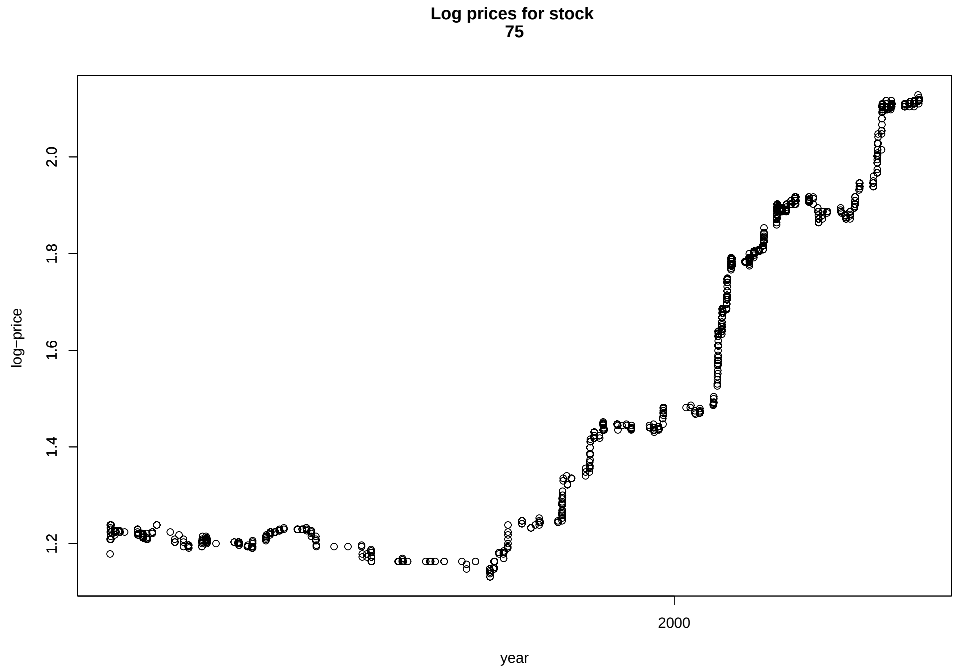

Figure 79: Log-prices for stock number 75

Stock number 75: A total of 901 transactions from 08/31/99 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0043	0.0043	0.0043	0.0043
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	9.2571	9.1718	9.1718	9.1718
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0052	0	0.0001
log-likelihood	3024.7920	3026.3080	3026.3080	3026.3080
$skewness$	-0.1993	-0.1140	-0.1140	-0.1140
$kurtosis$	7.0507	6.8824	6.8824	6.8824

Table 75: Comparison of models for stock number 75

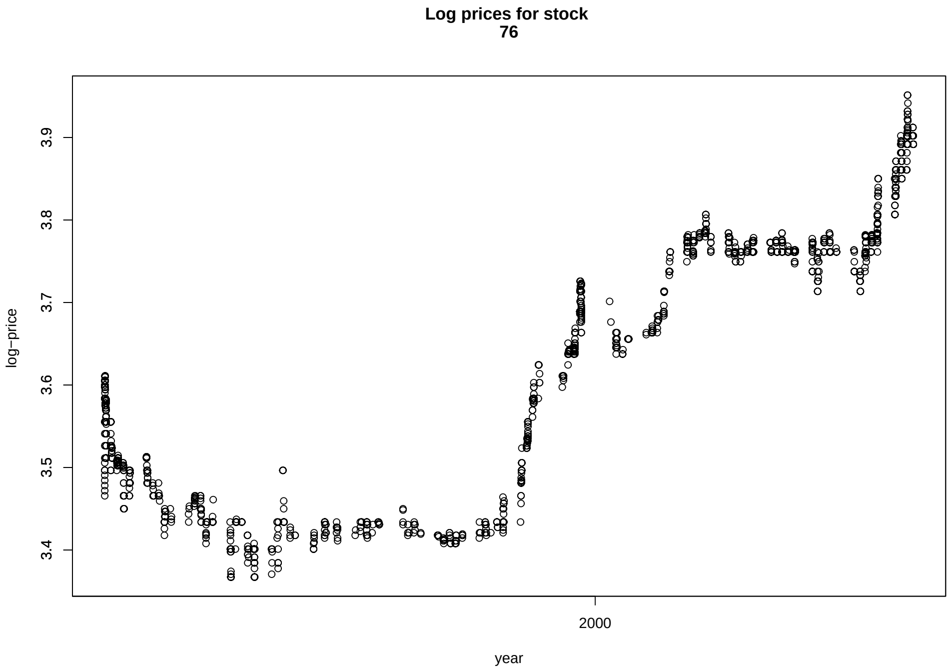

Figure 80: Log-prices for stock number 76

Stock number 76: A total of 1305 transactions from 10/11/99 to 02/24/00

	M_1	M_2	M_3	M_4
$\hat{\sigma}_\varepsilon$	0.0073	0.0073	0.0073	0.0073
$\hat{\sigma}_1/\hat{\sigma}_\varepsilon$	11.4990	11.4944	11.4944	11.4944
$\hat{\sigma}_2/\hat{\sigma}_\varepsilon x10^2$	0	0	0.0000	0.0000
$\hat{\gamma}$	0	∞	0	0.0000
$\hat{r}$	0	0.0029	0	0.0001
log-likelihood	3833.0180	3833.0990	3833.0990	3833.0990
skewness	0.3908	0.4042	0.4042	0.4043
kurtosis	9.7943	9.7852	9.7852	9.7851

Table 76: Comparison of models for stock number 76

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