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Capital Investment and Equilibrium Unemployment

By

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February 2013

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Capital Investment and Equilibrium Unemployment*

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February 2013

Abstract

Econometric analysis of cross-country data reveals a robust long-term relationship between capital investment and unemployment. This paper studies this relationship within a search and matching model of the labor market. In the model developed, firms employ labor for two purposes: for production of a final good and for production of capital. Quantitative analysis shows that an increase in growth of capital-production technology increases capital formation and employment in capital production, reducing unemployment in equilibrium. The model is therefore successful in generating the negative long-run investment-unemployment relationship found in macroeconomic data.

Keywords: Search and matching model, Unemployment, Investment, Technology growth
JEL Classification: E22, J23, J64, O30

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1 Introduction

Empirical research has revealed a robust medium to long-term negative relationship between unemployment and capital investment. This relationship, labeled the "Modigliani puzzle" in Blanchard (2000), referring to the findings presented in Modigliani (2000), has been the focal point in a recent but growing empirical literature. The results of this line of research indicates that investment in capital affects how unemployment evolves over the long term.

Several studies have explored the negative relationship between unemployment and investment using macroeconomic data.[1] Herbertsson & Zoega (2002) estimate this relationship using data for a panel of OECD countries. Their results show the relationship to be statistically significant and robust both over time and across countries. Furthermore, they account for the effects of labor market institutions on unemployment, a factor much emphasized in the literature on medium-term unemployment changes, but conclude that although taking theses institutional factors into account, investment comes through as one of the most important determinants of movements in unemployment. Karanassou et al. (2003) and Karanassou et al. (2004) find that decline in capital formation to be essential for understanding the unemployment experience within the European Union in the 1970s and 1980s. Similarly, Smith & Zoega (2009) conclude that investment has been the driving force of unemployment in the OECD countries since the 1960s. Karanassou et al. (2008) assess the investment-unemployment relationship using data for the Nordic countries, finding similar results.

Although this robust negative relationship has been explored in data, the explicit relationship has not been given much attention in theoretical modelling. A notable exception is Phelps (1994). In the book *Structural Slumps* he presents three models where investment affects the natural rate of unemployment. In the fixed-investment model, there are two production sectors: a capital-intensive sector that produces the consumption good and a labor-intensive sector which produces the capital-good. Efficiency wage is included in order to generate positive unemployment in equilibrium. In the model, an increase in the relative price of the capital-good raises both the real wage and employment in equilibrium. Although different in many aspects, the general idea of the fixed-investment model motivates how the relationship between capital investment and equilibrium unemployment is developed within the model presented in the current paper.

The model of search and matching has come to play a role as the workhorse

[1] Karanassou et al. (2008) provide a review of the studies exploring this empirical relationship.

for macroeconomic analysis of the labor market. This paper develops a search and matching model in the spirit of Diamond-Mortensen-Pissarides, as described in Pissarides (2000), but which is successful in generating a negative long-term relationship between capital formation and equilibrium unemployment. Previous exploration of this relationship in a context of a search and matching model is not known to the author. In the model developed, firms use both capital and labor for production of the final good and are large in the sense that they employ more than one worker. Firms carry out two production processes: firms employ labor for production of capital and for final-good production.

After a quantifiable steady-state growth model is developed, I calibrate the model and perform a comparative static analysis. Under reasonable parameter values, increased growth of technology in capital-production increases employment in the capital-production sector but decreases employment in the sector producing the final good. The overall effect is a reduction in equilibrium unemployment rate and increases capital formation, hence a negative investment-unemployment relationship. This negative relationship is robust to addition of growth in aggregate productivity and selection of parameter values.

To some economists a relationship between equilibrium unemployment and investment may be intuitive, or even obvious. However, authors of seminal contributions to labor market theory, notably Layard et al. (1991), have argued that capital investment does not affect long-term unemployment. In Layard et al. (1991), capital accumulation and technology change, which increase efficiency, are translated into higher wages bargained by unions. With an assumption of a Cobb-Douglas production function with unit elasticity of substitution between capital and labor, their framework ensures that the wage rises absorbing all the efficiency gains translate into increased investment in capital but unaffected equilibrium unemployment.[2] The present paper contributes to the literature on the investment-unemployment relationship by developing a framework that incorporates channels through which technology growth increases both capital investment and employment in equilibrium.

The remainder of the paper is organized as follows. Section 2 provides a motivation for the theoretical modelling that follows by presenting empirical evidence on the relationship between unemployment and investment using macroeconomic data for the OECD countries. Section 3 develops the model. In Section 4, the model is calibrated and a numerical steady-state equilibrium solution is presented as well as results from a comparative static analysis. Section 5 concludes.

[2]In the current paper, increased technology growth increases the bargained wage but the structure of the model generates a decrease in equilibrium unemployment.

2 The investment-unemployment relationship

This section presents empirical evidence on the stylized relationship between investment and unemployment using macroeconomic data for several OECD countries.

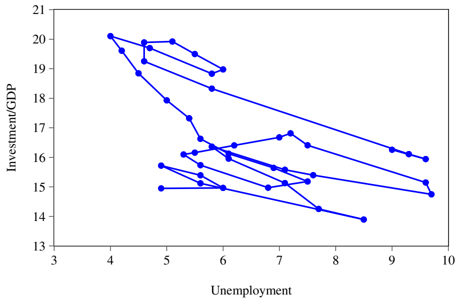
Figures 1 and 2 plot the relationship between unemployment and investment, defined as capital formation as the share of GDP, for twelve OECD countries over the period 1970-2011. The plots illustrate the stylized relationship between the two variables. In all cases a clear negative relationship can be detected, showing what appears to be a long-term property. Furthermore, this negative relationship is very stable over the sample period. The simple correlation between the two series ranges from -0.42 in Sweden to -0.91 in Finland with the mean correlation of about -0.67. The generality and stability of this long-term relationship seems similar to the Beveridge curve, a robust negative relationship between vacancies and unemployment.

To test for the empirical significance of the relationship between unemployment and investment, I perform a panel regression using data for 15 OECD countries over a period of 42 years, from 1970 to 2011.[3] I use data on unemployment, u_{it}, where i denotes a country, i = 1,2, ... , 15, and t denotes years, t = 1970, 1971 , ... , 2011, Δq_{it} denotes the growth rate of labor productivity, and I_{it} denotes investment measured as the ratio of gross domestic capital formation to GDP. In order to account for omitted country country-specific variables a country fixed effects are included. The following equation is estimated:

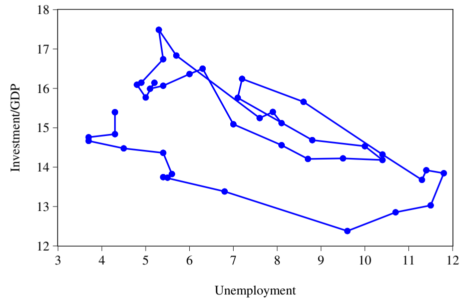
$$u_{it} = \alpha_i + \beta \Delta q_{it} + \gamma I_{it} + \epsilon_{it} \tag{1}$$

The estimation results are presented in Table 1. The search and matching literature has emphasized the role of productivity growth as a driving force of changes in unemployment, see *e.g.* Pissarides (2000). In the first column we assess this relation between growth in productivity and unemployment. The coefficient on productivity growth is negative and statistically significant. According to the results, a one percentage increase in productivity leads unemployment to fall by about a quarter of a percentage point. The second column reports and estimate for the relationship between investment and unemployment. The results show a negative and statistically significant relation and that the coefficient has a greater numerical value than the coefficient in the regression on productivity

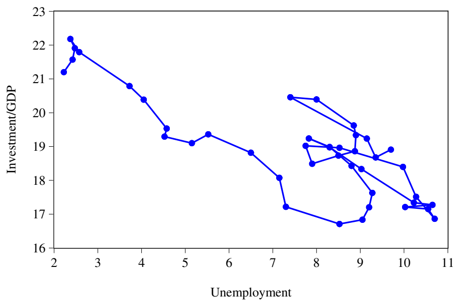
[3]This analysis uses data for 15 OECD countries which there were available data for the whole period 1970-2011. Those countries are: Australia, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, New Zealand, Norway, Spain, Sweden, United Kingdom, and United States.



(a) United States

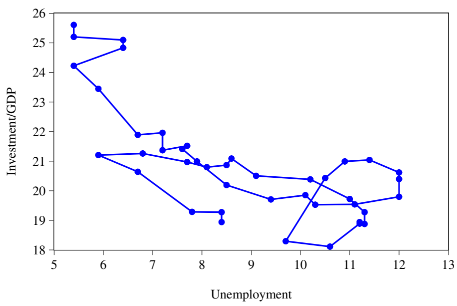


(b) United Kingdom

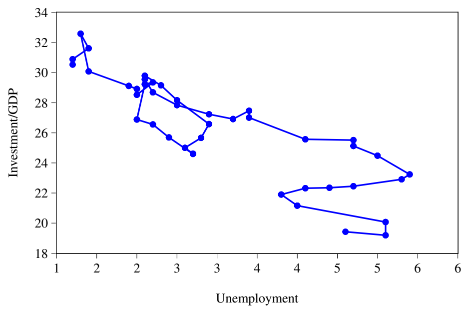


(c) France

(d) Germany



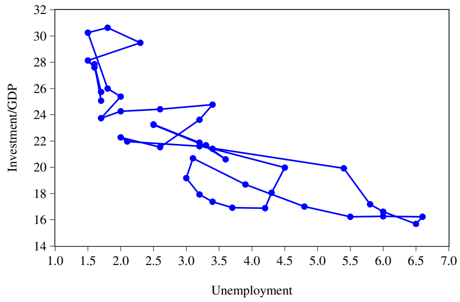
(e) Italy



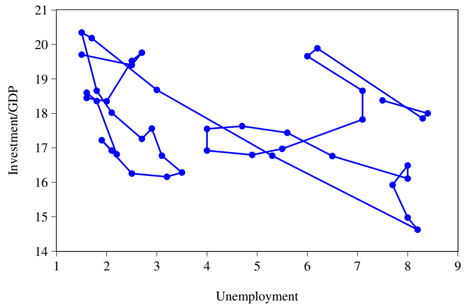
(f) Japan

Figure 1: Unemployment and Investment

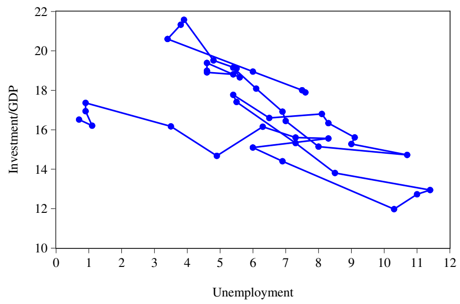
Notes: Data for the period 1970-2011. Investment is gross domestic capital formation as a share of GDP. Unemployment is the rate of unemployed workers to the labor force.
Sources: Macrobond, OECD Statistics.



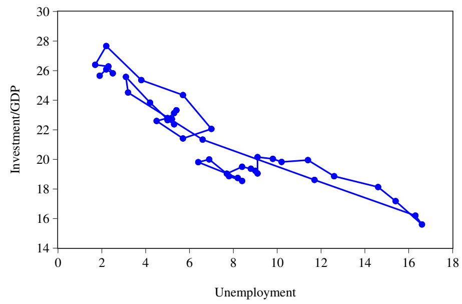
(a) Norway



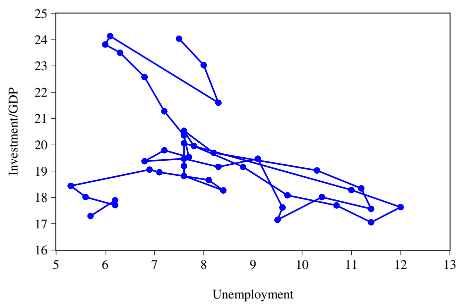
(b) Sweden



(c) Denmark



(d) Finland



(e) Canada

(f) Netherlands

Figure 2: Unemployment and Investment

Notes: Data for the period 1970-2011. Investment is gross domestic capital formation as a share of GDP. Unemployment is the rate of unemployed workers to the labor force.
Sources: Macrobond, OECD Statistics.

Table 1: Unemployment, productivity growth and investment

	(1)	(2)	(3)
Labor productivity growth	-0.268^*	-	-0.202^*
	(0.132)	-	(0.091)
Investment	-	-0.524^*	-0.512^*
	-	(0.051)	(0.055)
Country fixed effects	Yes	Yes	Yes
Observations	630	630	630
Adjusted R^2	0.46	0.56	0.57

Notes: Panel estimation with country fixed effects. Standard errors are in parenthesis. White cross-section standard errors & covariance. * denotes significance at the 5% level.

growth: an increase in investment by one percentage point will decrease unemployment by more than half a percentage point. The estimate reported in the final column accounts for both effects from investment and productivity growth, which both do negatively affect unemployment.

To summarize, although productivity growth is an important determinant of the evolution of the unemployment rate, investment comes through as an even more important factor in influencing the long-term changes in unemployment.

3 The model

Consider an economy which consists of a large number of homogeneous firms and a continuum of workers of a unit measure. Firms are large, in the sense that they can employ more than one worker. Workers are homogeneous from the perspective of their productivity. All agents in the economy are infinitely lived and maximize the present discounted value of their income stream with the rate of discount r. We denote the discount factor as $\beta = \frac{1}{1+r}$.

There are frictions in the process of search and matching in the labor market. We describe the outcome of workers' and firms' search, *i.e.* the number of matches $\mathcal{M}_t$, with a matching function where unemployed workers and vacancies match randomly:

$$\mathcal{M}_t(u_t, v_t) = \Omega u_t^{\xi} v_t^{1-\xi} \tag{2}$$

where u_t is the total number of unemployed workers at a given time, v_t is the number of vacancies, Ω is match efficiency and $0 < \xi < 1$ is the elasticity of matches with respect to unemployment. The choice of a Cobb-Douglas functional form is a standard form for the matching function in the literature. The tightness in the labor market is denoted as $\theta_t = \frac{v_t}{u_t}$. The probability of filling a vacancy is the same for all firms. The rate at which firms fill vacant positions is:

$$\frac{\mathcal{M}_t}{v_t} = q(\theta_t) \tag{3}$$

and the rate at which workers find jobs is:

$$\frac{\mathcal{M}_t}{u_t} = \theta_t q(\theta_t) \tag{4}$$

In their decision making, firms and workers take the probabilities of vacancy-filling and job-finding as given.

3.1 Firms

Firms produce a final good, y_t, using both capital and labor. We describe the production technology with a constant elasticity of substitution (CES) production function:

$$y_t = A\Big[\alpha k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_t^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

where A is aggregate productivity, $0 < \alpha < 1$ is the share of capital in production and $0 < \sigma < 1$ is the elasticity of substitution between capital and labor.

Capital accumulates according to:

$$k_{t+1} = (1-\delta)k_t + j_t \tag{6}$$

where δ is the constant rate of capital depreciation. A distinguishing feature of the model is the production of the capital good. In this model, firms engage in production of the final good y_t, but also in a labor-intensive production of the capital good k_t, where the capital good is used in the final-good production. We specify a process for production of the capital good where the output from capital production adds j_t to the stock of capital each period. Production of capital can be described with the following function:

$$j_t = \Phi_t m_t^{\pi} \tag{7}$$

where m_t is the amount of labor, or the share of labor force, employed in capital production and $\pi \leq 1$ is the return to labor in capital production. The state of technology in capital production is denoted with Φ_t. We assume that the evolution of technology is governed by an exogenous process where g is the growth rate:

$$\Phi_t = \Phi e^{gt} \tag{8}$$

where Φ is the initial level of technology. We assume that $g < r$, *i.e.* that the rate of growth is less than the rate of time discount, in order to ensure that the steady-state equilibrium and balanced growth we come to later will be well defined.

The total number of employed workers is denoted n_t. Since the labor force is normalized to one the unemployment rate is the following:

$$u_t = 1 - n_t \tag{9}$$

As we have described in equations (5) and (7), firms employ workers for two purposes: final-good production and production of a capital good. Labor is assumed perfectly substitutable between the two different uses:

$$n_t = l_t + m_t \tag{10}$$

and firms allocate labor in order to ensure its optimal usage. For simplicity, we assume that workers employed in one production sector in period t can be reallocated to the other production sector in period $t+1$ without any cost or delay.

The law of motion for employment is:

$$n_{t+1} = (1-\lambda)n_t + v_t q(\theta_t) \tag{11}$$

where λ is the exogenous rate at which workers separate from jobs. The costs of posting new vacancies are assumed to be convex in vacancies:[4]

$$C(v_t) = \frac{\psi v_t^2}{2} \tag{12}$$

where $\psi > 0$ is a parameter.

The firms' dynamic programming problem is to maximize expected profits by controlling the number of vacancies they post, how many workers to employ in capital production and by that the amount of capital it uses for final production.

[4]Convex vacancy-posting costs are assumed in many recent models, for example in Acemoglu & Hawkins (2010)

The dynamic programming problem is described with the following Bellman equation:

$$V_t = \max_{v_t, m_t} \left\{ A\Big[\alpha k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_t^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}} - w_t(l_t + m_t) - \frac{\psi v_t^2}{2} + \beta V_{t+1} \right\} \tag{13}$$

subject to:

$$n_t = l_t + m_t$$

$$n_{t+1} = (1-\lambda) n_t + v_t q(\theta_t)$$

$$k_{t+1} = (1-\delta) k_t + \Phi_t m_t^{\pi}$$

where V_t is the period t value of firms.

The first-order conditions to the dynamic programming problem are:

$$-\psi v_t + \beta V_{n,t+1} q(\theta_t) = 0 \tag{14}$$

$$-w_t + \beta V_{k,t+1} \pi \Phi_t m_t^{\pi-1} = 0 \tag{15}$$

where $V_{n,t+1}$ and $V_{k,t+1}$ denote, respectively, the derivative of the value function with respect to labor and capital, evaluated at period $t+1$.

The envelope condition for capital, corresponding to the dynamic programming problem, is:

$$V_{k,t} = A\Big[\alpha k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_t^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{1}{\sigma-1}} \alpha k_t^{-\frac{1}{\sigma}} + \beta V_{k,t+1}(1-\delta) \tag{16}$$

which describes the value of marginal capital to firms. Using equation (10), we similarly derive an envelope condition for labor:

$$V_{n,t} = A\Big[\alpha k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_t^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{1}{\sigma-1}} (1-\alpha) l_t^{-\frac{1}{\sigma}} - w_t + \beta V_{n,t+1}(1-\lambda) \tag{17}$$

describing the value of marginal labor to firms.

3.2 Workers

In this economy, workers can be either employed or unemployed. If a worker is unemployed he searches for a job and enjoys some real return z. When employed the worker receives a wage w_t. The value of an employed worker, W_t, satisfies the following Bellman equation.:

$$W_t = w_t + \beta[\lambda U_{t+1} + (1-\lambda)W_{t+1}] \tag{18}$$

The value of unemployed workers, searching for a job, is described with the following equation:

$$U_t = z + \beta[\theta_t q(\theta_t)W_{t+1} + (1 - \theta_t q(\theta_t))U_{t+1}] \tag{19}$$

3.3 Nash wage bargaining

Wages are determined by Nash bargaining between firms and workers over a share of the expected future surplus from the match. Wages are a solution to the following bargaining problem:

$$w_t = \arg\max_{w_t} \Big(W_t - U_t\Big)^{\gamma} \Big(V_{n,t}\Big)^{(1-\gamma)} \tag{20}$$

where γ is the workers' bargaining power, according to which the surplus is shared. From the first-order condition associated with the bargaining problem we get the following *wage-sharing rule*:

$$W_t - U_t = \gamma\Big(V_{n,t} - U_t + W_t\Big) \tag{21}$$

3.4 Steady-state equilibrium

In previous sections I have outlined the general structure of the model. As emphasized in Sections 1 and 2, we are interested in developing a model with channels that generate a long-term relation between investment and unemployment. Therefore our main focus will be on the model's steady state. The steady-state we derive is symmetric, in the sense that all firms will have the same level of technology. Furthermore, in equilibrium, all variables grow at the constant rate of technology g, hence a balanced growth equilibrium.

Along the balanced growth path the period gain in value of capital to the firm, $V_{k,t+1} - V_{k,t}$, is gV_k. Similarly the gain in value of employment is gV_n and the gain in value of employment and unemployment to workers is gW and gU, respectively. In order to ensure the existence of balanced growth path we assume that the exogenous variables of the model, representing the return enjoyed when unemployed, z, and cost of posting vacancies, $C(v)$, grow at the same rate as technology, g.[5]

[5] In the literature assessing the effect of productivity growth on unemployment, the existence of a balanced growth path is ensured by assuming that all the exogenous variables grow at the same rate as productivity. See, for example, Pissarides & Vallanti (2007) and Miyamoto (2010).

Following the above we are able to rewrite the value equations (16), (17), (18), and (19) as follows:

$$(1-\beta(1+g)(1-\delta))V_k = A\Big[\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{1}{\sigma-1}}\alpha k^{-\frac{1}{\sigma}} \tag{22}$$

$$(1-\beta(1+g)(1-\lambda))V_n = A\Big[\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{1}{\sigma-1}}(1-\alpha)l^{-\frac{1}{\sigma}} - w_t \tag{23}$$

$$(1-\beta(1+g))W = w + \beta\lambda[U-W] \tag{24}$$

$$(1-\beta(1+g))U = \Phi z + \beta\theta q(\theta)[W-U] \tag{25}$$

We are now ready to define a steady-state balanced growth equilibrium and derive the equilibrium equations.

Definition 1. *A steady-state balanced growth equilibrium is the tuple:*

$$\langle V_k, V_n, W, U, w, \theta, n, m\rangle$$

of positive values that satisfies:
(i) The aggregate matching rates (3) *and* (4)
(ii) The labor force allocation identity (10)
(iii) The wage-sharing rule (21)
(iv) The value functions (22), (23), (24) *and* (25)

The steady-state equilibrium does not specify the unemployment rate u. This is because the unemployment rate can be determined by other endogenous variables in the model. In steady-state the law of motion for employment (11) reduces to the following condition:

$$\lambda n = q(\theta)v \tag{26}$$

which implies that the flow into unemployment equals the flow into employment. We know that by definition $v = \theta u$ and $n = 1-u$. Using equation (26) we write an equation for unemployment in equilibrium:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{27}$$

where the equilibrium unemployment rate is determined by the two transition rates in and out of unemployment.

The law of motion for capital (6) associated with the capital-production process (7) yields the following condition for capital formation in steady state:

$$\delta k = \Phi m^{\pi} s^{1-\pi} \tag{28}$$

In the steady-state, firms' value equation for capital (22) can be rewritten as:

$$V_k = \frac{1}{1-\beta(1+g)(1-\delta)}\left[A\left(\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \alpha k^{-\frac{1}{\sigma}}\right] \tag{29}$$

Combination of (29) and the first-order condition (15) yields:

$$\frac{w}{\pi\Phi m^{\pi-1}} = \frac{\beta}{1-\beta(1+g)(1-\delta)}\left[A\left(\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \alpha k^{-\frac{1}{\sigma}}\right] \tag{30}$$

We refer to (30) as the *optimal capital-production condition*. The condition states that the marginal benefit from creation of new capital for final-good production equals the marginal cost of capital production.

The firm's value of employment (23) can be rewritten as:

$$V_n = \frac{1}{1-\beta(1+g)(1-\lambda)}\left[A\left(\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} (1-\alpha) l^{-\frac{1}{\sigma}}\right] \tag{31}$$

Combining (31) and (14) gives the *job creation condition*:

$$\frac{\Phi\psi v}{q(\theta)} = \frac{\beta}{1-\beta(1+g)(1-\lambda)}\left[A\left(\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} (1-\alpha) l^{-\frac{1}{\sigma}}\right] \tag{32}$$

According to the above condition the expected cost of opening a new vacancy equals the firm's share of the net marginal surplus from creation of the new job.

We now derive an equation for wages in steady state. First it should be noted that in steady-state the value equation (18) can be rewritten as:

$$W = \frac{w+\beta\lambda U}{1-\beta(1+g)(1-\lambda)} \tag{33}$$

Substitution of (33) and (23) into the wage-sharing rule (21) yields an equation for wages in equilibrium:

$$w = (1-\gamma)\Phi z + \gamma\theta\Phi v + \gamma\Big[p\Big(\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)l^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{1}{\sigma-1}}(1-\alpha)l^{-\frac{1}{\sigma}}\Big] \tag{34}$$

The equilibrium wage equation states that the wage bargained is the weighted average of the net continuation value of employment and workers' outside option where the weight is the workers' bargaining power γ.

4 Quantitative analysis

In this section I will use numerical methods to calculate the steady-state growth equilibrium derived in the previous section. First, I will calibrate the model to match the basic empirical facts about the US labor market and economy. The reason I choose the US economy is to facilitate comparison to the literature and the the fact that much empirical research has been carried out using US data, providing available parameter values.[6] Second, I will carry out a quantitative comparative static analysis by assessing the response in the calibrated steady-state equilibrium to a change in capital-production technology and productivity. I also assess and discuss the sensitivity of the results to the selection of parameter values.

4.1 Calibration

A time period in the model is one year and the parameter values are chosen accordingly. The discount factor β is set at 0.953 which corresponds to a discount rate, r, of about 5%. Capital depreciation rate, δ, is set at the standard 10% annual rate. The share of capital in production, α, equals the conventional value of $1/3$ and in the baseline calibration the elasticity of substitution between capital and labor in final-good production, σ, is set at 0.75. Aggregate productivity, A, is assumed constant in the baseline calibration and normalized to 1.

In the Cobb-Douglas matching function the elasticity of matching, ξ, is set at 0.5 based on estimates in Petrangolo & Pissarides (2001). Following Pissarides (2009), I target labor market tightness θ of 0.72 and set the match-efficiency parameter Ω at 0.70, the exogenous rate of separation λ at 0.036 and the parameter ψ in the convex vacancy-cost function at 4.8, in order to achieve that target.

The bargaining power of workers, γ, is chosen to be 0.5. Workers' outside option, the unemployment flow value z, is set at 0.8. I choose to set the return

[6] As highlighted in Section 2, the motivation for this research and the implications of the results are to be thought of in a context beyond the US economy.

Table 2: Parameter Values

Parameter	Description	Value
β	Discount factor	0.953
δ	Capital depreciation rate	0.10
α	Production function parameter	0.33
σ	Elasticity of substitution in production	0.75
A	Aggregate productivity factor	1.0
ξ	Elasticity of matches to unemployment	0.5
Ω	Scale parameter in the matching function	0.70
ψ	Parameter in vacancy posting cost equation	4.8
λ	Exogenous rate of separation	0.036
γ	Bargaining power	0.5
z	Unemployment flow value	0.8
π	Returns to labor in capital production	0.95
g	Growth rate of technology in capital production	0.01
Φ	Technology parameter in capital production	1.0

to labor in capital production, π, to be 0.95 in the benchmark calibration. The parameter Φ is normalized at 1 and in the benchmark calibration the growth rate of technology g is set at an annual rate of 1%. Table 2 summarizes the parameter values.

Using the parameter values, I calibrate a solution for the steady-state equilibrium. The model solution is presented in Table 3. Labor market tightness and the vacancy rate are at their targeted values which gives a steady-state unemployment rate of 5.7%. The benchmark allocation of labor between employment in final good production and capital production is 80/20.

4.2 Comparative statics

How does the steady-state respond to changes in the growth rate of technology in capital production? Does the model generate a long-term relationship between capital formation and unemployment? To answer these questions, a comparative static analysis can be conducted using the calibrated steady-state as a baseline.

Results from comparative statics are presented in Figure 3. An increase in the growth rate of technology, g, by 1 percentage point – from 1% annual growth to 2% – leads to a decrease in unemployment by 0.13 percentage points. In generating this result, there are two forces at play, working in opposite directions. Increased growth rate of technology decreases the relative cost of capital

Table 3: Model Steady-State Equilibrium Solutions

Variable	Description	Solution
θ	Labor market tightness	0.720
v	Vacancy rate	0.041
k	Capital	2.035
w	wage	0.970
n	Total employment	0.943
l	Employment in final-good production	0.758
m	Employment in production of capital	0.185
u	Unemployment rate	0.057

in production of the final good, increasing the returns to job creation in the capital-production sector. Since the cost of creating a new job is borne now but profits build up during the life time of the job, firms will post more vacancies and employ more workers in capital production. Hence, there is a positive job creation effect.[7] However, firms are now more willing to substitute capital for labor in production. As a result, firms will reallocate labor between its different uses, from production of the final good to production of capital, which reduces the firms' incentive to post new vacancies. This is the reallocation effect. As becomes clear in Figure 3, the reallocation effect counteracts the job creation effect. However, given the nature of capital formation process, the job creation effect outweighs the reallocation effect, generating an increase in vacancies and leading to an overall fall in unemployment associated with an increase in capital formation. These results are summarized as follows.

Proposition 1. *Increase in the technology growth rate g increases employment in the capital production sector but decreases employment in final-good production, partially through reallocation into capital production. Because the job creation effect more than offsets the reallocation effect, unemployment decreases.*

Figure 4 shows the relationship between capital formation and unemployment that results from increased growth rate of capital-production technology. An increase in technology growth by 1 percentage point – from 1% to 2% – leads to an increase in steady-state capital formation of 4.8% and a corresponding

[7] When discussing the effect on unemployment and job creation from changes in productivity, Pissarides (2000) refers to a negative effect of increased productivity on unemployment as the "capitalization effect". Since faster growth rate of productivity leads future income to be discounted at a lower rate, and that the cost of creating a job is borne now but recovered from revenues over the life of the job, the lower discount rate will increase job creation.

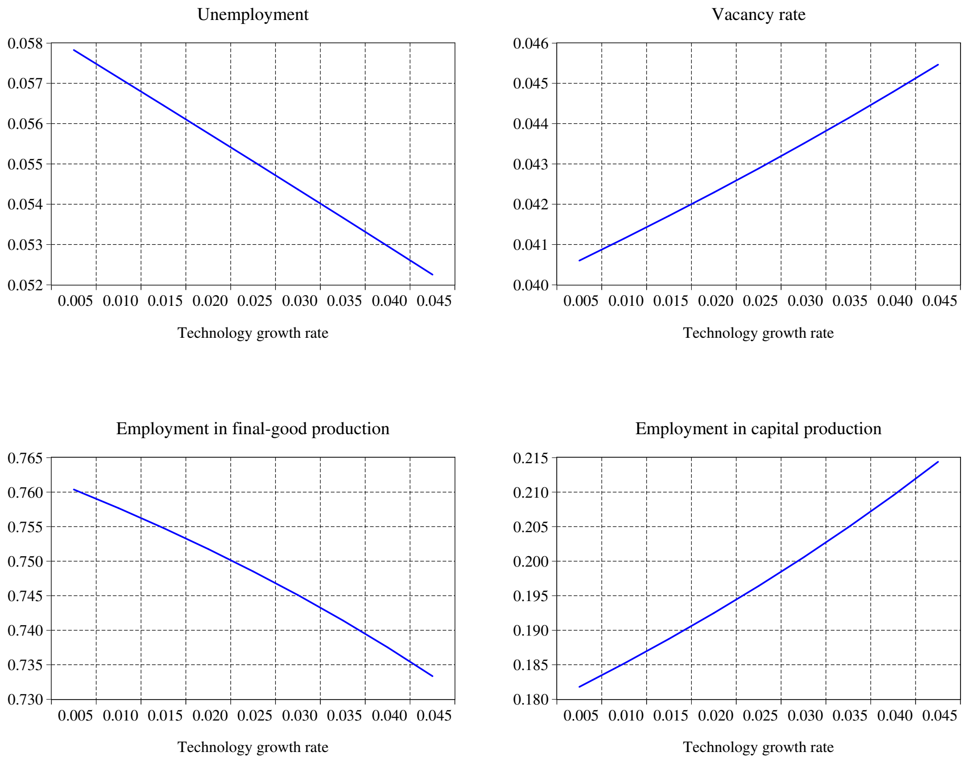


Figure 3: Comparative statics for the technology growth rate

decrease in steady-state unemployment by 0.13 percentage points. The model is able to generate a negative long-term relationship between unemployment and capital formation and therefore successful in replicating the empirical evidence found in macroeconomic data. This result is summarized in the following:

Proposition 2. *Increase in the technology growth rate g increases capital formation in equilibrium and decreases equilibrium unemployment.*

The search and matching literature has emphasized the role of changes in productivity as the driving force of changes in unemployment. As highlighted in the literature and demonstrated by the empirical analysis in Section 2, increased productivity has a negative effect on unemployment. In order to isolate the effect from a structural change in capital-production technology, we have assumed aggregate productivity, A, to be constant. We now assess the effect of a growth in productivity on unemployment in the model and assume that aggregate productivity evolves according to:

$$A_t = Ae^{gt}$$

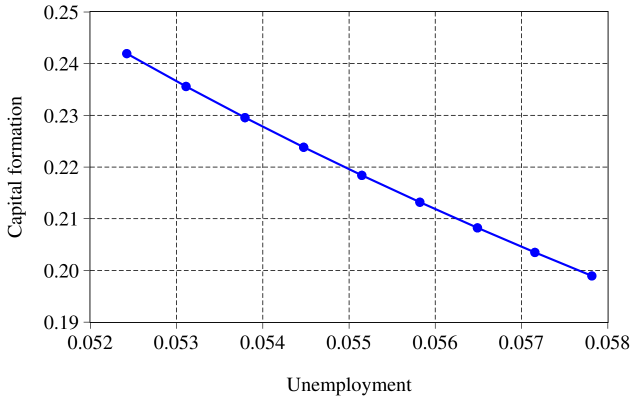


Figure 4: Investment-Unemployment relationship

where A is some initial level of productivity, normalized to one, and the growth rate of productivity in the final-good production, g, is the same as the growth in level of technology.

An increase in the growth rate g by 1 percentage point – from 1% annual growth to 2% – decreases unemployment by a half a percentage point. The negative effect from an increase in the growth rate of both productivity and capital-production technology on unemployment is greater than in the case when only capital-production technology grows at a higher rate. The reason is that increased growth in aggregate productivity generates a stronger job creation effect but the reallocation effect is mitigated since the productivity of both factors in the final-good production increases. However, there is still a strong and negative relationship between capital formation and unemployment in equilibrium.

4.3 Sensitivity analysis

The results, both the job-creation effect and, in particular, the strength of the reallocation effect, are sensitive to the selection of parameter values in the production function (5) and in the process for capital production (7). I discuss the sensitivity of the above results to values of the parameters σ and π.

The production function (5) is a standard CES production function. Like all standard CES production functions it nests the Cobb-Douglas production function when $\sigma \to 1$ and a linear production function with perfect factor substitution when $\sigma = 0$. Figure 5 plots the unemployment rate, employment in capital production and the investment-unemployment relation for three different values of σ: 0.4; 0.75 and 0.99. In the calibrations, the parameter ψ is chosen such that unemployment is 5.7% and the vacancy rate is 4.1% under a technol-

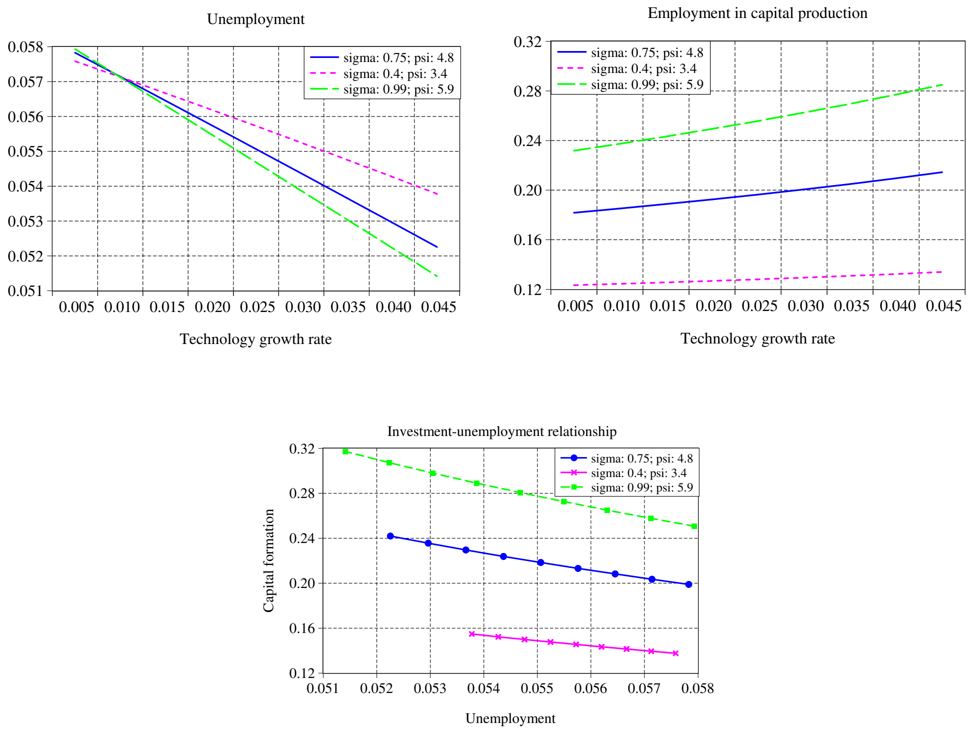


Figure 5: Sensitivity to elasticity of substitution in final-good production, σ
Note: The parameter ψ is chosen for each specification of the parameter σ in order to generate the same unemployment rate and vacancy rate under 1% growth of technology.

ogy growth rate of 1%. As Figure 5 shows, the sign of the relationship between investment and unemployment is always negative for different parameter specifications, indicating that the relationship is robust to changes in σ. However, we can see that the larger the value of σ, *i.e.* greater elasticity of substitution in final good production, the greater the impact of an increase in technology growth on employment in capital production and unemployment.

Next, I consider the sensitivity of the results to the choice of the value of π. In the benchmark calibration the returns to labor, π, was set at 0.95. However, the value of this parameter is open for discussion. In order to test the robustness of the results to the specification of this parameter I compare calibrations for two values of π, 0.95 and 0.5. Like before, the parameter ψ is chosen such that unemployment is 5.7% and the vacancy rate is 4.1% under a technology growth rate of 1% in the calibrations. Figure 6 plots the unemployment rate and employment in capital production for the different parameter specifications. We can see that the impact of an increase in technology growth on employment in capital production and unemployment is much greater when returns to labor

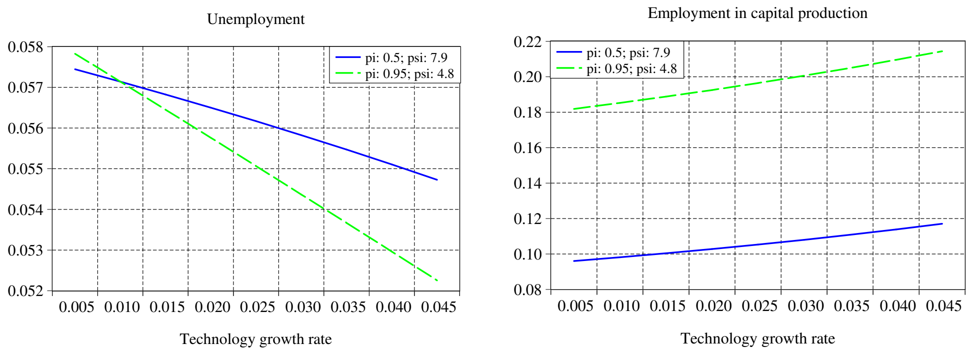


Figure 6: Sensitivity to returns to labor in capital production, π

Note: The parameter ψ is chosen for each specification of the parameter π in order to generate the same unemployment rate and vacancy rate under 1% growth of technology.

in capital production is high. However, the sign of the relationship between investment and unemployment is robust to selection of the parameter π.

5 Conclusion

The literature on medium to long-term changes in unemployment has to a large extent ignored the empirical relationship between unemployment and capital investment which is clear in macroeconomic data. Recently, researchers have been rediscovering this relationship and have emphasized its importance in explaining changes in unemployment. However, there has been limited theoretical investigation into this relationship.

In the paper I develop a search and matching model which incorporates channels through which there is a clear relationship between capital formation and unemployment in equilibrium. A setup with firms producing both a final good and a capital good generates an endogenous relationship between investment and job creation. Quantitative analysis shows how an increase in the growth rate of technology in production of capital leads to a decrease in equilibrium unemployment associated with an increase in capital formation. Although simple in structure, the model is successful in generating this relationship found in data.

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