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positive and normative implications**

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# Asymmetric Labor Market Institutions in the EMU: positive and normative implications<sup>1</sup>

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## **Abstract**

How do labor market institutions affect the volatility and persistence of inflation and unemployment in a monetary union? What are the implications for monetary policy? This paper sets up a DSGE currency union model with unemployment, hiring frictions and real wage rigidities. The model provides a rigorous but tractable framework for the analysis of the functioning of a currency union characterized by asymmetric labor market institutions. Positively, we find that inflation and unemployment differentials depend strongly on the underlying labor market structure: the hiring friction lowers the persistence and increases the volatility of the inflation differential whereas real wage rigidities imply more persistence and variability in output and unemployment differentials. Normatively, we find that macroeconomic stabilization is easier when labor market frictions are high and real wage rigidities are low. This has important implications for optimal monetary policy: The optimal inflation target should give a higher weight to regions with more sclerotic labor markets and more flexible real wages.

JEL classification: E32, E52, F41

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## 1 Introduction

The aim of the present paper is to develop a tractable currency union model to analyze, first, how different labor market institutions affect the volatility and persistence of inflation and unemployment differentials in a monetary union and, second, what the implications are for optimal monetary policy. To this purpose, we setup a DSGE currency union model that combines three key ingredients: (i) monopolistic competition and nominal rigidities in the goods market, which serve to give a role to monetary policy; (ii) hiring frictions in the labor market, which generate involuntary unemployment; (iii) real wage rigidities, which hinder wage adjustments and shift the labor market adjustment from prices to quantities.

Following the seminal contribution by Mundell (1961), the structure of labor markets has often been seen as a crucial element in determining the efficient adjustment of a currency union to asymmetric shocks. The old Mundell wisdom states that it may be very costly to abandon the devaluation tool when labor does not move easily and prices and wages are sticky. Adverse country-specific shocks, in this case, may trigger a long and painful adjustment process of low growth and rising unemployment, until the equilibrium is restored. Looking at Europe today, the general impression is that many States and politicians may have understated the economic costs of entering in a monetary union.

Three main elements seem to characterize European labor markets. First, unemployment is high and tends to be prolonged over time. Second, real wages seem to be rather inflexible. Third, labor market institutions are widely heterogeneous across countries. Indeed, the notion that labor market rigidities are at the core of the European unemployment problem has now become widely accepted among policy makers.

Recent research, by integrating labor market frictions “à la Mortensen-Pissarides (1994)” in otherwise standard closed-economy New Keynesian (NK) models<sup>1</sup>, has shown that the structure of labor markets substantially influences the transmission mechanism of monetary policy and, more generally, the overall adjustment of economic activity to shocks. Labor market institutions are, in fact, an important determinant of the dynamics of real wages and of marginal costs of firms, which are in turn the main drivers of inflation. The results seem to suggest that the introduction of a more realistic labor market structure is needed in order to overcome some of the well-known weaknesses of the traditional NK framework<sup>2</sup>.

A few currency union models have been proposed in recent years (see, among others, Benigno, 2004, Monacelli and Galí, 2005, Benigno and Lopez-Salido, 2002, Altissimo, Benigno and Palenzuela, 2005). The literature has focused its attention on the implications of different degrees of nominal rigidities in different member countries. The main result is that, when asymmetries in the degree of price stickiness are present, an inflation targeting strategy that gives higher weight to inflation in the “sticky price” region is nearly optimal (Benigno, 2004). Most of these works assume perfectly competitive labor markets and

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<sup>1</sup>E.g. Christoffel and Linzert (2005), Trigari (2005), Krause and Lubik (2005) and Walsh (2005).

<sup>2</sup>Hall (2005) shows that the introduction of sticky wages improves the behaviour of labour market models, as it increases the sensitivity of labour market conditions – and hence unemployment – to productivity shocks. Blanchard and Galí (2005) demonstrate that the introduction of real wage rigidities is a natural way to overcome one of the shortcomings of the standard New Keynesian model, namely the lack of a meaningful trade-off between inflation stabilization and output gap stabilization.

thus ignore a fundamental source of asymmetry among member countries, namely the wide heterogeneity in European labor market institutions.

Campolmi and Faia (2006) are the first to integrate labor markets frictions “à la Mortensen-Pissarides” into a DSGE currency union model. The currency union consists of two regions sharing the same currency and is characterized by a variety of frictions: matching frictions and wage rigidity in the labor market, monopolistic competition and price rigidity in the goods market. The result is a rich and quite complex model, which needs to be studied through calibration and simulations. The paper, which studies the link between inflation volatility differentials and different unemployment insurance coverage, represents an important first step towards an understanding of how the transmission mechanism of monetary policy works in the presence of asymmetries in the structure of labor markets<sup>3</sup>.

Following Blanchard and Galì (2006), we model labor market frictions by assuming the presence of hiring costs, which increase in the degree of labor market tightness. Real wage rigidities are introduced, following much of the literature, by employing a version of Hall’s (2005) notion of the wage norm. Using this very simple modelling of the labor market, we develop a currency union model that incorporates many realistic features, but is still tractable. Interestingly, the model can be reduced to a six-equation model - like standard open-economy models. We regard the tractability of our framework as the key advantage of our approach.

The model provides a rigorous framework for the analysis of the functioning of a currency union characterized by asymmetric labor market institutions. We use the model in three ways.

First, we analyse how different labor market structures influence the Phillips Curves of member countries. We distinguish among two types of labor market imperfections: *labor market frictions (LMF)*, which capture the institutions - like employment protection legislation, hiring costs and the matching technology - that limit the flows in and out of unemployment; and *real wage rigidities (RWR)*, intended to capture all the institutions - including wage staggering and the wage bargaining mechanism and legislation - which influence the responsiveness of real wages to economic activity. These two types of labor market rigidities are found to have very different effects on the incentives for firms to reset prices and thus on the Phillips Curve. A higher degree of LMF makes the Phillips Curve steeper. This effect is strong and highly non-linear. The introduction of RWR has instead two implications for the Phillips Curve. First, the Phillips Curve gets flatter, as inflation becomes less sensitive to unemployment changes. Second, as in Blanchard and Galì (2005, 2006) the introduction of RWR creates a trade-off of monetary policy between inflation stabilization and unemployment stabilization; a trade-off which is increasing in the degree of real wage rigidities. LMF and RWR are found to interact in a rich and complex way

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<sup>3</sup>Other contributions related to our paper include Andersen and Seneca (2007), Poilly and Sahuc (2007) and Dellas and Tavlas (2004). Andersen and Seneca (2007) discuss the effects of asymmetric size, market power and nominal wage rigidities for aggregate volatilities while Dellas and Tavlas (2004) and Poilly and Sahuc (2007) study respectively the implications of asymmetries for the costs of membership in a currency union and the implications of labor market reforms on the welfare of member countries in the presence of labor market asymmetries.

in determining the slope of the Phillips Curve. When they are substitutes (low RWR are associated with high LMF or vice versa), their effects tend to reinforce each other; when they are complements (high RWR are associated with high LMF), they tend to offset each other.

Second, we study how different labor market structures are likely to affect the efficient functioning of the currency union in response to both symmetric and asymmetric shocks. The main focus is on the evolution of inflation and unemployment differentials, as these reflect the way in which economic disturbances are absorbed in the currency union. A higher degree of LMF increases the response of inflation and reduces the response of unemployment: labor market rigidities make it more costly for the firm to hire new people and shift the adjustment from quantities to prices. A higher degree of RWR amplifies the response of the real economy to shocks: real wage rigidities shift adjustment from prices to quantities. As a consequence, different labor market structures have very different effects on the volatility and persistence of inflation and unemployment differentials. LMF increase the volatility but decrease the persistence of inflation differentials, while they decrease the volatility but increase the persistence of the unemployment differential. Our results suggest that LMF, by shifting the adjustment from quantities to prices, may actually improve the adjustment mechanism of the currency union to asymmetric shocks. RWR increase both the volatility and the persistence of both inflation and unemployment differentials: when real wages cannot adjust, the adjustment mechanism to asymmetric shocks worsens considerably.

We also find that labor market asymmetries matter: when member countries have different labor market structures, symmetric shocks (monetary policy or symmetric productivity shocks) may cause large and long-lasting inflation and unemployment differentials. All these differentials are inefficient.

Which are the consequences for monetary policy and welfare of asymmetric labor market rigidities? This is the question we answer in the third part of the paper. We first illustrate the trade-offs that monetary policy faces by calculating the Policy frontier of the central bank. We find that macroeconomic stabilization is easier in a currency union with more sclerotic labor markets, while RWR make macroeconomic stabilization more difficult. Intuitively, when LMF are higher, the Phillips Curve gets steeper and the central bank can stabilize inflation volatility incurring a smaller increase in unemployment volatility. The opposite happens when we increase the degree of real wage rigidities. Second, we ask ourselves whether the central bank should target only union inflation (inflation targeting) or both union inflation and union unemployment (mixed targeting). We find that strict inflation targeting is near optimal at high levels of LMF and lower levels of RWR, but it is clearly dominated by a mixed targeting strategy when the currency union is characterized by low LMF and high RWR. Finally, we restrict our attention to the class of inflation targeting rules and ask ourselves which is the optimal weight the central bank should give to home inflation and foreign inflation when member countries have different labor market structures. We find that the optimal inflation targeting rule should give a higher weight to a country with *higher* LMF but *lower* RWR. The result that it is optimal to target the region with flexible real wages is particularly interesting, because it seems apparently in contrast with the literature on sticky prices and sticky nominal wages, such as for example

in Erceg et al. (1999), whose main conclusion was to “target what is sticky”. Again, since the effects of LMF and RWR point in opposite directions, it is crucial to determine whether they are complements or substitutes: when LMF and RWR are complements, their effects tend to offset each other and to target union inflation is near optimal. When they are substitutes, their effects re-inforce each other and symmetric inflation targeting is strongly dominated by an inflation targeting rule that gives higher weight to the country with sclerotic labor markets and flexible real wages.

### 1.1 Labor Market Asymmetries in the EMU

Inflation and output growth differentials are a big concern for policy-makers<sup>4</sup>. Recent empirical evidence has shown that inflation and output growth differentials among Euro area countries - even when compared with the USA - are rather sizeable and, most importantly, very persistent over time<sup>5</sup>. This persistency has attracted substantial public attention, because it suggests that the adjustment mechanism in the single currency area may not be working efficiently. Indeed, as emphasized by Angeloni and Ehrmann (2004), the importance of these issues “can hardly be overemphasized, given the frequently voiced concern that heterogeneity dents the solidity of the monetary union itself”.

Labor market rigidities are one of the factors often blamed as one of the potential causes behind the inefficient and asymmetric adjustment of member countries to economic shocks. However, little work has been done, in the context of DSGE models, to analyse how labor market institutions affect the functioning of the currency area, and what are the implications for monetary policy<sup>6</sup>.

Three main elements seem to characterize European labor markets. First, unemployment is high and tends to be prolonged over time. As Blanchard notes in a recent paper, “being unemployed in Europe has always been a different experience from being unemployed in the United States (...) and has become increasingly so over time”<sup>7</sup>. Second, real wages seem to be rather inflexible. “Unemployment does eventually put some downward pressure on real wages in Europe, but a large share of the adjustment is borne by employment” (Mongelli 2002, p. 18). Third, labor market institutions are widely heterogeneous across countries. Indeed, the notion that labor market rigidities are at the core of the European unemployment problem has now become widely accepted among policy makers.

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<sup>4</sup>See, e.g., ECB (2003 and 2005), Angeloni and Ehrmann (2004), Benalal, Diaz del Hoyo, Pierluigi and Vidalis (2006) for some evidence on inflation and output differentials and for analyses of the potential causes and policy implications.

<sup>5</sup>It must be noticed that inflation and output differentials are not undesirable per se, as they are a part of the equilibrating adjustment process inside the monetary union. Whether they are desirable or not depends on the causes of such differentials. In fact, inflation differentials may also be the product of “misaligned fiscal policies, diverging wage developments and deep-seated structural inefficiencies such as nominal and real rigidities in product and factor markets (ECB 2005, p.61)”. The persistence of such differentials together with the empirical evidence on wage/price rigidities and on market imperfections, suggest that a large part of these divergences may indeed be inefficient and avoidable.

<sup>6</sup>The first attempts include Campolmi and Faia (2006), Dellas and Tavlas (2004) and Andersen (2007).

<sup>7</sup>Blanchard, “European Unemployment: The Evolution of Facts and Ideas” (2005, p. 6). The author notices that the proportion of long-term unemployment and unemployment’s average duration are much larger in Europe than in the US. For instance, average duration, which in the US is around three months, in France stands over a year.



	EPL	Wage Flexibility	Union Density	Wage coordin.	Benefit	Ben. Repl. Ratio	Benefit Duration	Unempl. rate
<b>Austria</b>	1.10	1.43	36.50	4.00	32.21	0.31	0.75	5.29
<b>Belgium</b>	1.00	0.65	55.60	4.00	38.74	0.46	0.76	7.78
<b>France</b>	1.40	2.22	9.70	2.00	40.24	0.58	0.51	9.64
<b>Germany</b>	1.30	0.55	25.00	4.00	28.16	0.36	0.60	7.98
<b>Ireland</b>	0.50	0.80	37.80	5.00	32.46	0.32	0.75	4.50
<b>Italy</b>	1.50	2.07	34.90	4.00	34.30	0.43	0.20	9.11
<b>Netherlands</b>	1.10	0.66	23.20	4.00	52.52	0.70	0.50	3.70
<b>Portugal</b>	1.70	n.a.	23.50	n.a.	42.87	0.65	0.38	5.44
<b>Spain</b>	1.40	0.17	13.90	n.a.	37.12	0.65	0.28	10.71
<b>Averages</b>	1.22	1.07	28.90	3.86	37.63	0.50	0.52	7.13
<b>UK</b>	0.35	0.98	31.20	1.00	16.59	0.22	0.73	5.19
<b>USA</b>	0.10	0.32	12.80	1.00	13.53	0.27	0.16	5.05
<b>Japan</b>	1.40	n.a.	21.50	5.00	10.65	0.31	0.00	4.89
<b>Average</b>	0.62	0.65	21.83	2.33	13.59	0.26	0.30	5.04

Figure 1: *Labor Market Characteristics of Selected Euro Area Countries*

Table 1<sup>8</sup> shows some evidence of these regularities. Euro Area countries seem to be characterized by more heavily regulated labor markets and more generous unemployment benefit systems than our control group (UK, USA and Japan). As a consequence, the average unemployment rate is higher in Europe than in other developed countries - even though it has recently improved.

Looking only at the “averages”, however, can be highly misleading. Labor market institutions vary considerably across Euro area countries. The unemployment rate, since the launch of the Euro, has been on average around 4% in Ireland and the Netherlands but around 10% in France and Spain. Employment protection legislation is typically extremely tight in countries like Italy, Portugal, France and Spain, but very loose in Ireland. Wage flexibility (measured as the percentage increase in wages in response to a 1 percent decrease in the unemployment rate) in France and Italy is estimated to be ten times bigger than in Spain and four times bigger than in Germany<sup>9</sup>. Large heterogeneity is also present in the degree of wage coordination and unionization, as well as in the generosity of the unemployment benefits system.

<sup>8</sup>EPL refers to Employment protection legislation such as constructed by Nickell (2001). Wage flexibility represent the percentage increase in wages in response to a one percentage point fall in the unemployment rate (Source: Nickell (1997)). The unemployment rate refers to the average from 1999Q1 to 2005Q4, The other data refers to the last date of availability in the dataset from Nickell 2001 or the OECD (1998 for EPL, 1995 for the benefit replacement rate and the benefit duration; 2000 for union density and wage coordination).

<sup>9</sup>The fact that some countries like Italy and France show large degrees of real wage flexibility might be surprising, but one should keep in mind that the link between nominal and real wage rigidity is non-trivial and depends on the underlying wage setting mechanism. For example, we would argue that in countries with high degrees of nominal wage indexation, real wages are relatively sticky.

How do asymmetric labor market structures influence the responses of member countries to shocks? What are the implications for monetary policy? In this paper we try to assess these questions, focusing on two particular types of labor market imperfections that have attracted the attention of the most recent literature: labor market frictions, which capture the institutions that limit the flows in and out of unemployment (employment protection legislation, hiring costs and firing costs, the matching technology); and real wage rigidities, intended to capture the institutions which influence the responsiveness of real wages to economic activity (e.g. wage staggering, the wage bargaining mechanism and legislation).

The remainder of the paper is organised as follows: Sections 2-5 describe the model. Section 6 studies how the New Keynesian Philips Curve changes with the labor market structure. Section 7 studies the positive behavior of the model under different calibrations. Section 8 carries out the normative analysis and Section 9 concludes.

## 2 The Model

A currency union is a group of regions or countries sharing the same currency, with a single central bank entitled to conduct the monetary policy. To keep things as simple as possible, we consider a currency union that consists of two regions, home and foreign, taken of the same size (normalised to 1). Each economy, which is populated by identical, infinitely lived households, is specialised in the production of a bundle of differentiated goods. There is no migration across regions. Capital markets are complete. Wages are set in an individual bargaining between the employer and the employee. The labor market is characterised by hiring costs, leading to involuntary unemployment in equilibrium<sup>10</sup>. Countries are symmetric for everything apart from labor market institutions.

In this section, we describe the main features of our framework, concentrating on the principal elements of departure from the previous treatments. The complete derivation of the model is described in the appendix.

### 2.1 Households

The representative household within a country is thought of as a very large extended family with names on the unit interval. In equilibrium, some members will be employed and others not; to avoid distributional issues, we assume that households pool their income and consumption.

The representative household in country  $i$  ( $i = H$  or  $F$ ) maximizes a standard lifetime utility, which depends on the household's consumption and disutility of work:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi \frac{(N_t^H)^{1+\phi}}{1+\phi} \right\}, \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t^* - \chi^* \frac{(N_t^F)^{1+\phi}}{1+\phi} \right\} \quad (1)$$

where variables with star are referred to the foreign country.  $N_t^i$  denotes the number of employed individuals in the representative household of country  $i$  while  $C_t$  and  $C_t^*$  are

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<sup>10</sup>The basic framework of the currency union is inspired by the work of Benigno (2004) and Galí-Monacelli (2006). The structure of the labor market builds on Blanchard-Gali (2006).

the composite consumption indexes for the home and foreign country respectively, defined as:

$$C_t = \frac{(C_t^H)^{1-\alpha} (C_t^F)^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}, \quad C_t^* = \frac{(C_t^{F,*})^{1-\alpha} (C_t^{H,*})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha} \quad (2)$$

$C_t^{j,i}$  is the quantity of the good produced in country  $j$  and consumed by residents of country  $i$ .  $\alpha \in [0, 1]$  is the weight on the imported goods in the utility of private consumption; a value for  $\alpha$  strictly less than  $\frac{1}{2}$  reflects the presence of home bias in consumption.

The production sectors are characterised by monopolistic competition. The index of country  $i$ 's consumption of the good produced in country  $j$ ,  $C_t^{j,i}$ , is given by the usual CES aggregator. The parameter  $\epsilon$  ( $\epsilon^*$ )  $> 1$  is the elasticity of substitution between varieties produced within home (foreign) country.  $P_t^H$  and  $P_t^F$  are the Dixit-Stiglitz domestic prices indexes of the home and foreign countries. Since the law of one price holds,  $P_t^i$  represents both the price index for the bundle of goods imported from country  $i$  as well as  $i$ 's domestic price index.

Utility maximization for the home household is subject to a sequence of budget constraints which, conditional on optimal allocation of expenditures across varieties, is given by:

$$P_t C_t + E_t \{ Q_{t,t+1}^H D_{t+1}^H \} \leq D_t^H + W_t^H N_t^H - T_t^H$$

where  $P_t = (P_t^H)^{1-\alpha} (P_t^F)^\alpha$  is the home CPI index,  $D_t^H$  is the nominal payoff in period  $t$  of the portfolio held at the end of period  $t-1$ ;  $W_t^H$  is the nominal wage and  $T_t^H$  denotes lump-sum taxes. We assume complete securities markets;  $Q_{t,t+1}^H$  is the stochastic discount factor for one-period ahead nominal payoffs, which is common across countries. Implicit in the budget constraint is the assumption that the law of one price holds across the union. Similar conditions hold for the foreign country.

## 2.2 The Terms of Trade and the Real Exchange Rate

In this section we introduce some definitions and identities that are used extensively below. First, we define the bilateral term of trade between the home and foreign countries as the ratio of the price of the composite good produced in country F in terms of country H's good:

$$S_t = \frac{P_t^F}{P_t^H} \quad (3)$$

The terms of trade, which represent an index of competitiveness, play a central role in our model. Movements in the terms of trade are crucial for understanding the response of the economy to asymmetric shocks and the transmission mechanism of monetary policy.

As the law of one price holds for all goods - which implies  $P_t^F = P_t^{F,*}$  and  $P_t^H = P_t^{H,*}$  - the *CPI* and the *domestic* price indexes in the two regions are related according to:

$$P_t = P_t^H (S_t)^\alpha, \quad P_t^* = P_t^F (S_t)^{-\alpha}$$

The real exchange rate  $V_t$  is defined as the ratio between foreign and home CPIs and is related to the terms of trade according to:

$$V_t = \frac{P_t^*}{P_t} = (S_t)^{1-2\alpha}$$

### 2.3 International Risk Sharing

Capital markets are complete: each household has access to a complete set of contingent claims, traded internationally. Combining the first order conditions relative to state contingent securities in the two countries, we obtain the usual result:

$$V_t = \psi \frac{u'(C_t^*)}{u'(C_t)} = \psi \frac{C_t}{C_t^*} \quad (4)$$

where  $\psi = V_0 \frac{u'(C_0^*)}{u'(C_0)} = \frac{P_0^*}{P_0} \frac{u'(C_0^*)}{u'(C_0)}$  is a constant, reflecting initial conditions regarding relative net asset positions. If PPP holds - i.e. for  $\alpha = 1/2$  -, the real exchange rate  $V_t = 1$  and the marginal utilities of consumption are equated up to a constant  $\psi$ . In general, movements in the real exchange rate are reflected in different consumption rates: even with complete financial markets, it is not efficient to equalize consumption across countries when there is a home bias in consumption ( $\alpha < \frac{1}{2}$ ). Henceforth, to keep the analysis as simple as possible, we assume initial conditions are such that  $\psi = 1$ .

### 2.4 Firms and the labor market

The setup of the supply side of the economy follows Blanchard and Galí (2006).

The production sector in each country is composed by a continuum of firms, indexed by  $j \in [0, 1]$ . Each firm in a country produces a differentiated good with an identical technology:

$$Y_t^i(j) = A_t^i N_t^i(j), \text{ for } i = H, F^*$$

where the variables  $A_t^i$  represent the state of technology in country  $i$ .

In each period a fraction  $\delta^i$  of the employed loses their jobs and joins the unemployment pool. Employment in firm  $j$  evolves according to:

$$N_t^i(j) = (1 - \delta^i) N_{t-1}^i(j) + h_t^i(j), \text{ for } i = H, F^*$$

where  $h_t^i(j)$  is the the number of new hires for firm  $j$  in country  $i$ . The job destruction rate  $\delta^i$  is exogenously given.

We assume all unemployed in the family look passively for a job. The analysis thus abstracts from any transition of people in and out the labor force, which we assume to be constant and equal to 1<sup>11</sup>.

The number of searching workers who are available for hire in country  $i$ ,  $U_t^i$  is defined as

$$U_t^i = 1 - (1 - \delta^i) N_{t-1}^i, \text{ for } i = H, F^*$$

<sup>11</sup>See, e.g., Merz (1995) for a similar assumption. Alternatively, we could have assumed, as in Blanchard and Galí (2006), that the equilibrium wage is set at a level such that at all times all individuals are either employed or willing to work. The choice of one or the other assumption does not change our main results.

“After hiring” unemployment, instead, is defined as the fraction of the population who are left without a job after hiring takes place,  $u_t^i = 1 - N_t^i$ .

Aggregate hiring  $h_t^i \equiv \int_0^1 h_t^i(j) dj$  evolves according to

$$h_t^i = N_t^i - (1 - \delta^i)N_{t-1}^i$$

Where  $N_t^i \equiv \int_0^1 N_t^i(j) dj$  denotes aggregate employment.

Firms face a cost of searching and recruiting new workers “à la Howitt”<sup>12</sup>. Hiring costs for firm  $j$  in country  $i$  are:

$$G_t^i h_t^i(j), \text{ for } i = H, F^*$$

where  $G_t^i$  is the cost per hire in country  $i$  (expressed in terms of the domestic CES bundle of goods), which is taken as given by the individual firm. Following Blanchard–Galì (2006), we assume

$$G_t^i = A_t^i B^i \left( \frac{h_t^i}{U_t^i} \right)^\varphi, \text{ for } i = H, F^*$$

where  $\varphi > 0$  and  $B^i$  is a positive scaling parameter that may be influenced by the authorities. The marginal cost of hiring is increasing in the aggregate hiring rate  $h_t^i$ : this captures the idea that a high rate of hiring may force firms to increase their search intensity. The marginal cost is decreasing in  $U_t^i$ : a high aggregate unemployment makes it easier and cheaper for firms to find willing and competent workers. Notice that there are two externalities at work in the model. When a firm hires new workers, she does not internalize the effect that her action has on the cost of hiring for other firms through  $h_t^i$  and  $U_t^i$ . In this framework, the presence of hiring costs creates a friction in the labor market very similar to that of standard search models.

If we define the labor market tightness index as

$$x_t^i \equiv \frac{h_t^i}{U_t^i}, \text{ for } i = H, F^* \quad (5)$$

i.e. as the ratio of aggregate hires to the employment rate, we can rewrite the cost per hire for H and F as

$$G_t^i = A_t^i B^i (x_t^i)^\varphi, \text{ for } i = H, F^*$$

Recruitment costs are increasing in the labor market tightness index. Since by assumption firms can hire workers only from the pool of unemployed,  $x_t^i \in [0, 1]$ .

Note that, from the viewpoint of the unemployed,  $x_t$  can be interpreted as the probability of finding a new job in period  $t$ , i.e. as the job-finding rate.

It is possible to show that the clearing of all markets imply, for the home and the foreign country respectively<sup>13</sup>:

$$Y_t = C_t(S_t)^\alpha + G_t^H h_t^H; \quad Y_t^* = C_t^*(S_t)^{-\alpha} + G_t^F h_t^F$$

<sup>12</sup>Cfr. Howitt (1988).

<sup>13</sup>Implicit in the market clearing condition is the assumption that hirings are homogeneous with the final goods. See also Ravenna and Walsh (2007), p. 6.

Finally, notice that the assumption of Cobb-Douglas preferences over the home and foreign goods allows us to derive a simple relation between the terms of trade and relative output<sup>14</sup>:

$$S_t = \frac{P_t^F}{P_t^H} = \frac{Y_t - G_t^H h_t^H}{Y_t^* - G_t^F h_t^F} \quad (6)$$

Equation (6) simply states the relative price of domestic (foreign) goods is inversely related to the quantity produced in the two regions (net of aggregate hiring costs)<sup>15</sup>.

In this paper, by determining inflation and output fluctuations in terms of deviations from the corresponding efficient levels, we are able to focus on the cyclical, undesired, components of inflation and output differentials. To this purpose, we need to analyse three different equilibrium allocations: the constrained efficient, the flexible price and the sticky price allocations. This is the task to which we turn.

### 3 The Social Planner's Problem

In this section we derive the so-called “constrained efficient allocation”. Following Blanchard and Gali’ (2006), we assume that the social planner maximizes the welfare of the union, taking as given the technological constraints and the labor market frictions that are present in the decentralised economy. In other words, the social planner cannot eliminate or reduce hiring costs; he can, however, internalize the effects of variations in employment on labor market tightness and, hence, on hiring costs<sup>16</sup>.

Consider for instance the home country. Solving the social planner’s problem with

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<sup>14</sup>The Cobb-Douglas assumption in fact implies that the percentage variation in relative prices is equal, and opposite in sign, to the percentage variation in relative quantities. Equation (6) allows us to pin down the steady state level of  $S_t$ :

$$S = \frac{A^H N^H (1 - g\delta)}{A^F N^F (1 - g^*\delta^*)}$$

where  $g = B(x^H)^\varphi$  and  $g^* = B^*(x^F)^\varphi$ .

<sup>15</sup>This expression allows us to highlight one simple, but interesting, point. First, note that in this model inflation differentials are simply represented by percentage variations in the terms of trade, i.e.  $\Delta s_t = \pi_t^F - \pi_t^H$ . Suppose that in a long run equilibrium the Home and the Foreign country are characterised by different productivity growth rate, which we call  $\gamma_a$  and  $\gamma_a^*$ . Assume employment is constant in the long run equilibrium; it is easy to check that inflation differentials evolves according to:

$$\Delta s_t = \pi_t^F - \pi_t^H = \gamma_a - \gamma_a^*$$

This shows one possible explanation for the persistent inflation differentials we see in the European Union: persistent differences in the productivity growth rates. Inflation differentials, in this case, are an equilibrium phenomenon, performing a positive role: they allow the clearings of all markets. In other words, inflation differentials are not undesirable per se. As Ahearne and Pisani-Ferry puts it, “whether the observed differences are desirable or undesirable, depends in large part on the nature of shocks that are causing that divergences” (Ahearne and Pisani-Ferry 2006, p. 3).

<sup>16</sup>Blanchard and Gali (2006), p.9

respect to home employment, we obtain<sup>17</sup>:

$$\chi \frac{(N_t^H)^\phi C_t^{H,W}}{A_t^H} \leq 1 - B(1 + \varphi) (x_t^H)^\varphi \quad (7)$$

$$+ \beta(1 - \delta) E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}}{A_t} B \left[ \varphi (x_{t+1}^H)^\varphi + (1 - x_{t+1}^H) \right] \right\}$$

which must hold with strict equality if  $N_t^H < 1$ .  $C_t^{H,W} = C_t^H + C_t^{H,*} = C_t(S_t)^\alpha$  denotes world consumption of the home good.

The above condition states that the marginal rate of substitution between labor and consumption (the left hand side) has to be less or equal to the corresponding marginal rate of transformation (the right hand side) - both normalised by productivity. Hiring an additional worker at time  $t$  has three effects, captured by the right hand side of eq. (7). First, it generates one unit of additional output. Second, it increases the recruitment costs at time  $t$ . This effect is represented by the term  $-B(1 + \varphi) (x_t^H)^\varphi$ . Third, it reduces the costs of hiring new workers in period  $t + 1$ . This effect is captured by the last term on the right hand side.

The important point to note is that equation (7) implies a *level of employment that is invariant to productivity shocks*<sup>18</sup>. This invariance is a consequence of the assumption of a log utility function, which implies offsetting income and substitution effects on the labor supply. The fact that employment is constant is a very useful result, since it allows us to say that all fluctuations in employment are inefficient.

## 4 Equilibrium under Flexible Prices

In this section we derive the equilibrium under the assumption that prices are flexible. We first describe the optimal price setting of a firm, given the wage. We then characterize the equilibrium that emerges with Nash bargained wages. Finally, we introduce real wage rigidities in the form of a Hall (2005) type wage norm. We focus on the home country; the solution for the foreign country is completely symmetric.

### 4.1 Optimal Price Setting

Under flexible prices, the optimal price setting rule takes the form of a markup  $\mu = \frac{\epsilon}{\epsilon - 1}$  over the real marginal cost:

$$\frac{P_t^H(i)}{P_t^H} = \mu M C_t^H \quad (8)$$

<sup>17</sup>See the Appendix for the details of the derivation. Similar conditions and the same conclusions hold for the Foreign country.

<sup>18</sup>To see this, notice that world consumption of the home good is proportional to productivity

$$C_t^{H,W} \equiv C_t^H + C_t^{H,*} = A_t^H (N_t^H - B (x_t^H)^\varphi h_t^H)$$

It follows that the optimality condition does not depend on the productivity levels prevailing at Home (or Foreign), and thus both the employment level and the labor market tightness indicator are constant under the constrained efficient solution. Blanchard and Galí (2006, p. 9-11) get the same result in the context of a one-country model.

where the firm's real marginal cost is (expressed in terms of domestic goods):

$$MC_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha + B(x_t^H)^\varphi - \beta(1-\delta)E_t \left\{ \frac{C_t(S_t)^\alpha}{C_{t+1}(S_{t+1})^\alpha} \frac{A_{t+1}^H}{A_t^H} B(x_{t+1}^H)^\varphi \right\} \quad (9)$$

and  $W_t^{H,R} = \frac{W_t^H}{P_t}$  is the real wage expressed in terms of the consumption good.

The key difference between the supply side in our model and in a standard New Keynesian model with a neoclassical labor market is the behaviour of the real marginal cost. In a model with a competitive labor market the real marginal cost is strictly related to the evolution of the real wage:

$$MC_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha$$

In our model, which embeds the NK model as a special case, the presence of hiring costs creates a wedge between the real wage and the marginal costs relevant for the firm, which in turn are essential to explain inflation dynamics. This wedge consist of two terms. The first,  $B(x_t^H)^\varphi$ , represents the additional cost the firm faces to hire a new worker; the second - the last term in (9) - reflects the savings in future hiring costs resulting from increasing the number of employees today. The cyclical behaviour of marginal costs in a model with labor market frictions can thus depart substantially from that of real wages<sup>19</sup>.

In a symmetric equilibrium,  $P_t^H(i) = P_t^H$  for all  $i \in [0, 1]$ , and hence the optimal price setting implies:

$$MC_t^H = \frac{1}{\mu}$$

for all t. When shocks occur, each firm varies its prices and hiring decisions to keep the marginal cost constant.

To get a full characterization of the equilibrium, we now need to specify a mechanism of wage determination.

## 4.2 Equilibrium with Nash Bargained Wages

In this model, the presence of hiring costs creates a positive rent for existing employment relationships. Following much of the literature, we assume that wages are bargained to split this rent between the firm and the employee, according to their respective bargaining power.

Let  $\eta$  denote the relative weight of workers in the Nash bargaining for the home country<sup>20</sup>. It can be shown (see the Appendix for details) that the *Nash wage schedule for home* is given by:

<sup>19</sup>Krause and Lubik (2005) make a similar argument comparing a standard NK model with a model with search and matching frictions in the labor market. See p.10-11.

<sup>20</sup>If we denote by  $\zeta$  the relative bargaining power of workers, it is easy to show that

$$\eta = \frac{\zeta}{1-\zeta}$$



$$\frac{W_t^{H,Nash}(S_t)^\alpha}{A_t^H} = \chi \frac{C_t^{H,W} (N_t^H)^\phi}{A_t^H} + \eta \left\{ B(x_t^H)^\varphi - \beta(1-\delta)E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} \left[ (1-x_{t+1}^H) B(x_{t+1}^H)^\varphi \right] \right\} \right\} \quad (10)$$

where  $W_t^{H,Nash}$  denotes the Nash bargained wage (in real terms).

Intuitively, the Nash wage depends on the reservation wage (here given by the the marginal rate of substitution between leisure and consumption,  $\chi \frac{C_t^{H,W} (N_t^H)^\phi}{A_t^H}$ ) plus a “wage premium”, which depends on the size of the rents for existing employment relationships (the term in curled brackets) and on the workers’ relative share of the surplus,  $\eta$ .

Substituting this wage rule in (9), we obtain the *equilibrium under Nash bargaining*:

$$\frac{\chi C_t^{H,W} (N_t^H)^\phi}{A_t^H} = \frac{1}{\mu} - (1+\eta)B(x_t^H)^\varphi + \beta(1-\delta)E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} \left[ (1+\eta(1-x_{t+1}^H)) B(x_{t+1}^H)^\varphi \right] \right\} \quad (11)$$

This condition determines the evolution of (un)employment under flexible prices and wages. It is easy to verify that the decentralised equilibrium with Nash bargained wages involves a constant job-finding rate and, hence, a *constant level of unemployment*<sup>21</sup>.

Combining the equilibrium under Nash bargaining and the Nash wage rule, we can determine the equilibrium behaviour of real wages:

$$\frac{W_t^{H,Nash}}{A_t^H} (S_t)^\alpha = \Theta \quad (12)$$

where  $\Theta = \frac{1}{\mu} - [1 - \beta(1-\delta)] B(x_M^H)^\varphi$  is the (constant) equilibrium real wage, which depend on the separation rate  $\delta$ , on the markup  $\mu$  and on the (constant) equilibrium job-finding rate  $x_M^H$ <sup>22</sup>.

The equilibrium wage moves one for one with  $A_t^H (S_t)^{-\alpha}$ . Notice also that, since the employment level is constant at home and abroad, the terms of trade  $S_t$  vary proportionally to  $\frac{A_t^H}{A_t^F}$ . Under flexible prices and wages, asymmetric productivity shocks at home or at foreign are neutralized by changes in the wage rate and thus do not affect firms’ incentives to hire people; as a result, unemployment is unchanged. In other words, movements in the term of trade, reflected in movements in the real wage, imply that employment does not vary with productivity shocks.

<sup>21</sup>To see this, notice that  $C_t^{H,W} = C_t^H + C_t^{H,*} = A_t^H (N_t^H - B(x_t^H)^\varphi h_t^H)$ . It follows that  $\frac{C_t^{H,W}}{A_t^H}$  does not depend on  $A_t^H$ . Again, this result derives from the assumption of a utility function that is log in consumption.

<sup>22</sup>The labor market tightness under flexible wages,  $x_M^H$ , is solution of (11). It can be shown that  $x_M^H$  is invariant to Home and Foreign technology shocks.

Compare the equilibrium under the efficient allocation (7) and under the decentralised equilibrium (11). While the (un)employment level is constant in both cases, these levels generally differs. Mainly due to the monopolistic distortions, the employment level under the efficient allocation is higher than the one prevailing in the decentralised solution. It is easy to verify that the conditions under which the two equilibria correspond, are the following<sup>23</sup>:

1. Perfect competition in the goods market, i.e.  $\mu = 1$  (or alternatively a production subsidy offsetting the distortions arising from monopolistic competition in goods market).
2.  $\varphi = \eta$ , i.e. the share of the surplus that goes to workers has to coincide with the elasticity of hiring costs with respect to the job-finding rate (the Hosios condition).

### 4.3 Introducing Real Wage Rigidities

As Christoffel and Linzert note, “sudden and significant shifts in the aggregate wage level are not observed. Due to collective wage bargaining agreements, wage changes only take place on a quite infrequent basis. Therefore, a wage that can be freely adjusted each period assumes a degree of wage flexibility that is hardly consistent with actual practises”.<sup>24</sup> Moreover, Hall (2005) shows that the introduction of sticky wages improves the behaviour of labor market models, as it increases the sensitivity of labor-market conditions - and hence unemployment - to productivity shocks.

Accordingly, and following much of the recent literature, we introduce real wage rigidity by employing a version of Hall’s (2005) notion of wage norm. A wage norm may arise as a result of social conventions that constrain wage adjustment for existing and newly hired workers. One way to model this is to assume that the real wage  $W_t^{H,R}$  is a weighted average of the Nash bargained wage  $W_t^{H,Nash}$  and a wage norm  $\bar{W}^H$ . Specifically, we assume the real wage is determined as follows:

$$W_t^{H,R} = \left(W_t^{H,Nash}\right)^{1-\gamma} (\bar{W}^H)^\gamma, \quad W_t^{F,R} = \left(W_t^{F,Nash}\right)^{1-\gamma^*} (\bar{W}^F)^{\gamma^*} \quad (13)$$

For home and foreign respectively. The wage norm  $\bar{W}^H$  ( $\bar{W}^F$  for foreign) is simply the wage prevailing in steady state while  $\gamma$  ( $\gamma^*$ ) is an index of the real wage rigidities present in the economy, with  $0 \leq \gamma^i \leq 1$ . The introduction of such a wage rule modifies the decentralised equilibrium solution<sup>25</sup>. As shown before, under flexible wages the Nash bargained wage varies proportionally to  $A_t^H (S_t)^{-\alpha}$  and thus neutralizes the effect of productivity changes on employment and on the labor market tightness indicator, which

<sup>23</sup>Blanchard-Gali (2006) obtain the same conditions in the context of a one country model.

<sup>24</sup>Christoffel and Linzert (2005), p. 17-18.

<sup>25</sup>To see this, consider for instance the Home country. In equilibrium:

$$\frac{\left(W_t^{H,Nash}\right)^{1-\gamma} (\bar{W}^H)^\gamma}{A_t^H} (S_t)^\alpha = \frac{1}{\mu} - B \left(x_t^H\right)^\varphi + \beta(1-\delta)E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} B \left(x_{t+1}^H\right)^\varphi \right\}$$

Even if  $W_t^{H,Nash}$  varies proportionally to  $A_t^H (S_t)^{-\alpha}$ , the real wage  $W_t^{H,R} = \left(W_t^{H,Nash}\right)^{1-\gamma} (\bar{W}^H)^\gamma$  varies less than proportionally to  $A_t^H (S_t)^{-\alpha}$ . As a consequence, wages do not move enough to neutralize the effects of productivity shocks on employment, and employment is not invariant to productivity shocks.

are both constant. When real wage rigidities are present, instead, the wages do not move enough to absorb the impact of technology shocks. As a result, in a decentralised equilibrium with sticky wages, employment and the labor market tightness will *not* be constant. As in Blanchard and Gali (2005 and 2006), the presence of real wage rigidities introduces a substantial difference between the constrained efficient solution (where employment is constant) and the decentralised solution (where employment varies with productivity shocks). For this reason, to the extent that  $\gamma$  or  $\gamma^*$  are different from zero, it is not possible for the monetary authority to stabilize simultaneously inflation and unemployment. There is no “divine coincidence”.

## 5 Introducing Sticky Prices

We introduce nominal price rigidity using the formalism à la Calvo (1983). Each period, firms may reset their prices with a probability  $\theta$  (independent of the time elapsed since the last revision of prices). The expected time over which the price is fixed is therefore  $\frac{1}{1-\theta}$ .

Log-linearizing around a zero inflation steady state the optimal price setting rule and the price index equation  $P_t^H = \left[ (1-\theta)(\tilde{P}_t^H)^{1-\epsilon} + \theta(P_{t-1}^H)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$ , we get the New Keynesian Phillips Curve:

$$\hat{\pi}_t^H = \beta E_t \hat{\pi}_{t+1}^H + \lambda \widehat{m}c_t^H \quad (14)$$

where  $\hat{\pi}_t^H$  is domestic (i.e. producer prices’) inflation,  $\widehat{m}c_t^H$  represent the log deviation of real marginal cost from its steady state value and  $\lambda = (1 - \beta\theta)(1 - \theta)/\theta$ . Note that, while (14) looks like a standard New Keynesian Phillips Curve, the dynamics of the real marginal costs are now substantially different from the ones of a standard NK model, as they are deeply affected by the labor market institutions. In fact, log-linearizing eq. (9) we can rewrite marginal costs as:

$$\widehat{m}c_t^H = h_0 \hat{n}_t^H + h_1 \hat{x}_t^H - h_2 E_t \{ \hat{x}_{t+1}^H \} - \gamma h_3 (\hat{a}_t^H - \alpha \hat{s}_t) \quad (15)$$

where variables with hat denote log-deviations from steady state and the coefficients  $h_i$  are functions of the structural parameters characterizing the economy: workers’ bargaining power, hiring costs, separation rates, markups, degree of nominal stickiness or of real wage rigidity, and so on. The introduction of hiring costs and real wage rigidities substantially change the dynamics of the marginal costs, which in turn influence the firms’ optimal price setting and the inflation dynamics<sup>26</sup>.

Equation (15) highlights the determinants of marginal costs. Marginal costs increase with employment ( $\hat{n}_t^H$ ) as the firm has to pay higher wages to persuade households to provide more labor. This is the only channel at work in the standard NK model. The worsening of labor market conditions at time  $t$  (i.e. an increase of  $\hat{x}_t^H$ ) increases marginal costs through two channels. A more tight labor market, in fact, increase both the hiring costs and the bargained wage, as the rents associated to an existing employment relationship are higher. An expected increase of  $E_t \hat{x}_{t+1}^H$ , instead, has the opposite effect, as it

<sup>26</sup>It can be shown that  $h_0 = \mu \chi_0 (N^H)^{1+\phi} (1 + \phi) (1 - \gamma)$ ,  $h_1 = g\mu\varphi(1 + \eta(1 - \gamma))$ ,  $h_2 = g\mu\beta \left[ \varphi + \eta(1 - x)(1 - \gamma) \left( \varphi - \frac{x^H}{1-x^H} \right) \right]$ , and  $h_3 = \mu W^H (S)^\alpha$ .

becomes convenient for the firm to hire at time  $t$  in order to be ready for a more difficult labor market in time  $t + 1$ . Finally, when the real wage adjust sluggishly to the economic activity (i.e.  $\gamma > 0$ ), marginal costs depend negatively on productivity shocks ( $\hat{a}_t^H$ ) and positively on the terms of trade ( $\hat{s}_t$ ). The reason is that when there is a productivity shock (or a change in the terms of trade), wages do not increase (decrease) as much as under Nash bargaining, and hence marginal costs decrease (increase) by more.

## 6 Equilibrium Fluctuations under sticky prices

If prices are sticky, monetary policy matters. In a closed economy model, the presence of staggered price setting typically leads to an inefficient dispersion of inflation and output across resources produced using the same technology. In an open economy model, price stickiness creates an additional source of distortion: as prices are not free to adjust, the terms of trade typically follow an inefficient path in response to asymmetric disturbances<sup>27</sup>. In this model, two new elements deeply affect the economy: the presence of labor market imperfections and real wage rigidities. In the following we show that these elements matter substantially for the dynamic behaviour of the economy and, in particular, for the transmission mechanism of monetary policy.

Given a variable  $X$ , we denote with  $\hat{X}$  the deviation of a variable from its steady state and union-wide variables are defined as  $X^U = \frac{X^H + X^F}{2}$ . Denoting with  $\hat{u}_t^i = u_t^i - u^i$  the deviations of (after-hiring) unemployment from its steady state level  $u$  and using the approximation  $\hat{u}_t^i = -(1 - u^i) \hat{n}_t^i$ , it is possible to express the union-wide IS equation as:

$$\hat{u}_t^{Ag} = E_t \hat{u}_{t+1}^{Ag} + (\hat{i}_t - E_t \hat{\pi}_{t+1}^U - E_t \Delta \hat{a}_{t+1}^U) \quad (16)$$

where  $\hat{\pi}_t^U$  and  $\hat{a}_t^U$  are union-wide inflation and productivity<sup>28</sup> and, to simplify the notation, we define aggregate unemployment as a weighted average of the unemployment rates prevailing in each country:  $\hat{u}_t^{Ag} = \frac{1}{2} \left( \frac{\hat{u}_t^H}{1-u^H} + \frac{\hat{u}_t^F}{1-u^F} \right)$ <sup>29</sup>. Solving (16) forward, we get

$$-\hat{u}_t^{Ag} = -E_t \sum_{j=0}^{\infty} [(\hat{i}_{t+j} - E_t \hat{\pi}_{t+j+1}^U) - E_t \Delta \hat{a}_{t+j+1}^U]$$

Hence, the unemployment fluctuations are driven by the expected future path of real interest rates and productivity differentials (the latter mimic the natural interest rate

<sup>27</sup>In an open economy model, this problem has typically a (at least partial) solution: the exchange rate. Movements in the exchange rate in fact may provide some additional flexibility to the terms of trade. This instrument, however, is absent in a monetary union. See, for instance, Benigno (2004) and Pappa (2002) for a discussion of the welfare properties of monetary unions.

<sup>28</sup>Note that expected changes in productivity shocks enter the IS equation because in equilibrium:

$$\hat{c}_t^U = \hat{a}_t^U + \hat{n}_t^U = \hat{a}_t^U - \hat{u}_t^{Ag}$$

Note also that  $\hat{i}_t$  is defined as the deviation of the nominal interest rate from the interest rate in a zero inflation steady state:

$$\hat{i}_t = i_t - i$$

where  $i = (\log \beta^{-1})$ .

<sup>29</sup>It can be shown that  $\hat{u}_t^{Ag} = -\hat{n}_t^U$ .

that would prevail under the constrained efficient allocation). While the real interest rate affects aggregate (union) unemployment, terms of trade movements distribute production among the two countries and explain unemployment and consumption differentials:

$$\begin{aligned}\hat{s}_t - \bar{s}_t &= \frac{1}{1 - u^F} \hat{u}_t^F - \frac{1}{1 - u^H} \hat{u}_t^H \\ \hat{c}_t &= \hat{c}_t^* + (1 - 2\alpha) \hat{s}_t\end{aligned}\quad (17)$$

where  $\bar{s}_t = \hat{a}_t^H - \hat{a}_t^F$  denotes the evolution of the terms of trade under the constrained efficient solution. In this model, inefficient terms of trade movements ( $\hat{s}_t - \bar{s}_t$ ) are reflected in an inefficient allocation of production among the two regions - and thus inefficient unemployment fluctuations. Notice also that, notwithstanding the presence of complete markets, as long as there is home bias in consumption (i.e.  $\alpha < \frac{1}{2}$ ) the PPP does not hold and consumption is not equated in equilibrium.

The supply block of the model contains the aggregate supply equations for home:

$$\hat{\pi}_t^H = \beta E_t \hat{\pi}_{t+1}^H - \kappa_0 \hat{u}_t^H + \kappa_1 \hat{u}_{t-1}^H + \kappa_2 E_t \hat{u}_{t+1}^H - \gamma \kappa_3 \hat{a}_t^H + \alpha \gamma \kappa_3 \hat{s}_t \quad (18)$$

and foreign:

$$\hat{\pi}_t^F = \beta E_t \hat{\pi}_{t+1}^F - \kappa_0^* \hat{u}_t^F + \kappa_1^* \hat{u}_{t-1}^F + \kappa_2^* E_t \hat{u}_{t+1}^F - \gamma^* \kappa_3^* \hat{a}_t^F - \alpha \gamma^* \kappa_3^* \hat{s}_t \quad (19)$$

Note that the coefficients are functions of the structural parameters characterizing the two economies: workers' bargaining power, hiring costs, separation rates, markups, degree of nominal stickiness or of real wage rigidity, and so on<sup>30</sup>. The introduction of hiring costs and real wage rigidities substantially change the dynamics of the marginal costs, which in turn influence the firms' optimal price setting and the inflation dynamics.

In open economy models, an implicit inertia in the inflation rate is inherent. In fact, from the definition of the terms of trade  $S_t = \frac{P_t^F}{P_t^H}$  we get the following relationship between the terms of trade and the domestic inflation rates:

$$\hat{s}_t - \hat{s}_{t-1} = \hat{\pi}_t^F - \hat{\pi}_t^H \quad (20)$$

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<sup>30</sup>It can be shown that, for  $i = H$  or  $F$ :

$$\begin{aligned}\kappa_0^i &= \frac{1}{1 - u^i} \lambda^i \left( g^i \Psi_0^i b_0^i + g^i \Psi_1^i b_1^i + \Psi_2^i \right) \mu^i \\ \kappa_1^i &= \frac{1}{1 - u^i} \lambda^i \mu^i g^i \Psi_0^i b_1^i \\ \kappa_2^i &= \frac{1}{1 - u^i} \lambda^i \mu^i g^i \Psi_0^i b_0^i \\ \kappa_3 &= \lambda \mu W^{H,R} (S)^\alpha; \quad \kappa_3^* = \lambda^* \mu^* W^{F,R} (S)^{-\alpha} \\ b_0^i &= \frac{1}{\delta^i} \\ b_1^i &= \frac{1}{\delta^i} (1 - \delta^i) (1 - x^i)\end{aligned}$$

where  $\Psi_0^i = \varphi (1 + \eta^i (1 - \gamma^i))$ ,  $\Psi_1^i = \beta \left[ \varphi + \eta^i (1 - \gamma^i) (1 - x^i) \left( \varphi - \frac{x^i}{1 - x^i} \right) \right]$  and  $\Psi_2^i = (1 - \gamma^i) \chi^i (N^i)^{1+\phi} (1 + \phi)$ .

As Benigno (2004) notes, “If monetary policy is not able to eliminate the link between the inflation rate and the terms of trade, inflation itself will be a function of its past values”.<sup>31</sup>

Equations (16), (17), (18), (19), (20), together with a specification of monetary policy, completely characterize our equilibrium dynamics.

In a currency union with labor market imperfections and nominal rigidities, not all unemployment and inflation differentials are a desirable equilibrium phenomenon. Our model permits to distinguish exactly the portion of these differentials that is efficient from the inefficient ones. It is easy to check that the efficient evolution of inflation differentials is determined by:

$$\bar{\pi}_t^F - \bar{\pi}_t^H = \Delta \hat{a}_t^H - \Delta \hat{a}_t^F \quad (21)$$

The efficient response of inflation at home and at foreign requires that the change in prices exactly offsets the change in technology, leaving employment unaffected. Inflation differentials that depart from this solution are inefficient and are due to the interactions between price stickiness, real wage rigidities and labor market frictions. In the following, we will use equation (21) to determine the portion of the inflation differentials that is inefficient and analyse how this relates to the underlying economic structure.

## 6.1 Baseline calibration

Before proceeding, let’s discuss our calibration strategy. In our baseline calibration, we assume that home and foreign are perfectly symmetric. The parameters are chosen to be largely consistent with those standard in the New Keynesian literature. The following table summarises the values for the key parameters of our model (for  $i = H$  or  $F$ ):

Preferences	$\beta$	$\phi^i$	$\epsilon^i$	$\mu^i$	$\alpha$
	0.99	1	6	1.2	0.2
Technology	$A^i$	$\varphi^i$			
	1	1			
Labor market	$u^i$	$x^i$	$\delta^i$	$\eta^i$	
	0.08	0.5	0.087	1	
Price and Real Wage rigidities	$\theta^i$	$\gamma^i$			
	0.75	0.5			
Shocks’ Persistence and Volatility	$\rho_a^i$	$\rho_{a,a^*}$	$\sigma_a^i$	$\sigma_\varepsilon$	
	0.95	0.258	0.0085	0.002	

*Preferences:* Time is taken as quarters. The discount factor  $\beta$  is set equal to 0.99, which implies a riskless annual return of about 4 percent. We assume the labor supply elasticity to be  $\phi^i = 1$ , as in Galì (2002). The elasticity of substitution between differentiated goods  $\epsilon^i$  is set equal to 6, corresponding to a markup  $\mu^i = 1.2$ . Finally, the home bias parameter  $\alpha$ , representing the share of imported goods on total consumption, is set to 0.2.

*Technology:* Following Blanchard and Galì (2006) we set the parameter  $\varphi^i$  in the hiring cost function, representing the sensitivity of hiring costs to labor market conditions, to be

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<sup>31</sup>See Benigno (2004), p. 11.

$\varphi^i = 1^{32}$ . The steady state level of productivity  $A^i$  is normalized to 1.

*The labor market:* In the baseline calibration, we set unemployment in country  $i$  to be  $u^i = 0.08$ , which is roughly consistent with the average unemployment in Europe. The job-finding rate  $x^i$  is set to 0.5, which corresponds approximately to a monthly rate of 0.15. Given  $u^i$  and  $x^i$ , it is possible to determine the separation rate using the relation  $\delta^i = u^i x^i / ((1 - u^i)(1 - x^i))$ . We obtain a value  $\delta^i = 0.087$ . The relative bargaining power  $\eta^i$  is set to 1, which implies that firms and workers have the same bargaining power. The scaling parameter  $B^i$  is chosen such that hiring costs represent a 1 percent fraction of steady state output, as in Walsh (2005)<sup>33</sup>. The parameters  $\chi^i$  can then be determined using steady state identities.

*The degree of Price rigidity*  $\theta^i$  is set equal to 0.75, as in Galì (2002), implying an average duration of price contracts of one year. In the baseline calibration, following Campolmi and Faia (2006) and Blanchard and Galì (2006), we set the degree of real wage rigidity  $\gamma^i$  equal to 0.5.

*Shocks:* Following Backus, Kehoe and Kydland (1992) we set the correlation between the productivity shocks  $\rho_{a,a^*}$  to 0.258 and the standard deviation of productivity shocks  $\sigma_a^i$  to 0.0085. Following Walsh (2005), we set the standard deviation of the policy shock  $\sigma_\varepsilon = 0.002$ .

## 6.2 Real Wage Rigidities and the Inflation/Unemployment Trade-off

In this section, we characterize the dynamics of a currency union composed by two countries that are perfectly symmetric. This is not a realistic scenario, but it allows us to highlight some interesting facts of the model and it can constitute a good benchmark for comparing more general frameworks.

The “complete symmetry” assumption implies that the two regions have the same parameter values:

$$\kappa_i^* = \kappa_i \text{ and } \gamma = \gamma^*$$

In this case, we can find a simple expression for the union-wide NK Phillips Curve:

$$\hat{\pi}_t^U = \beta E_t \hat{\pi}_{t+1}^U - \kappa_0 \hat{u}_t^U + \kappa_1 \hat{u}_{t-1}^U + \kappa_2 E_t \hat{u}_{t+1}^U - \gamma \kappa_3 \hat{a}_t^U \quad (22)$$

where union wide variables are defined as before and we use the fact that, under complete symmetry,  $\hat{u}_t^U = \hat{u}_t^{Ag}$ . Equation (22) has the same interpretation as a closed-economy AS equation, in which all variables are substituted with their union correspondents. This is the equation the common central bank takes into consideration when it implements monetary policy. What is important to note is that in our model - even in the case of complete symmetry - stabilizing inflation *does not* stabilize the output gap. To see this, consider first a “Strict Inflation Targeting” strategy, i.e. a strategy aimed at stabilizing inflation at all horizons ( $\hat{\pi}_t^U = 0$  at all  $t$ ). It is well known that, under this strategy,

<sup>32</sup>In order to calibrate  $\varphi^i$ , Blanchard and Galì exploit a simple mapping between their model and the standard search and matching model. See Blanchard and Galì (2006), p. 28.

<sup>33</sup>To pin down  $B^i$ , we use the fact that in steady state hiring costs represent a fraction  $\delta^i g^i = \delta^i B^i (x^i)^{\varphi^i}$  of GDP.

firms have no incentive to change their prices<sup>34</sup>. Accordingly, the dynamics of unemployment replicate exactly the dynamics under flexible prices. From (22) it follows that the unemployment gap evolves according to:

$$\kappa_0 \hat{u}_t^U = \kappa_1 \hat{u}_{t-1}^U + \kappa_2 E_t \hat{u}_{t+1}^U - \gamma \kappa_3 \hat{a}_t^U \quad (23)$$

It is useful to highlight two facts. First, unemployment displays a substantial degree of inertia. In fact, on the one side, the expectations about future labor market conditions affect today's firms' decisions; on the other side, today's hiring costs depend on the employment level at time  $t - 1$ , which is inherited from the past. Second, if real wages are sticky (i.e.  $\gamma$  is different from zero), productivity shocks at home or foreign influence the *unemployment gap*, i.e. affect the wedge between the “natural” and the “efficient” unemployment level.

Secondly, consider a “Strict Unemployment Targeting” policy, a strategy aimed at stabilizing the unemployment gap in each period, i.e.  $\hat{u}_t^U = 0$  at all  $t$ . Iterating forward (22), one gets:

$$\hat{\pi}_t^U = -\gamma \kappa_3 \sum_{s=0}^{\infty} \beta^s E_t \hat{a}_{t+s}^U$$

A “Strict Unemployment Targeting” strategy is unable to stabilize inflation in face of productivity shocks.

Therefore, a positive productivity shock necessarily leads to a reduction in inflation and/or a negative unemployment gap. The reason is that, with real wage rigidities, real wages do not increase one for one with productivity: on the one hand, if monetary policy stabilizes unemployment, marginal costs and hence inflation decrease; on the other hand, if monetary policy stabilizes inflation, unemployment will decrease up to a point where marginal costs are constant. Therefore, as in Blanchard and Galì (2005, 2006), a strategy that is exclusively focused on inflation stabilization is likely not to be optimal anymore, as it may lead to large and persistent unemployment fluctuations.

In the “complete symmetry” special case it is also possible to find a simple expression for inflation differentials in terms of unemployment differentials and shocks:

$$\hat{\pi}_t^R = \beta E_t \hat{\pi}_{t+1}^R - \kappa_0 \hat{u}_t^R + \kappa_1 \hat{u}_{t-1}^R + \kappa_2 E_t \hat{u}_{t+1}^R - \gamma \kappa_3 (1 - 2\alpha) (\hat{a}_t^H - \hat{a}_t^F) + 2\alpha \gamma \kappa_3 \hat{s}_t \quad (24)$$

where  $\hat{\pi}_t^R = \hat{\pi}_t^H - \hat{\pi}_t^F$  and  $\hat{u}_t^R = (\hat{u}_t^H - \hat{u}_t^F)$  denote respectively inflation and unemployment differentials.

Equation (24) shows the existence of a strict relationship between inflation and unemployment differentials, a relationship that depends on the (common) economic structure of the currency union, and is influenced by the degree of real wage rigidity and by the labor market institutions characterizing the currency union.

Notice in particular that, as long as  $\gamma > 0$  and for a given unemployment level, a positive productivity shock in the home country generates a competitiveness gain for home ( $\hat{\pi}_t^R$  decreases): as wages do not fully absorb the increase in productivity, firms' marginal costs decrease when  $\hat{a}_t^H$  increases. Notice however that productivity shocks also have an

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<sup>34</sup>See Galì (2002) for a discussion of this point.



indirect impact through the terms of trade  $\hat{s}_t$ . Thus, a home productivity shock not only influences (24) directly, it also generates - through the identity (20) - a terms of trade depreciation, partially offsetting the direct impact of the shock. The relative strength of the impact of these two factors depends on the degree of trade openness between the two countries, as represented by the home bias parameter  $\alpha$ .

Labor market institutions also affect equation (24) through their impact on the parameters  $\kappa_i$ , which determine the slope of the New Keynesian Phillips Curve and the persistence of differentials. In the following sections, we analyse how different labor market institutions influence these parameters and thus the size and persistence of inflation and unemployment differentials.

### 6.3 Labor market frictions, Real Wage Rigidities and the Slope of the Phillips Curve

How do labor market structures influence the slope of the Phillips Curve? To answer this question, we distinguish between two types of labor market imperfections: *labor market frictions*, which capture the institutions - like employment protection legislation, hiring costs and the matching technology - that limit the flows in and out of unemployment; and *real wage rigidities*, intended to capture all the institutions - including wage staggering and the wage bargaining mechanism and legislation - which influence the responsiveness of real wages to economic activity. These two types of labor market rigidities, while often associated, are likely to have different effects on the dynamics of the economy: in the first case, the rigidity is in “labor quantities”, while in the second case, are “labor prices” that cannot adjust.

Consider for instance the NK Phillips Curve of the home country, which we rewrite here for convenience:

$$\hat{\pi}_t^H = \beta E_t \hat{\pi}_{t+1}^H - \kappa_0 \hat{u}_t^H + \kappa_1 \hat{u}_{t-1}^H + \kappa_2 E_t \hat{u}_{t+1}^H - \gamma \kappa_3 \hat{a}_t^H + \alpha \gamma \kappa_3 \hat{s}_t \quad (25)$$

As already mentioned, different degrees of labor market frictions and real wage rigidities affect the Phillips Curve through their impact on the parameter  $\kappa_i$ . The parameters on lagged, current and future unemployment reflect the impact of unemployment on inflation. In our calibration, the parameters on lagged and future unemployment are small relative to the parameter on  $\hat{u}_t^H$ . Therefore, following Ravenna and Walsh (2007), we identify the slope of the Phillips Curve with  $\kappa_0$ .

Before proceeding, some details on the calibration strategy are needed. To study the role of different degrees of real wage rigidities, we simulate the model varying the index of real wage rigidities  $\gamma$  from 0 to 0.9.

Calibrating the degree of labor market frictions is somehow a more challenging task, as the overall degree of “rigidity” in the labor market does not depend only on one parameter but on all the configuration of the labor market, as captured by the interplay of different parameters. Following Blanchard and Galì (2006), we characterize the degree of labor market frictions by calibrating the steady state unemployment and steady state job-finding rates ( $u^i$  and  $x^i$ ). We define a labor market as “flexible” when the job-finding rate is high and the unemployment rate low; the opposite holds in a “sclerotic” labor market. To perform simulations, we vary simultaneously  $u^i$  and  $x^i$ ; the job-finding rate is then

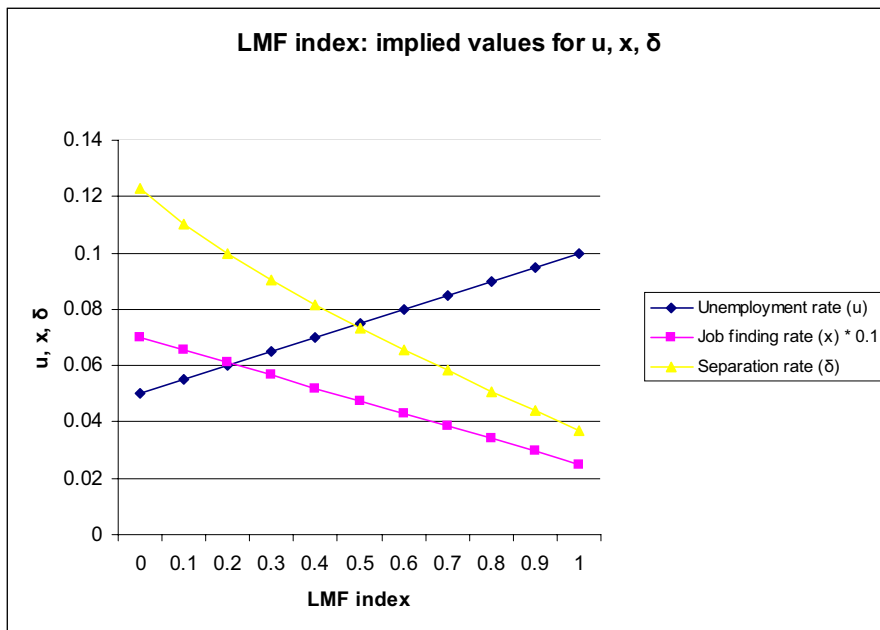


Figure 2: *Calibration of Labor Market Frictions (LMF)*

determined through the steady state relationship  $\delta^i = u^i x^i / ((1 - u^i)(1 - x^i))$ . Figure 2 displays the evolution of the three parameters implied by our calibration strategy (the job finding rate is rescaled by dividing it by 10). Notice that to any particular value of labor market rigidity corresponds a different steady state and that in a rigid economy, as in real data, a low job-finding rate is associated with a low separation rate and a high unemployment rate<sup>35</sup>.

Figure 3 shows how the slope of the Phillips Curve (*PC*) of a country changes for varying degrees of labor market frictions (*LMF*) and real wage rigidities (*RWR*). The picture that emerges is rich of interesting insights.

Ceteris paribus, a higher degree of LMF has a strong, positive and highly non-linear effect on the slope of the PC. For instance, for a degree of RWR as in the baseline calibration, the slope of the PC increases non-linearly with the degree of rigidity in the labor market, passing from around 0.2 for low levels of labor market frictions to almost 3 for high levels. This result is mainly explained by the evolution of the separation rate  $\delta^i$ ,

<sup>35</sup>The unemployment rate vary between 0.05 and 0.10 and the job-finding rate between 0.7 and 0.25; the implied separation rate goes from around 0.12 to 0.04. We decided to calibrate directly the job-finding rate and the unemployment rate because these are more easily estimated than the reservation wage or the separation rate. Notice that the main results do not rely on the particular calibration chosen: in fact, the same results are obtained when we just vary the job-separation rate while maintaining all other parameters constant. The particular calibration adopted has the advantage that it ensures the consistency of the steady state unemployment rate with economic data and economic intuition.

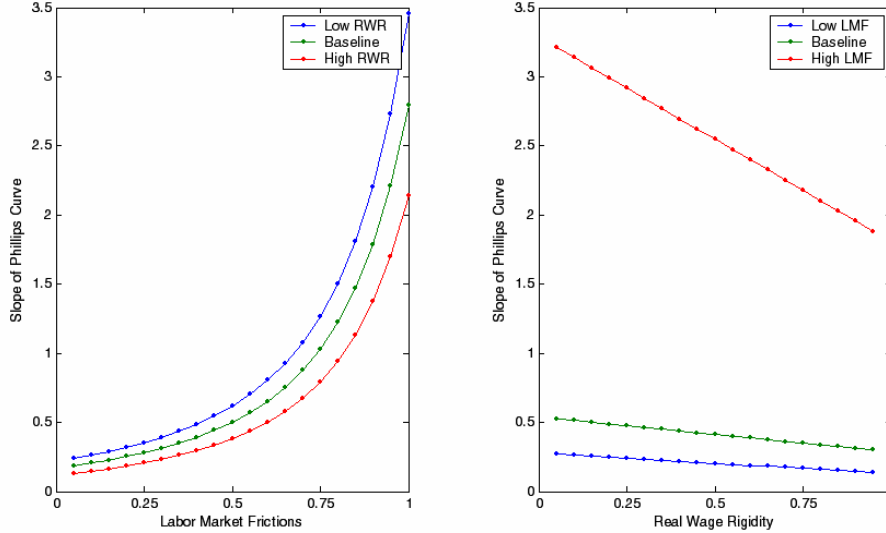


Figure 3: *LMF, RWR and the Phillips Curve*

which is higher the more flexible the economy. As the probability of exogenous separation gets higher, fewer matches survive from one period to the other and employment becomes more sensitive to labor market conditions. This in turn implies that marginal costs and hence inflation become less sensitive to unemployment changes<sup>36</sup>.

Real wage rigidities have the opposite effect on the slope of the Phillips Curve: higher degrees of real wage rigidities lower the sensitivity of real wages and inflation to unemployment changes. The sensitivity of the slope of the PC to the degree of real wage rigidities is smaller than the effect of labor market frictions, at least in the baseline calibration: the slope of the PC decreases linearly from more than 0.5 for the case of no RWR to around 0.3 when the degree of RWR is at its maximum. It is important however to remind here that even if RWR have a smaller impact on the slope of the PC, they determine the size of the trade-off coefficient  $\gamma\kappa_3$  and thus determine to what extent productivity shocks enter as a (negative) cost push shock in the PC, to be absorbed through variations of inflation and/or unemployment<sup>37</sup>.

Looking at the effect of one type of rigidity while maintaining the other constant is informative, but it ignores the existence of important interactions between LMF and RWR on the slope of the PC. Figure 3 highlights how the effect of LMF on the supply side of the economy is magnified when the degree of RWR is low and (especially) how the effect of high RWR is magnified when the economy has flexible labor markets. A very interesting and important result emerges: the effects of RWR and LMF on the PC's slope tend to

<sup>36</sup>See Ravenna and Walsh (2007) for a similar argument in the context of a closed-economy DSGE model with search in the labor market.

<sup>37</sup>It can be shown instead that labor market frictions have a negligible impact on the trade-off coefficient  $\gamma\kappa_3$ .

offset each other when the two types of rigidities are *complements* (in the sense that high RWR are associated with high LMF, or vice versa) while they tend to reinforce each other when they are *substitutes* (in the sense that countries with high LMF have flexible real wages). The slope of the Phillips Curve is maximal when LMF are high and real wages are flexible, while it is minimal when LMF are low and RWR high. Intermediate cases can be determined by different combinations of LMF and RWR. As we will show in the last part of the paper, determining whether wage rigidities and labor market frictions are substitutes or complements will have important implications for monetary policy.

**Result 1 (RWR, LMF and the Phillips Curve):**

*a) Different labor market rigidities have very different effects on the Phillips Curve. LMF have a strong, positive and highly non linear effect on the slope of the Phillips Curve but (almost) no effect on the trade-off coefficients. RWR have a (smaller) negative effect on the slope of the Phillips Curve; they determine, however, the size of the trade-off coefficient  $\gamma\kappa_3$  and thus determine to what extent productivity shocks enter as a (negative) cost push shock in the Phillips Curve.*

*b) LMF and RWR interact in a rich and complex way in determining the slope of the Phillips Curve. When they are substitutes (low RWR are associated with high LMF or vice versa), their effects tend to reinforce each other; when they are complements (high RWR are associated with high LMF), they tend to offset each other.*

## 7 Positive Analysis: The Adjustment Mechanism

A key issue in the debate about the monetary union has concerned how individual countries adjust to common or country-specific shocks, and how the authorities should tackle eventual competitiveness and growth problems (see, e.g., EEAG report 2007). Indeed, after seven years of the Euro, the marked and persistent divergence of growth and inflation among euro-area economies seem to suggest that the adjustment process inside the currency area may not be working efficiently.

In this positive analysis we study how different labor market structures are likely to affect the efficient functioning of a currency union. The main focus is on the evolution of inflation and unemployment differentials, as these reflect the way in which shocks are absorbed in the currency union.

Intuitively, labor market rigidities can affect inflation and unemployment differentials in two main ways. First, the presence of labor market rigidities may affect the size and persistence of unemployment and inflation differentials following asymmetric shocks. Second, symmetric shocks may have asymmetric effects when the two regions have different labor market structures. How do these effects operate? Are they likely to be important or negligible? These are the questions we try to address in the following impulse response analysis.

### 7.1 The Dynamics of the Currency Union

In this section we describe the dynamic behaviour of the model in response to two types of shocks: productivity shocks (symmetric and asymmetric) and monetary policy shocks.

The monetary authority is assumed to follow a Taylor-type interest rate rule:

$$\hat{i}_t = \rho_m \hat{i}_{t-1} + (1 - \rho_m) (1.5 \hat{\pi}_t^U - 0.25 \hat{u}_t^U) + \varepsilon_t \quad (26)$$

and where  $\rho_m = 0.9$ , a value consistent with the empirical evidence on policy rules<sup>38</sup>.

We restrict our attention to two cases that we find particularly interesting. First, we look at asymmetric productivity shocks, and consider how the transmission mechanism of the union changes when both the countries have more rigid wages or more rigid labor markets. Secondly, we look at symmetric productivity and monetary policy shocks, and analyse to what extent asymmetric labor market structures can cause common shocks to be propagated differently in member countries.

### 7.1.1 Asymmetric Shocks and Labor Market Rigidities

Figure 4 displays the responses of unemployment and inflation to a positive technology shock in the home country. Consider first the *baseline calibration* (in red on the graph). On impact, home inflation ( $\hat{\pi}^H$ ) decreases while home unemployment ( $\hat{u}^H$ ) increases. The latter is due to the presence of price rigidity. Because of sticky prices, not all firms are able to reduce prices as they would do under flexible prices. The productivity increase allows firms that cannot reset prices to produce the same amount with less work; consequently, unemployment rises on impact. This unemployment increase is short-lived, as over time more firms can reset their prices and the effect of the productivity shock fades away.

The home productivity shock is transmitted to the foreign country through movements of the terms of trade and through monetary policy: following the reduction in the prices of the home goods, foreign prices also decrease. At the same time, in response to the the reduction in union inflation, the central bank lowers the interest rate to avoid too much deflation. This reduction in interest rates has a stimulating effect on the foreign economy and hence foreign unemployment decreases.

At the aggregate level, union inflation decreases while union unemployment increases<sup>39</sup>. Notice that the effects are very persistent, as they do not fade away after 20 quarters. This persistency come from the interactions of some realistic elements of our framework: nominal prices and real wage rigidities, interest rate smoothing in monetary policy, the presence of labor market frictions.

A positive technology shock at home causes a negative inflation differential (as inflation at home decreases more than at foreign) and a positive unemployment differential (as  $\hat{u}^H$  increases while  $\hat{u}^F$  decreases). To what extent are these differentials a welcome equilibrium adjustment mechanism in the face of asymmetric shocks? To answer this question, notice first that (as a consequence of our simplifying assumption, that imply a constant unemployment rate under the efficient allocation), all unemployment fluctuations in our model are inefficient. Inflation differentials, on the contrary, are partly a welcome

<sup>38</sup>See, e.g, Clarida et al. (2000).

<sup>39</sup>The fact that unemployment increases on impact is consistent with empirical evidence: Galí (1999) shows that aggregate hours decreases on impact in response to productivity shocks.

In Blanchard and Galí (2006) unemployment decreases on impact. The reason is that we have persistence in the Taylor rule (which they dont) and hence interest rates decrease less on impact. Note that our main conclusions are robust to different persistence parameters.

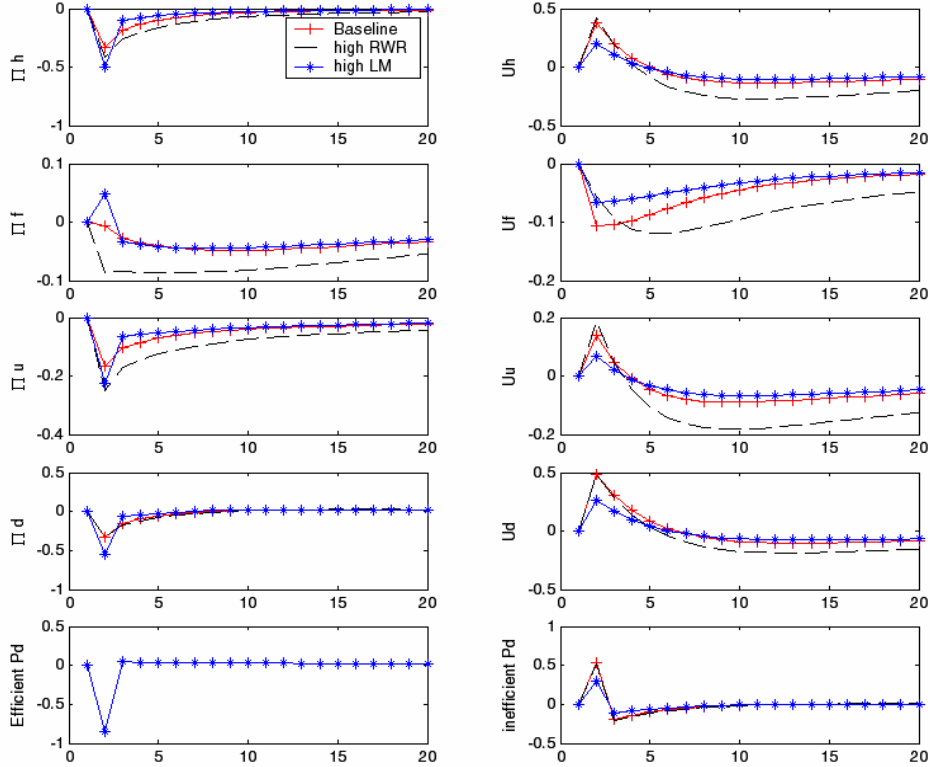


Figure 4: *Impulse Responses to an Asymmetric Productivity Shock (Home)*

equilibrium phenomenon: in fact, by changing the relative demand for home and foreign goods, inflation differentials may help to absorb asymmetric shocks and allow to keep unemployment and output at their efficient level<sup>40</sup>. The last quadrant on the left of Fig. 4 shows the efficient response of the inflation differential: following a home technology shock, prices at home should jump down causing a big (but very short-lived) negative inflation differential. In the model, instead, the reaction of  $\hat{\pi}^d$  is too small on impact, but then persists inefficiently over time. This explains the evolution of the inefficient portion of the inflation differentials, which are initially positive but then turn negative and persists for some period.

The labor market structure has an important influence on the adjustment mechanism of the monetary union to asymmetric shocks. Consider the case in which both countries

<sup>40</sup>Note, however, that in the face of price rigidities, non-zero inflation has important welfare consequences because it leads to inefficient dispersion of employment across varieties. This tension between relative price adjustment and volatility of inflation is an important cost of currency unions. In the follow up we refer to efficient inflation differentials as to the inflation differential that prevails in the absence of price rigidities.

are characterized by a higher degree of RWR (in black in Fig. 4)<sup>41</sup>. A higher degree of RWR amplifies substantially the size and persistency of the response of union inflation and union unemployment to productivity shocks. The reason, as explained before, is that a higher degree of RWR influences directly the trade-off coefficient of the Phillips Curve, which is increasing in  $\gamma$ . The response of inflation differentials is not much affected - even if the response of the member countries changes considerably. Unemployment differentials instead persist much longer when real wages are more rigid.

The effect of a higher degree of labor market frictions, in blue in Fig. 4, is very different. When the countries forming the currency union are characterized by more sclerotic labor markets<sup>42</sup>, the reaction of union inflation is larger than in the baseline calibration, but the response of union unemployment is smoother. Similarly, a higher degree of LMF increases the impact response of inflation differentials while it decreases considerably the size of the unemployment differentials. Interestingly, the inefficient portion of the inflation differential is much reduced by the higher degree of labor market rigidity. This result, while it may seem at odds with conventional wisdom, is very intuitive. When labor markets are more rigid, firms find it more convenient to absorb shocks by changing prices rather than by changing quantities. Since the efficient reaction of the currency union would require a large change in relative prices that maintains the quantities produced (and thus unemployment) unchanged, having more rigid labor markets leads firms to a behaviour that is closer to the efficient one.

**Result 2 (Asymmetric Productivity Shocks):**

*a) A higher degree of RWR amplifies the response of union inflation, of union unemployment and of the unemployment differentials following an asymmetric productivity shock.*

*b) A higher degree of LMF leads to a larger but less persistent response of union inflation and inflation differentials, and to a smaller reaction of union unemployment and unemployment differentials.*

*c) The size of the portion of the inflation differentials which is inefficient is only slightly affected by the degree of RWR, but decreases significantly when the member countries have higher LMF.*

**7.1.2 Symmetric Shocks and Asymmetric Labor Markets**

In this section we analyse the dynamic responses of the currency union following symmetric productivity and monetary policy shocks. We distinguish three cases:

1. Baseline Case: the member countries share the same economic structures
2. Asymmetry in RWR: in the home country real wages are more sticky, i.e. respond less to market forces, than in the foreign country ( $\gamma = 0.7$  while  $\gamma^* = 0.3$ )
3. Asymmetry in LMF: the home country has more rigid labor markets than the foreign country, where labor market “rigidity” means lower job-finding and separation rate. Specifically, the parameters are calibrated as follows<sup>43</sup>:

<sup>41</sup>The impulse responses are computed for  $\gamma = \gamma^* = 0.7$ .

<sup>42</sup>The simulations are performed setting  $u^i = 0.10$  and  $x^i = 0.25$  in both countries.

<sup>43</sup>The calibration of the rigid and the flexible economy follows Blanchard and Galí (2006).

	Separation Rate $\delta^i$	Job-Finding Rate $x^i$
Home	0.04	0.25
Foreign	0.12	0.7

The results are displayed in Figure 5. The interpretation of the dynamics under the baseline calibration is straightforward. Following a union technology shock, union inflation decreases while aggregate unemployment initially increases and then decreases. The effects are again quite persistent. A negative monetary policy shock, represented by a one standard deviation increase in the interest rate, reduces union inflation and increase unemployment. When member countries are perfectly symmetric, the transmission mechanism of shocks is identical and differentials do not arise.

Consider instead the case in which member countries differ in the degree of RWR (in black on the graph). At the aggregate level, after a common productivity shock, asymmetries in the degree of RWR increase the reaction of both union inflation and union unemployment. Inflation and unemployment react much more in the “rigid wages” country, causing very persistent inflation and unemployment differentials. Following a common demand shock, instead, in the rigid country inflation decreases less while unemployment increases by more, causing positive inflation and unemployment differentials.

Notice that in the case of common shocks any adjustment in relative prices, and thus any inflation differential that arises, is inefficient.

Consider now the case in which the asymmetry is in the labor market (in blue in the graph). Again, when the member countries have different labor market institutions, symmetric shocks can have large and long-lasting asymmetric effects. Following a productivity shock, on impact inflation reacts more and unemployment less in the country with “rigid” labor markets than in the flexible economy. The same happens following a monetary policy shock: inflation on impact decreases by more in the rigid country (home) while unemployment increases by more in the flexible country (foreign).

Interestingly, asymmetries in the degree of RWR seem to have a bigger effect on the transmission mechanism of productivity shocks, while LMF have a bigger impact on the transmission mechanism of monetary policy. This is a consequence of the different impact that the two types of rigidities have on the Phillips Curve of the member countries: while RWR are crucial in determining the extent to which productivity shocks enter as negative “cost push shock” in the Phillips Curve, LMF are the main determinant of the slope of the Phillips Curve, which plays a key role in the transmission mechanism of the demand shocks.

More generally, these results suggest that understanding the labor market structure and the sources of rigidities in the wage determination mechanism is crucial to understand the transmission mechanism of shocks in member countries. The response of member countries to changing economic conditions depends substantially on the source of the rigidity. Intuitively, when the rigidity lies in the wage determination mechanism, real wages cannot fully adjust and shocks tend to be absorbed through changes in quantities - unemployment in our case. A higher degree of real wage rigidities thus amplifies the response of the real economy to shocks. When the rigidity lies in the labor market, it is more costly for firms to hire new workers; therefore, shocks tend to be absorbed mainly



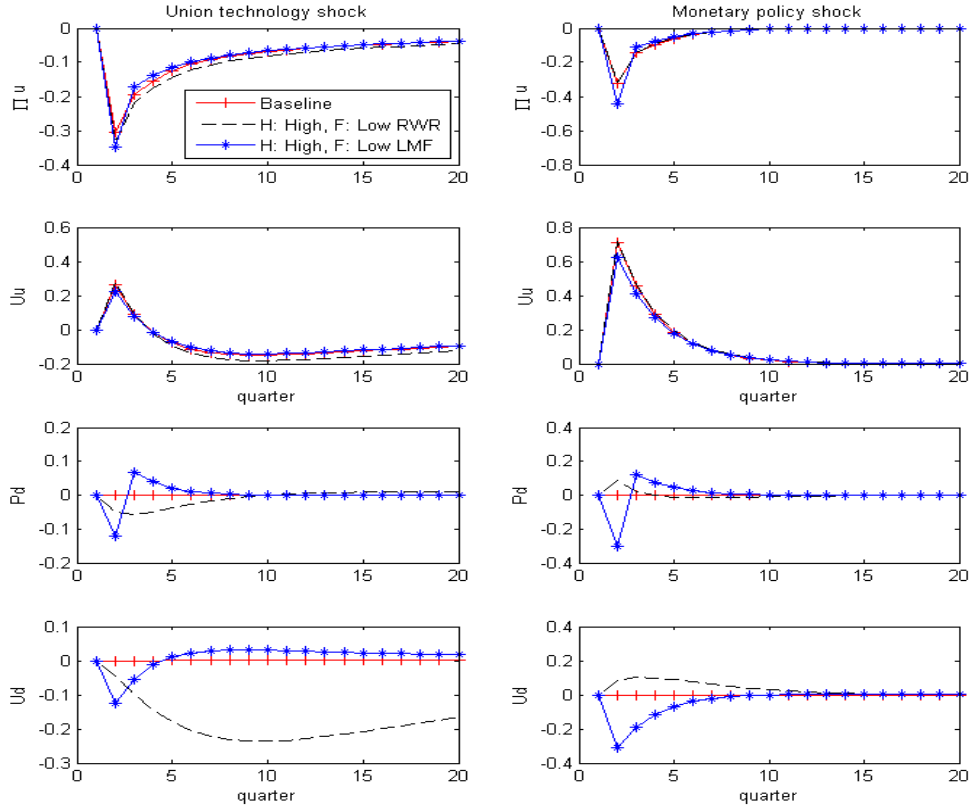


Figure 5: *Impulse Responses to Symmetric Productivity and Monetary Policy Shocks*

through changes in prices, and unemployment does not vary as much as it would in a more flexible economy.

**Result 3 (Asymmetric Effects of Symmetric Shocks):**

a) *When member countries have different labor market structures, symmetric shocks (monetary policy or symmetric productivity shocks) may cause large and long-lasting inflation and unemployment differentials. All these differentials are inefficient.*

b) *A higher degree of RWR amplifies the response of the real economy to shocks: real wage rigidities shift adjustment from prices to quantities.*

c) *A higher degree of LMF increases the response of inflation and reduces the response of unemployment: when it is more costly for the firms to hire new workers, they shift the adjustment from quantities to prices.*

## 7.2 Volatility and Persistence of Differentials

Labor market rigidities are often blamed as one of the possible causes of large and long-lasting inflation and unemployment differentials in the European Monetary Union. In this section, by analysing the effects of different labor market structures on the volatility and persistence of these differentials, we study to what extent this is true. As in the previous section, to make our point clearer we distinguish among two cases. First, we analyse how the differentials depend on the overall degree of rigidity of a currency union formed by symmetric regions. This permits to focus the attention on the functioning of the currency union following asymmetric shocks, as when the regions are symmetric differentials can arise only as a consequence of country-specific shocks. Secondly, we introduce asymmetries in real wages and labor markets to investigate how asymmetries affect the adjustment mechanism of the union.

### 7.2.1 The Role of RWR and LMF

Figure 6 shows the standard deviations and persistence of unemployment and inflation differentials for different calibrations of the labor market structure<sup>44</sup>. Unemployment and inflation differentials are found to depend importantly from the average degree of labor market rigidities in the currency union. Labor market rigidities do not necessarily lead to an increase of the persistence and volatility of differentials: again, it is crucial to distinguish among the institutions that constrain the “quantity” adjustment (LMF) from the ones that constrain the “price” adjustment (RWR).

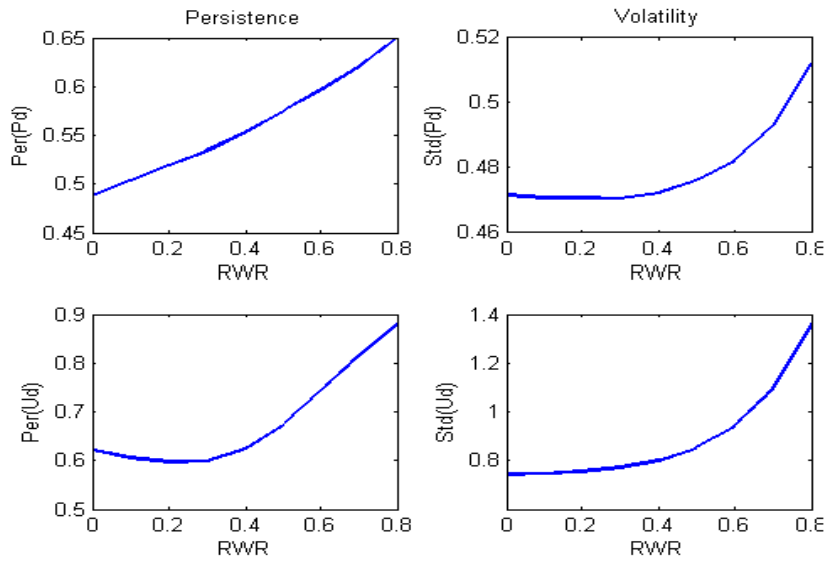
A higher degree of RWR unambiguously worsens the adjustment mechanism of the member countries to shocks, as it increases the persistence and volatility of both inflation differentials and unemployment differentials. The effect on persistence is hardly surprising, as when real wages adjust sluggishly to economic conditions, shocks tend to have longer effects on real variables and these effects are spread, through monetary policy and the endogenous response of firms, to inflation<sup>45</sup>. Regarding the increase in unemployment volatility, the intuition is the same as in Hall (2005): higher real wage rigidities imply that the firms’ share of the match surplus increases strongly with productivity and hence hiring and unemployment is strongly related to movements in productivity. At the same time, the presence of real wage rigidities makes marginal costs and inflation more volatile. This may sound counterintuitive, but the reason is that with real wage rigidities, the firms surplus from a match and hence the costs of hiring an additional worker are much more procyclical.

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<sup>44</sup>Persistence is measured as the sum of the AR coefficients in a univariate AR(5) model (as in Christoffel and Linzert, 2005). Volatility is measured as standard deviations of the simulated series. Notice that since all the other factors characterizing the dynamics of the union (shocks, monetary policy etc.) are maintained constant, we are able to perfectly isolate the effect of different degrees of real rigidities on business cycles.

<sup>45</sup>To understand why RWR increase the persistence of the “boom and bust cycle”, is important to remind that the trade-off coefficient in the Phillips Curve is increasing in  $\gamma$ . As a consequence, the higher is the degree of RWR, the more productivity shocks enter as a cost push shock in the Phillips Curve, to be absorbed through variations of inflation and unemployment. Therefore, the higher is  $\gamma$  the more the differentials inherit the persistence coming from the interaction among nominal rigidities, the persistence of the productivity shocks and the interest rate smoothing in monetary policy.

### The Role of RWR



### The Role of LMF

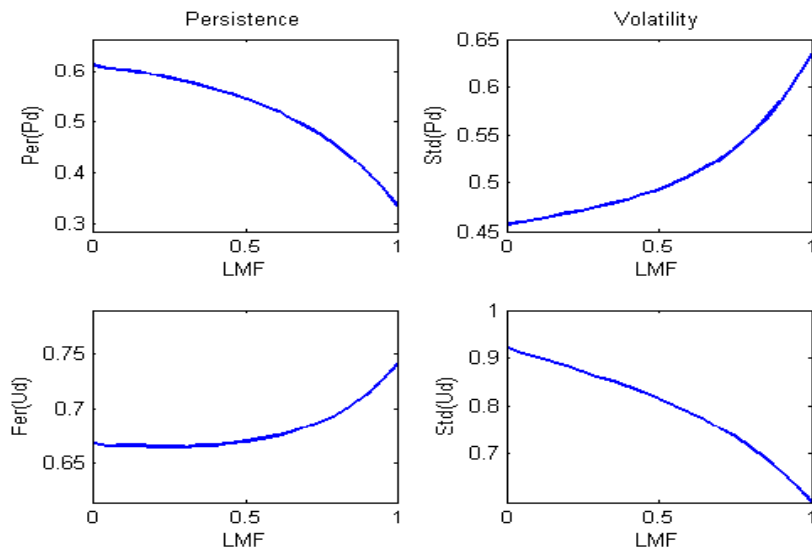


Figure 6: Labor Market rigidities and the Differentials

When the rigidity is on the quantity side of the labor market, the picture is less clear-cut. A higher degree of LMF increases the volatility and reduces the persistence of inflation differentials, while it reduces the volatility but increases the persistence of unemployment differentials. These two results can be reconciled looking at the impulse response functions (see the previous section). When labor markets are more rigid, monetary or productivity shocks are mainly absorbed through a large (but short-lived) change in inflation. Intuitively, when hiring new workers becomes more costly, firms find it relatively more convenient to absorb a shock through changes in prices rather than through changes in the quantities produced. As a consequence, inflation reacts a lot to shocks while the response of output and unemployment is smaller. For this reason, it does not seem that labor market frictions are necessarily worsening the adjustment mechanism of the currency union to asymmetric shocks. In fact, to the extent that movements of relative prices are a desirable stabilization mechanism in a currency union, a higher degree of LMF, by limiting unemployment volatility and increasing the adjustment of the relative prices, may actually improve the adjustment of member countries to asymmetric shocks<sup>46</sup>.

**Result 4 (Labor Market Rigidities and the Volatility and Persistence of Differentials):**

*Different labor market structure have very different effects on the volatility and persistence of inflation and unemployment differentials. RWR increase both the volatility and the persistence of both inflation and unemployment differentials: when real wages cannot adjust, the adjustment mechanism to asymmetric shocks worsens considerably. LMF increase the volatility but decrease the persistence of inflation differentials, while they decrease the volatility but increase the persistence of the unemployment differentials.*

**7.2.2 Are Asymmetries in Labor Markets Important?**

Should European authorities worry about the asymmetries that characterize the member countries? To answer this question, in this section we analyse how labor market asymmetries affect the volatility and persistence of differentials. In order to focus the attention on the effects arising from asymmetries in the economic structures, we construct an index of asymmetry of RWR and LMF that maintains constant the average level of RWR and LMF in the union. The index starts out at 0 where both countries are perfectly symmetric and are calibrated according to the baseline parameters. As the index increases towards 1, the two countries become increasingly asymmetric but the average degree of RWR and LMF does not change.<sup>47</sup>

The results are displayed in Figure 7. Asymmetries in the degree of RWR are found to increase substantially both the persistence and volatility of inflation and unemployment differentials. The reason is extremely simple and intuitive: when asymmetries are present, symmetric shocks are transmitted differently across member countries and, as a consequence, inflation and unemployment differentials arise. What is important to note,

<sup>46</sup>This point should not be taken as an endorsement of a higher degree of labor market frictions. Indeed, labor market frictions may be strongly undesirable on other grounds, from which we abstract here (for instance, strong LMF may have undesirable effects on the equilibrium unemployment level of the economy).

<sup>47</sup>For example, for asymmetric RWR, at 0, both countries have  $\gamma = 0.5$ , but then  $\gamma \rightarrow 1$  and  $\gamma^* \rightarrow 0$  as in the index goes to 1. See Benigno (2004) and Andersen and Seneca (2007) for similar assumptions.

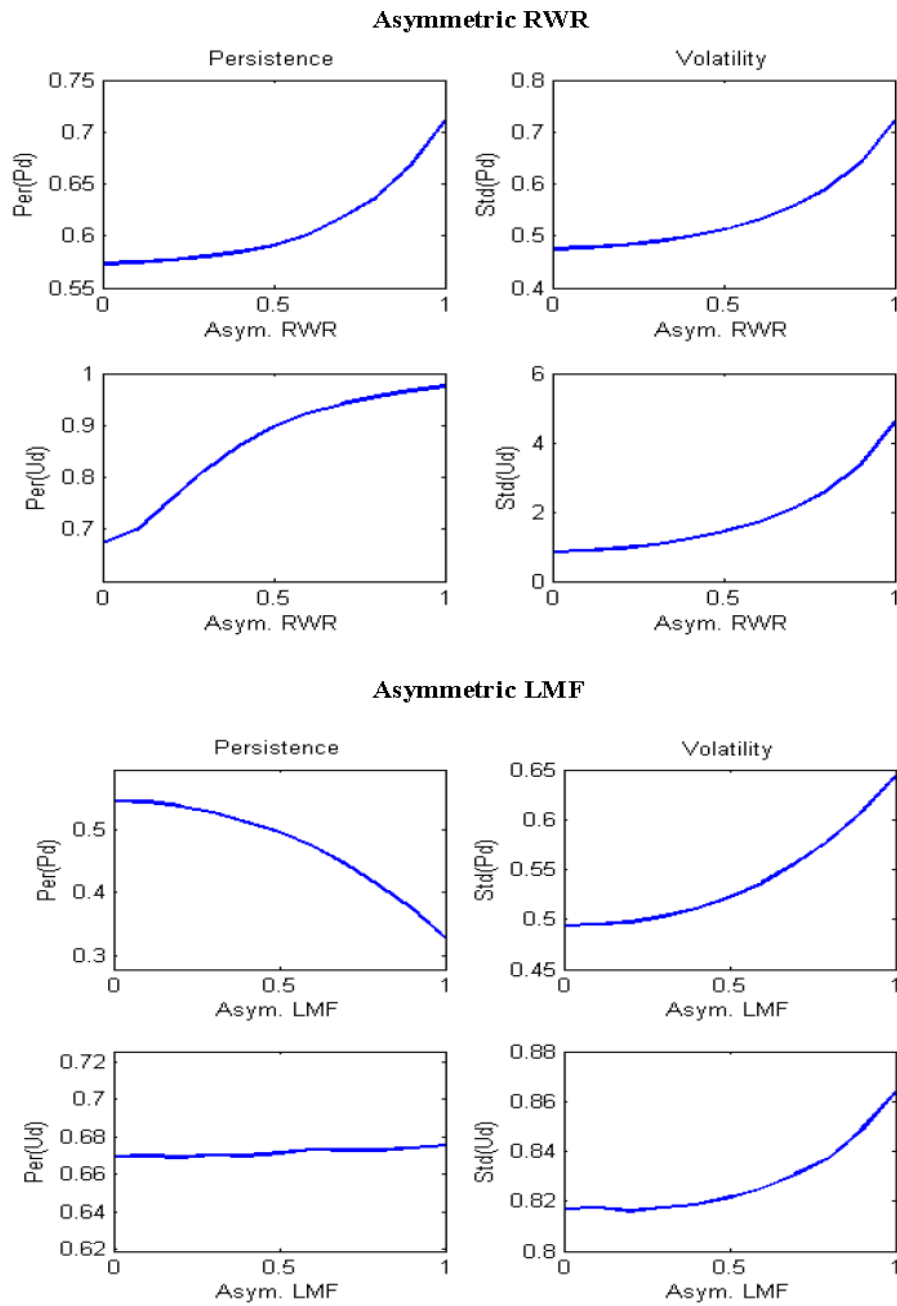


Figure 7: *The Importance of Labor Market Asymmetries for Inflation and Unemployment Differentials*

however, is that for plausible parameter values the effects can be very large. In particular, notice the strong increase in the volatility and persistence of unemployment differentials in the case of strong RWR asymmetries.

Asymmetries in the degree of LMF also matter, even though the effects are somehow weaker. LMF are found to increase the volatility but decrease the persistence of inflation differentials. The volatility and persistence of unemployment differentials are only slightly increased.

These results suggest that the monetary authority should carefully analyse and monitor the development in the labor markets of member countries. Indeed, it can be shown that when member countries are symmetric, the terms of trade is perfectly insulated from monetary policy and the central bank has no means to influence inflation and unemployment differentials arising from asymmetric shocks<sup>48</sup>. When member countries have asymmetric economic structures, the transmission mechanism of monetary policy differs among member countries, and interest rate changes affect the terms of trade. Can the central bank benefit from using “selectively” its policy instrument, that is by trying to move the terms of trade? This is one of the question we try to adress in the following normative part.

**Result 5 (Asymmetric Labor Markets and the Volatility and Persistence of Differentials):**

*Asymmetries in the labor market structures of member states are likely to have sizeable implications for the persistence and volatility of unemployment and inflation differentials. Asymmetries in the degree of RWR increase substantially both the persistence and volatility of inflation and unemployment differentials. LMF are found to increase the volatility but decrease the persistence of inflation differentials. The volatility and persistence of unemployment differentials are only slightly increased.*

## 8 Optimal monetary policy and welfare losses

What is optimal monetary policy in a currency union characterized by different and asymmetric labor market rigidities? What are the optimal targets for the policy maker? To analyze these questions, we derive a loss function from the welfare criterion of the currency union, which we define as the utilitarian social welfare function. A second order approximation to the welfare criterion delivers the utility loss from steady state utility

$$L_0 \simeq E_0 \sum_{t=0}^{\infty} \beta^t \left[ \omega_p (\hat{\pi}_t^H)^2 + \omega_p^* (\hat{\pi}_t^F)^2 + \omega_u (\hat{u}_t^H)^2 + \omega_u^* (\hat{u}_t^F)^2 \right] + t.i.p$$

where  $\omega_p = \frac{\epsilon}{\lambda^2}$ ,  $\omega_p^* = \frac{\epsilon^*}{\lambda^{*2}}$ ,  $\omega_u = \frac{1}{2}(1+\phi)\chi_0(1-u^H)^{\phi-1}$ ,  $\omega_u^* = \frac{1}{2}(1+\phi)\chi_1(1-u^F)^{\phi-1}$  and where t.i.p. refers to “terms independent of policy”. Notice that the monetary authority of the currency union cares about the variability of inflation at home and at foreign because inflation leads to dispersion in output across varieties. The weight on unemployment is increasing in the curvature of utility over consumption (with unit elasticity of intertemporal substitution) and employment ( $\phi$ ).<sup>49</sup> The loss is reported in percentage deviation from

<sup>48</sup>See, e.g., Benigno (2004) for a careful discussion of this point.

<sup>49</sup>Note that our results are robust, across different levels of  $\phi$ . The reason is that increasing  $\phi$ , increases not only the weight on unemployment in the loss function but also it reduces the volatility of unemployment

steady state consumption. To simplify the analysis, the approximation of the welfare criterion has been derived under the assumption that the steady state allocation of the decentralized economy corresponds to the constrained efficient allocation. The steady state allocation corresponds to the constrained efficient allocation if two conditions are met: 1. Hosios condition:  $\eta = \varphi$  2. The elimination of the monopolistic distortion through a labor subsidy.

Optimal monetary policy can be derived in a two step procedure<sup>50</sup>:

1. Find the optimal path for  $\{\hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^H, \hat{u}_t^F\}_{t=0}^{\infty}$  subject to the two Phillips Curves (equations 18 and 19) and the inflation differential equation (equation 20).
2. From the Euler equation, back out the interest rate that implements the path chosen in (1.).

There is no analytical solution to the problem of optimal monetary policy, even in the case of complete symmetry<sup>51</sup>.

## 8.1 The policy frontier

Before evaluating suboptimal rules and their welfare losses, we illustrate the trade-offs that monetary policy faces by calculating the Pareto policy frontier, that is the locus of the best combination of union inflation and union unemployment volatility that the central bank can achieve. The frontier is calculated by computing the volatility of union inflation and unemployment under the optimal monetary policy and for arbitrary weights in the loss function<sup>52</sup>. Figure 8 shows how the Pareto policy frontiers varies with different degrees of RWR and LMF. An increase in LMF changes the slope of the policy frontier,

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in the economy (because it makes the reasevation wage more procyclical). It turns out that these two effects balance eachother, such that the prescriptions for optimal monetary policy do not vary strongly with  $\phi$ .

<sup>50</sup>Note that our specification differs slightly from the specification of Blanchard and Gali (2006) in terms of how we define the wage rule and in the way we introduce wage rigidity.

<sup>51</sup>Even under optimal policy under commitment, in this model the monetary authority is unable to replicate the efficient allocation and the losses  $L > 0$ . We evaluate the welfare losses of suboptimal policy rules by using

$$L_0^{subopt} - L^{timeless}$$

where  $L_0^{subopt}$  refers to the Loss under the suboptimal policy rule and  $L^{timeless}$  refers to the Loss under the optimal policy under commitment (timeless perspective as in Woodford (2003)).

<sup>52</sup>We use the following two loss functions:

1. To calculate the policy frontier among union inflation and unemployment volatility

$$L1 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[ \omega \left( \hat{\pi}_t^H \right)^2 + \omega \left( \hat{\pi}_t^F \right)^2 + (1 - \omega) \left( \hat{u}_t^H \right)^2 + (1 - \omega) \left( \hat{u}_t^F \right)^2 \right]$$

2. To calculate the policy frontier of a central bank that only cares about home and foreign inflation

$$L2 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[ \omega \left( \hat{\pi}_t^H \right)^2 + (1 - \omega) \left( \hat{\pi}_t^F \right)^2 \right]$$

where we compute the optimal monetary policy for all  $\omega \in [0, 1]$ .

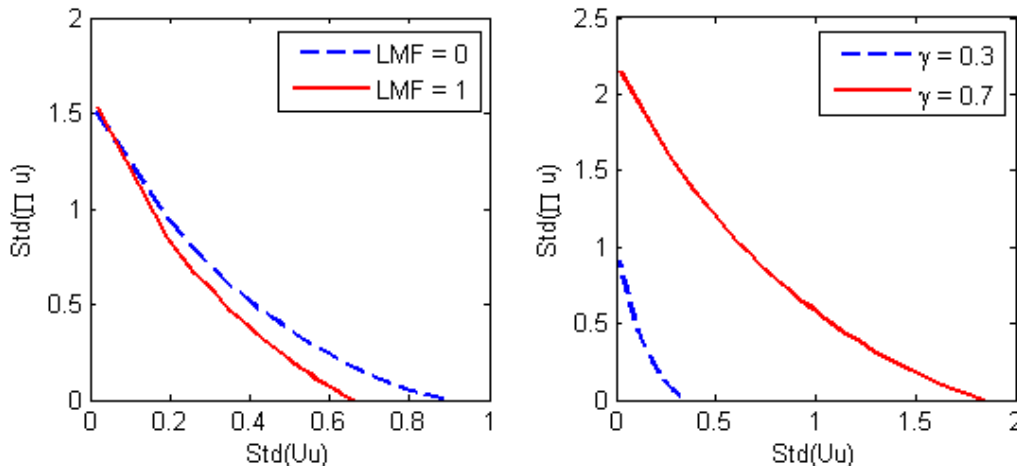


Figure 8: *Policy frontier between union inflation and unemployment*

which shift inside. Intuitively, when labor market are more sclerotic, inflation becomes more sensitive to labor market conditions and the Phillips Curve becomes steeper (as explained above). In terms of monetary policy, this implies that the central bank can reduce inflation volatility incurring in a smaller increase in unemployment volatility: the trade-off of monetary policy gets less severe. To put it in simple words, macroeconomic stabilization is easier in a more sclerotic currency union.

An increase in RWR, on the contrary, shifts outside the Pareto frontier (as it increases the trade-off of monetary policy in face of productivity shocks) and flattens its slope (as the Phillips Curve gets flatter). Both effects tend to increase the costs in terms of union inflation and unemployment volatilities: RWR make macroeconomic stabilization much more difficult.

These results may serve to formulate some implications of labor market reforms. Our analysis suggests that the reform of European labor markets could make macroeconomic stabilization more difficult. Of course, labor market reform have important effects on the level of (steady state) unemployment, but nevertheless it is important to keep in mind the negative effect on volatilities. A reduction in the degree of wage rigidities, on the contrary, would not affect the steady state unemployment level (at least in this stylized model) but would render the job of the central bank easier.

Figure 9 shows the policy frontier of a central banker who only cares about the variability of home and foreign inflation. In the cases in which we consider the two regions to be asymmetric, home is the rigid country and foreign the flexible one. The Figure shows that the slope of the policy frontier is in favor of the home country in the presence of asymmetric LMF, but in favor of the foreign country with asymmetric RWR. The intuition from the symmetric case can be transferred here: macroeconomic stabilization is easier in the country with low RWR and high LMF. Again, the reason is related to the slope of the Phillips Curve: inflation is more sensitive to labor market conditions when



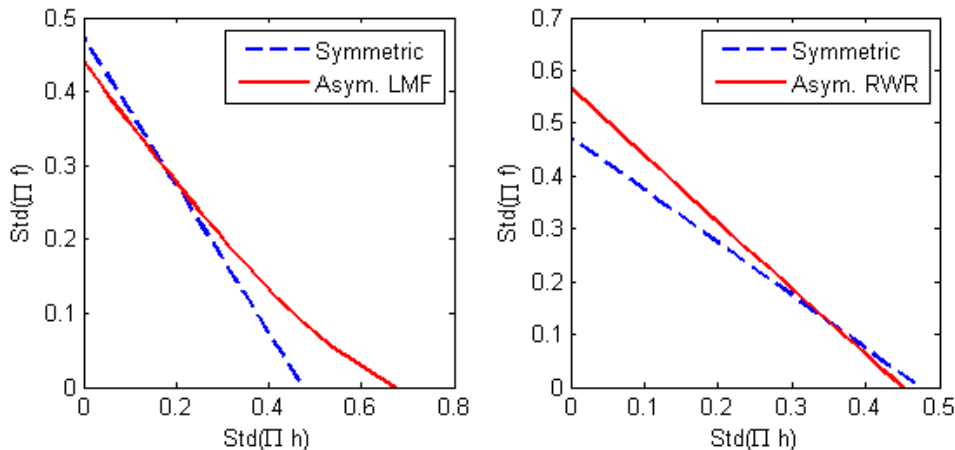


Figure 9: *Policy frontier between home and foreign inflation*

LMF are high and real wages are relatively flexible.

**Result 6 (Policy frontier)**

a) *The policy trade-off between inflation and unemployment is increasing in RWR but decreasing in LMF. Macroeconomic stabilization is easier in a currency union with more sclerotic labor markets and/or more flexible real wages.*

b) *The policy trade-off between inflation of the two countries is in favor of the country with low RWR and high LMF.*

**8.2 Evaluating suboptimal policy rules**

In this section we evaluate the welfare losses of the following targeting rules:

1.  $\hat{\pi}_t^U = 0$  (IT)
2.  $\hat{\pi}_t^U - \alpha_u \hat{u}_t^U = 0$  (MT)
3.  $\alpha_\pi \hat{\pi}_t^H + (1 - \alpha_\pi) \hat{\pi}_t^F = 0$  (ITopt)

The symmetric inflation targeting rule (IT) is a special case of the optimal inflation targeting rule (ITopt) and of the mixed targeting rule (MT) when  $\alpha_u = 0$  or  $\alpha_\pi = 0.5$ .

Mixed targeting (MT) may improve upon IT when the welfare losses due to variations in unemployment are high; ITopt may improve upon IT when strong asymmetries in the labor market structures are present. Note that we find the optimal weight  $\alpha_x$  on the target variable(s) in the policy rules and then compute the welfare losses under these optimized rules.

**Real wage rigidities and labor market frictions in a symmetric CU** First, let us define a policy to be near optimal if the welfare losses associated with deviating from the optimal rule under commitment are smaller than 0.001 percentage points of steady

state consumption. Figure 10 shows the losses from IT and MT for different combinations of real wage rigidities and labor market frictions. We use different combinations of RWR and LMF to emphasize interaction effects.

The first observation is that the optimal weight on unemployment is increasing in RWR but decreasing in LMF. This result is intuitive as it is optimal to put more weight on unemployment in economies where the volatility of unemployment is high relative to inflation. Note, however, that the weight on unemployment in our mixed targeting rule is low for all parameter values<sup>53</sup>. The reason is that our micro-founded loss function puts considerably more weight on inflation, by a factor of 30<sup>54</sup>.

It is also interesting to observe that welfare losses from IT and MT are *both* increasing in RWR and decreasing in LMF. This suggests that, while MT improves upon IT, it is not the optimal policy<sup>55</sup>.

Moreover, there are strong interaction effects between the two rigidities. At low levels of real wage rigidities, MT and IT are near optimal for all levels of LMF. On the contrary, at high levels of real wage rigidities welfare losses depend strongly on LMF and are greatest when LMF are low.

**Result 7 (Symmetric Currency Union)**

*a) IT is near optimal at lower levels of RWR and high levels of LMF, that is when the volatility of unemployment is low relative to the volatility of inflation. There are strong interaction effects between the two rigidities: welfare losses from IT are largest in economies with high RWR and low LMF.*

*b) MT dominates IT in economies with high RWR and low LMF. There are strong interaction effects between RWR and LMF: welfare losses from IT compared to MT are largest in economies with high RWR and low LMF.*

**Asymmetries in labor market structures** Let suppose now that the monetary authority is committed to a form of inflation targeting and therefore compares the strict IT rule ( $\hat{\pi}_t^U = 0$ ) with the optimal Inflation Targeting rule  $\alpha_\pi \hat{\pi}_t^H + (1 - \alpha_\pi) \hat{\pi}_t^F = 0$ .

What is the optimal inflation target when countries have asymmetric labor market rigidities? Figure 11 shows welfare losses and the optimal weight in the inflation target for different calibrations. The same index of asymmetry is used as in previous sections, and we analyze what happens to welfare losses when we increase this index from 0 to 1, holding other parameters constant. Note, however, that unlike in previous sections, we do not use the baseline calibration, but instead distinguish between two cases: 1. Rigidities are complements or 2. Rigidities are substitutes. To give two examples:

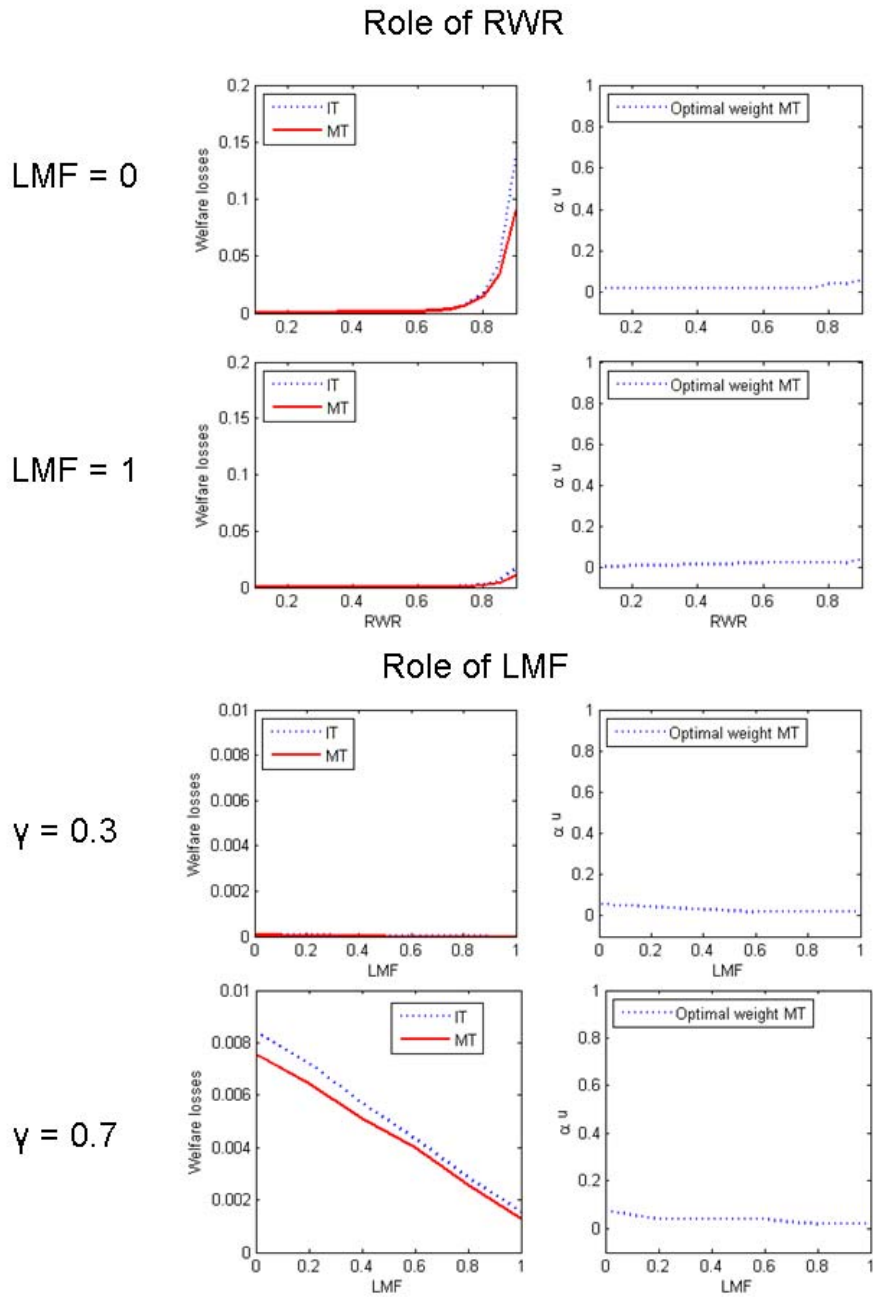
1. Asym. RWR, Compl. LMF refers to a situation where: the country in which we increase RWR has high LMF (= 1) and the other country where we reduce RWR has low LMF (= 0).

<sup>53</sup>For calibrations that increase the weight in the loss function on unemployment, MT dominates IT also for lower values of real wage rigidities. Note, however, that this is not generally true for  $\phi$ : an increase in  $\phi$  implies a higher weight on unemployment in the loss function, but it also lowers the volatility of unemployment because it makes the reservation utility more procyclical (and hence also real wages).

<sup>54</sup>This is consistent with Woodford (2003), who finds a weight on inflation that exceeds the weight on the output gap by a factor 20. His model has no unemployment, but is similar in many other respects.

<sup>55</sup>Note that it can be shown that the optimal policy is a function of all lagged and current states.

Figure 10: *Welfare losses in a symmetric currency union*



2. Asym. LMF, Subst. RWR refers to a situation where: the country in which we increase LMF has low RWR ( $\gamma = 0.3$ ) and the other country where we reduce LMF has high RWR ( $\gamma = 0.7$ ).

The results, shown in Figure 11, suggest that the optimal inflation targeting rule should give a higher weight to a country with *lower* RWR but *higher* LMF. The reason for this result is related to the slope of the Phillips Curve: it is optimal to reduce inflation volatility in the country where inflation is more sensitive to labor market conditions (i.e. where the Phillips Curve is steeper). This will increase the volatility of inflation in the country where inflation is less sensitive to labor market conditions, but by less than one to one. Therefore, because inflation is more sensitive to labor market conditions when real wages are flexible and labor markets sclerotic, it is optimal to put more weight on those economies in the optimal inflation targeting rule.<sup>56</sup>

The result that it is optimal to target the region with flexible real wages is interesting, but seems to be in contrast with the literature on sticky prices and sticky nominal wages, such as for example in Erceg et al. (1999). In this literature, welfare losses arise because in the presence of price and nominal wage staggering, inflation leads to an inefficient dispersion of output across firms and/or inefficient labor supply across households. As a consequence, the weights in the loss function are larger for sticky price countries (or sectors) and hence it is optimal to put more weight on that country in the optimal inflation targeting rule. On the contrary, in our framework, inflation bears the same costs in both countries because countries are symmetric in terms of price rigidities. Moreover, real wages are the same across firms, but institutional barriers limit the extent to which real wages can adjust and hence determine how responsive inflation is to labor market conditions. As a consequence, real wage rigidity does not affect the weights on inflation or unemployment in the loss function, but only the way in which shocks are transmitted in member states (as captured by the Phillips Curve of the member states). However, asymmetric real wage rigidities affect relative volatilities of inflation and affect the trade-off in favor of the country with flexible real wages.

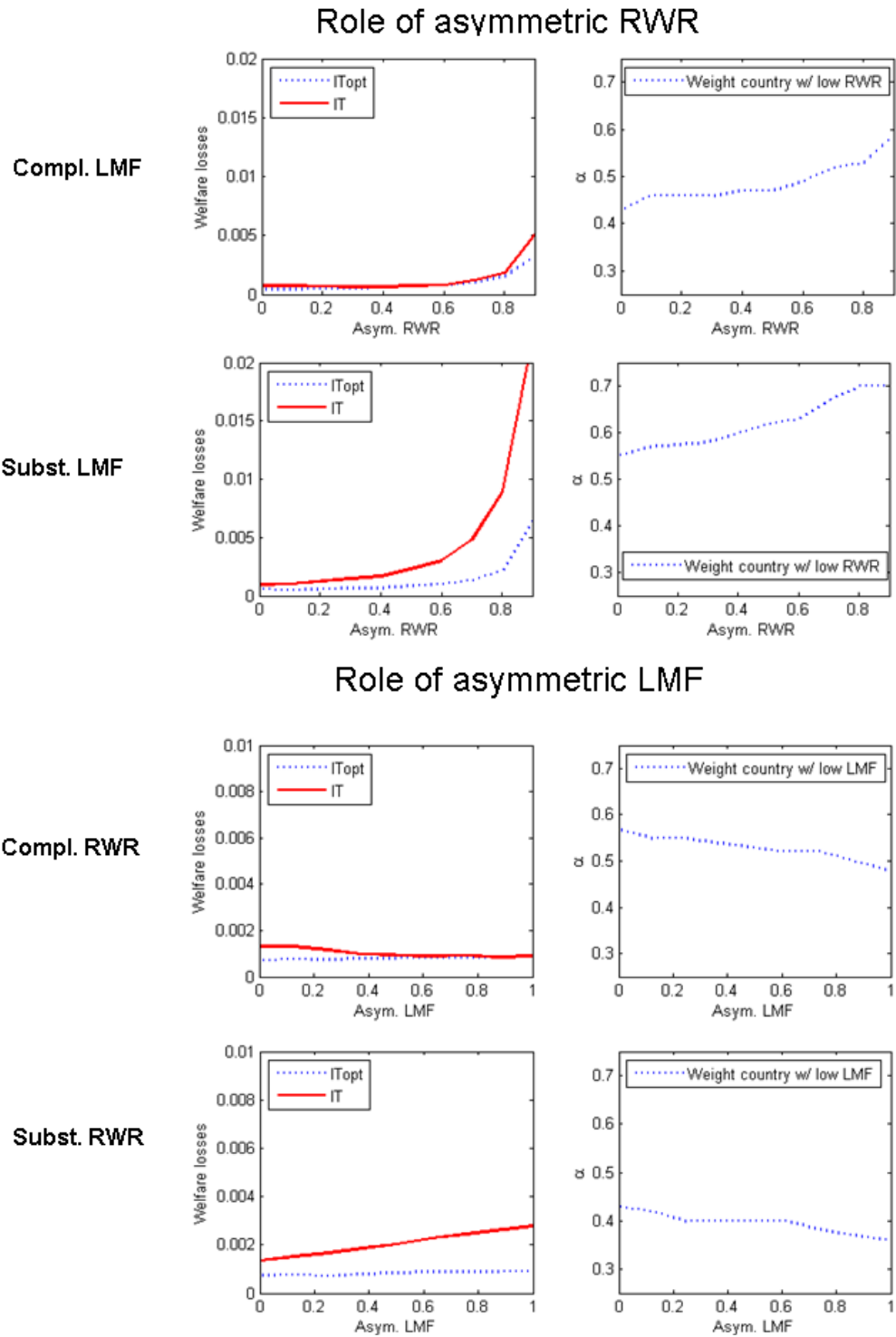
Rigid wages and rigid labor markets are thus found to lead to opposite prescriptions for monetary policy. Moreover, as shown in Figure 11, there are strong interaction effects between the two rigidities. On the one hand, when rigidities are complements, the effects mentioned above might cancel out. Indeed for some combinations of LMF and RWR the optimal weight is 0.5, i.e. complete symmetry in the targeting rule. On the other hand, when rigidities are substitutes, the effects above reinforce each other. Figure 11 shows that, for low values of RWR and high LMF, the welfare losses from following a strict IT rule are largest. In this case, the asymmetric inflation targeting rule (IT<sub>opt</sub>) clearly improves upon IT by putting more weight on the country with flexible real wages and sclerotic labor markets.<sup>57</sup>

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<sup>56</sup>Note that the fact that the trade-off in the Philips curve depends on the real wage rigidity, gives an additional motive for targeting the flexible real wage country. The reason is that targeting the inflation of the country with more sticky real wages - i.e. the country with a larger trade-off - creates much more volatility in unemployment.

<sup>57</sup>Note also that asymmetric price rigidities affect the optimal weight in the inflation target much stronger than real rigidities. The reason is that weights on inflation in the loss function are very sensitive to different

Figure 11: *Welfare losses in an asymmetric currency union*



### **Result 8 (Asymmetric currency union)**

*a) Targeting the appropriate inflation target is near optimal except when asymmetries in real wage rigidities are large. The appropriate inflation target gives a higher weight to regions with more flexible real wages and higher labor market frictions.*

*b) There are strong interaction effects between RWR and LMF: If rigidities are substitutes, symmetric inflation targeting is strongly dominated by  $IT_{opt}$ . If rigidities are complements,  $IT$  and  $IT_{opt}$  are both near optimal.*

## **9 Conclusion**

In this paper we have introduced labor market frictions and real wage rigidities in a standard DSGE currency union model. As in Blanchard and Gali (2006), labor market frictions are introduced by assuming the presence of hiring costs, which increase in the degree of labor market tightness. The introduction of hiring costs has two important consequences. First, it leads to involuntary unemployment. Second, it fundamentally changes the nature of marginal costs, which now depends not only on the evolution of real wages and productivity, but also on the evolution of marginal hiring costs. The presence of real wage rigidities, on the other side, hinders wage adjustments and hence increases the adjustments on the labor quantity side. The degree of labor market frictions and real wage rigidities, by influencing the incentives and constraints faced by firms, significantly affect the transmission mechanism of shocks in general, and of monetary policy in particular. Three main conclusions emerge from our analysis:

First, the two types of labor market rigidities are found to have very different effects on the incentives for firms to reset prices and thus on the Phillips Curve. A higher degree of LMF makes the Phillips Curve steeper whereas real wage rigidity makes the Phillips Curve flatter. The basic intuition is that inflation is more sensitive to labor market conditions when the quantities are constrained, so that firms adjust prices rather than quantities in response to a shock. The opposite holds when the constraints are on the price side of the labor market. Moreover, LMF and RWR are found to interact in a rich and complex way in determining the slope of the Phillips Curve. When they are substitutes (low RWR are associated with high LMF or vice versa), their effects tend to reinforce each other; when they are complements (high RWR are associated with high LMF), they tend to offset each other.

Second, labor market rigidities have a deep impact on the adjustment mechanism within a currency union: even when the countries have similar structures, large and long-lasting differentials can arise as a consequence of asymmetric shocks. The size of these differentials is influenced by the degree of real wage rigidities and labor market frictions. Moreover, symmetric shocks can have large and persistent effects on inflation and unemployment differentials in the presence asymmetric labor market frictions and real wage rigidities. In general, the hiring friction lowers the persistence and increases the volatility of the inflation differential whereas real wage rigidities imply more persistence and variability in output and unemployment differentials.

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degrees of price rigidity, whereas the weights do (almost) not depend on real rigidities. As explained above, real rigidities matter for the optimal weight insofar they affect the relative volatilities of inflation (and unemployment).

Finally, we find that macroeconomic stabilization is easier when labor market frictions are high and real wage rigidities are low. This has important implications for optimal monetary policy: except for large degrees of real wage rigidities, a strict inflation targeting is optimal. The inflation target should give a higher weight to regions with more flexible real wages but more sclerotic labor markets. Moreover, there are strong interaction effects between LMF and RWR. If rigidities are substitutes, symmetric inflation targeting is strongly dominated by an asymmetric targeting rule. If rigidities are complements, both policies are near optimal.

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## Appendix

### A The basic setup

In this section, we describe the main features of our framework, focusing on the principal elements of departure from the previous treatments.

#### A.1 Households

The representative household within a country is thought of as a continuum of members with names on the unit interval. Each household purchases consumption goods, holds money and supplies labor. Wages are fixed by bargaining, and, given the presence of involuntary unemployment, the labor supply is not binding. Household members can be employed or unemployed. To avoid distributional issues, we assume that households pool their income and consumption.

The representative household in country  $i$  ( $i = H$  or  $F$ ) seeks to maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_0 \frac{(N_t^H)^{1+\phi}}{1+\phi} \right\}, \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t^* - \chi_1 \frac{(N_t^F)^{1+\phi}}{1+\phi} \right\} \quad (27)$$

where variables with star are referred to the foreign country.  $N_t^i$  denotes the number of employed individuals in the representative household of country  $i$  while  $C_t$  and  $C_t^*$  are the composite consumption indexes for the home and foreign country respectively, defined

as:

$$C_t = \frac{(C_t^H)^{1-\alpha} (C_t^F)^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}, \quad C_t^* = \frac{(C_t^{F,*})^{1-\alpha} (C_t^{H,*})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha} \quad (28)$$

$C_t^{j,i}$  is the quantity of the good produced in country  $j$  and consumed by residents of country  $i$ .  $\alpha \in [0, 1]$  is the weight on the imported goods in the utility of private consumption; a value for  $\alpha$  strictly less than  $\frac{1}{2}$  reflects the presence of home bias in consumption.

The production sectors are characterised by monopolistic competition. The index of country  $i$ 's consumption of the good produced in country  $j$ ,  $C_t^{j,i}$ , is given by the usual CES aggregator:

$$C_t^{j,i} = \left( \int_0^1 (C_t^{j,i}(z))^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \quad i = H \text{ or } F; j = H \text{ or } F$$

The parameter  $\epsilon$  ( $\epsilon^*$ )  $> 1$  is the elasticity of substitution between varieties produced within home (foreign) country.

Utility maximization for the home household is subject to a sequence of budget constraints of the form<sup>58</sup>:

$$\int_0^1 P_t^H(j) C_t^H(j) dj + \int_0^1 P_t^F(j) C_t^F(j) dj + E_t \{ Q_{t,t+1}^H D_{t+1}^H \} \leq D_t^H + W_t^H N_t^H - T_t^H$$

for  $t = 0, 1, 2, \dots$ , where  $P_t^i(j)$  is the price of good  $j$  produced in country  $i$  (expressed in the units of the single currency).  $D_t^H$  is the nominal payoff in period  $t$  of the portfolio held at the end of period  $t-1$ ;  $W_t^H$  is the nominal wage and  $T_t^H$  denotes lump-sum taxes.

We assume complete securities markets;  $Q_{t,t+1}^H$  is the stochastic discount factor for one-period ahead nominal payoffs, which is common across countries. Implicit in the budget constraint is the assumption that the law of one price holds across the union.

The demands for the generic goods produced at home and foreign are respectively:

$$C_t^{H,i}(z) = \left( \frac{P_t^H(z)}{P_t^H} \right)^{-\epsilon} C_t^{H,i}; \quad C_t^{F,i}(z) = \left( \frac{P_t^F(z)}{P_t^F} \right)^{-\epsilon^*} C_t^{F,i} \quad (29)$$

for  $i = H, F^*, z \in [0, 1]$ . The appropriate domestic (producer) price indexes of the home and foreign countries are:

$$P_t^H = \left( \int_0^1 (P_t^H(z))^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}; \quad P_t^F = \left( \int_0^1 (P_t^F(z))^{1-\epsilon^*} dz \right)^{\frac{1}{1-\epsilon^*}}$$

Since the law of one price holds,  $P_t^H$  represents both the price index for the bundle of goods imported from country H as well as H's domestic price index. From the

<sup>58</sup>The utility maximization problem for the foreign household is completely analogous.

demand functions (29), we get (for the home country):  $\int_0^1 P_t^H(j) C_t^H(j) dj = P_t^H C_t^H$  and

$$\int_0^1 P_t^F(j) C_t^F(j) dj = P_t^F C_t^F.$$

Furthermore, the optimal allocation of expenditures by country of origin implies, for the home country:

$$P_t^H C_t^H = (1 - \alpha) P_t C_t, \quad P_t^F C_t^F = \alpha P_t C_t \quad (30)$$

while for the foreign country:

$$P_t^{H,*} C_t^{H,*} = \alpha P_t^* C_t^*, \quad P_t^{F,*} C_t^{F,*} = (1 - \alpha) P_t^* C_t^* \quad (31)$$

where  $P_t = (P_t^H)^{1-\alpha} (P_t^F)^\alpha$  and  $P_t^* = (P_t^{H,*})^\alpha (P_t^{F,*})^{1-\alpha}$  are respectively the home and the foreign CPI indexes. As usual with Cobb-Douglas preferences, households allocate a fixed proportion of income to each consumption bundle.

Under the assumption of “home bias” in consumption (i.e.  $\alpha < \frac{1}{2}$ ) different regions consume goods in different proportions; therefore, even if the Law of One Price holds for all goods, the Purchasing Power Parity (PPP) may not hold at the aggregate level ( $P_t \neq P_t^*$ ).

Combining all previous results, we can write total consumption expenditures by home’s households as  $P_t^H C_t^H + P_t^F C_t^F = P_t C_t$ . Thus, conditional on optimal allocation of expenditures, the period budget constraint is given by:

$$P_t C_t + E_t \{ Q_{t,t+1}^H D_{t+1}^H \} \leq D_t^H + W_t^H N_t^H - T_t^H \quad (32)$$

The remaining optimality conditions for country i are given by:

$$\beta R_t E_t \left( \frac{C_t^i}{C_{t+1}^i} \frac{P_t^i}{P_{t+1}^i} \right) = 1 \quad (33)$$

$$\chi_i C_t^i (N_t^i)^\phi \leq \frac{W_t^i}{P_t^i} \quad (34)$$

where  $R_t = \frac{1}{E_t Q_{t,t+1}^H}$  is the (gross) nominal interest rate.

## A.2 The Terms of Trade and the Real Exchange Rate

In this section we introduce some definitions and identities that are used extensively below. First, we define the bilateral term of trade between the home and foreign countries as the ratio of the price of goods produced in country F to that produced in country H:

$$S_t = \frac{P_t^F}{P_t^H} \quad (35)$$

As the Law of One Price holds for all goods - which implies  $P_t^F = P_t^{F,*}$  and  $P_t^H = P_t^{H,*}$  - the CPI and the domestic price indexes in the two regions are related according to:

$$P_t = P_t^H (S_t)^\alpha, \quad P_t^* = P_t^F (S_t)^{-\alpha} \quad (36)$$

Let domestic (i.e. producer prices') inflation be defined as the rate of change of domestically produced goods, i.e. as  $\pi_t^i \equiv \log \frac{P_t^i}{P_{t-1}^i} = p_t^i - p_{t-1}^i$ , where  $p_t^i = \log P_t^i$ . Taking logs of the above identities, we obtain a relation between Domestic and CPI inflation:

$$\pi_t = \pi_t^H + \alpha \Delta s_t, \quad \pi_t^* = \pi_t^F - \alpha \Delta s_t \quad (37)$$

for the home and the foreign country respectively<sup>59</sup>.

Finally, the real exchange rate  $V_t$  is defined as the ratio between foreign and home CPIs and is related to the terms of trade according to:

$$V_t = \frac{P_t^*}{P_t} = (S_t)^{1-2\alpha} \quad (38)$$

### A.3 International Risk Sharing

Capital markets are complete: each household has access to a complete set of contingent claims, traded internationally. Combining the first order conditions relative to state contingent securities in the two countries, we obtain the usual result:

$$V_t = \psi \frac{u'(C_t^*)}{u'(C_t)} = \psi \frac{C_t}{C_t^*} \quad (39)$$

where  $\psi = V_0 \frac{u'(C_0^*)}{u'(C_0)} = \frac{P_0^*}{P_0} \frac{u'(C_0^*)}{u'(C_0)}$  is a constant, reflecting initial conditions regarding relative net asset positions. If PPP holds (and this will occur in this model for  $\alpha = 1/2$ ), the real exchange rate  $V_t = 1$  and the marginal utilities of consumption are equated up to a constant  $\psi$ . In general, movements in the real exchange rate will be reflected in different consumption rates:

$$C_t = \frac{1}{\psi} V_t C_t^* \quad (40)$$

Therefore, even with complete financial markets, it is not efficient to equalize consumption across countries when there is a home Bias in consumption ( $\alpha < \frac{1}{2}$ ).

Henceforth, to keep the analysis as simple as possible, we assume initial conditions are such that  $\psi = 1$ .

### A.4 Firms and the Labor market

The setup of the supply side of the economy follows Blanchard-Galì (2006). The production in each country is composed by a continuum of firms, indexed by  $j \in [0, 1]$ . Each firm in a country produces a differentiated good with an identical technology:

$$Y_t^i(j) = A_t^i N_t^i(j), \text{ for } i = H, F^* \quad (41)$$

where the variables  $A_t^i$  represent the state of technology in country  $i$ .

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<sup>59</sup>Notice that the distinction between CPI inflation and domestic inflation, while important at the country level, vanishes for the monetary union as a whole. In fact, summing up the equation for the logs in prices, yields the equality  $p_{CPI,t}^U = p_t^U$ .

In each period a fraction  $\delta^i$  of the employed loses their jobs and joins the unemployment pool. Employment in firm  $j$  evolves according to:

$$N_t^i(j) = (1 - \delta^i)N_{t-1}^i(j) + H_t^i(j), \text{ for } i = H, F(*) \quad (42)$$

where  $H_t^i(j)$  is the number of new hires for firm  $j$  in country  $i$ . We assume that the job destruction rate  $\delta^i$  is exogenously given.

We denote by  $U_t^i$  the number of unemployed workers in country  $i$

$$U_t^i = 1 - (1 - \delta^i)N_{t-1}^i, \text{ for } i = H, F(*) \quad (43)$$

“After hiring” unemployment, instead, is defined as the fraction of the population who are left without a job after hiring takes place,  $u_t^i = 1 - N_t^i$ .

Aggregate hiring  $H_t^i \equiv \int_0^1 H_t^i(j) dj$  evolves according to

$$H_t^i = N_t^i - (1 - \delta^i)N_{t-1}^i \quad (44)$$

Where  $N_t^i \equiv \int_0^1 N_t^i(j) dj$  denotes aggregate employment<sup>60</sup>.

Firms face a cost of searching and recruiting new workers “à la Howitt”<sup>61</sup>. Hiring costs for firm  $j$  in country  $i$  are:

$$G_t^i H_t^i(j), \text{ for } i = H, F(*) \quad (47)$$

where  $G_t^i$  is the cost per hire in country  $i$  (expressed in terms of the domestic CES bundle of goods), which is taken as given by the individual firm. Following Blanchard–Galì (2006), we assume

$$G_t^i = A_t^i B^i \left( \frac{H_t^i}{U_t^i} \right)^\varphi, \text{ for } i = H, F(*) \quad (48)$$

where  $\varphi > 0$  and  $B^i$  is a positive scaling parameter that may be influenced by the authorities.

If we define the labor market tightness index as

$$x_t^i \equiv \frac{H_t^i}{U_t^i}, \text{ for } i = H, F(*) \quad (49)$$

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<sup>60</sup>We define the aggregate output for Home as  $Y_t^i = \left( \int_0^1 (Y_t^i(j))^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$ . The amount of labor employed is thus given by:

$$N_t^i = \int_0^1 N_t^i(j) dj = \frac{Y_t^i Z_t^i}{A_t^i} \quad (45)$$

where  $Z_t^i = \int_0^1 \frac{Y_t^i(j)}{Y_t^i} dj$ . It can be shown that equilibrium variations in  $z_t^i = \log Z_t^i$  around the perfect foresight steady state are of second order. Thus, up to a first order approximation,

$$y_t^i = a_t^i + n_t^i \quad (46)$$

<sup>61</sup>Cfr. Howitt (1988).

i.e. as the ratio of aggregate hires to the employment rate, we can rewrite the cost per hire for H and F as

$$G_t^i = A_t^i B^i (x_t^i)^\varphi, \text{ for } i = H, F^* \quad (50)$$

Note that, from the viewpoint of the unemployed,  $x_t^i$  can be interpreted as the probability of finding a new job in period t, i.e. as the job-finding rate.

### A.5 Market Clearing Conditions

Consider the home country. The clearing of the market for good j requires:

$$\begin{aligned} Y_t(j) &= C_t^H(j) + C_t^{H,*}(j) + G_t^H h_t^H(j) \\ &= \left( \frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon} [C_t^H + C_t^{H,*} + G_t^H h_t^H] \\ &= \left( \frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon} [C_t(S_t)^\alpha + G_t^H h_t^H] \\ &= \left( \frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon} Y_t \end{aligned} \quad (51)$$

Plugging (51) in the definition of aggregate output,  $Y_t = \left( \int_0^1 (Y_t(j))^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}$ , we obtain the aggregate goods market clearing condition for home:

$$Y_t = C_t(S_t)^\alpha + G_t^H h_t^H$$

Similar conditions hold for the foreign country:

$$Y_t^*(j) = \left( \frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon} Y_t^* \quad (52)$$

$$Y_t^* = C_t^*(S_t)^{-\alpha} + G_t^F h_t^F = C_t(S_t)^{-(1-\alpha)} + G_t^F h_t^F \quad (53)$$

The assumption of Cobb-Douglas preferences over the home and foreign goods allows us to derive a simple relation between the terms of trade and relative output<sup>62</sup>:

$$S_t = \frac{P_t^F}{P_t^H} = \frac{Y_t - G_t^H h_t^H}{Y_t^* - G_t^F h_t^F} \quad (54)$$

Equation (54) simply states the relative price of domestic (foreign) goods is inversely related to the quantity produced in the two regions (net of aggregate hiring costs)<sup>63</sup>.

<sup>62</sup>The Cobb-Douglas assumption in fact imply that the percentage variation in relative prices is equal, and opposite in sign, to the percentage variation in relative quantities.

<sup>63</sup>Notice that equation (54) allows us to pin down the steady state level of  $S_t$ :

$$S = \frac{AN(1-g\delta)}{A^F N^F (1-g^* \delta^*)}$$

where  $g^i = B^i (x^i)^\varphi$  for  $i = H, F^*$ .

## B The Social Planner's Problem

In this section we derive the so-called “constrained efficient allocation”. Following Blanchard and Gali’ (2006), we assume that the social planner maximizes the welfare of the Union, taking as given the technological constraints and the labor market frictions that are present in the decentralised economy. In other words, the social planner cannot eliminate or reduce hiring costs, which are simply taken as a fact of life; he can, however, internalize the effects of variations in employment on labor market tightness and, hence, on hiring costs.

Given symmetry in preferences and technology, the social planner chooses an equilibrium in which the goods, in each countries, are produced and consumed in identical quantities  $C_{jt}^i(z) = C_{jt}^i$ .

Hence, the Union’s optimal allocation can be described as the solution of the following social planner’s problem:

$$\begin{aligned} & \text{Max} \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_0 \frac{(N_t^H)^{1+\phi}}{1+\phi} + \log C_t^* - \chi_1 \frac{(N_t^F)^{1+\phi}}{1+\phi} \right\} \\ & \text{s.t.} \\ & C_t^H + C_t^{H,*} = A_t^H N_t^H - G_t^H h_t^H \\ & C_t^F + C_t^{F,*} = A_t^F N_t^F - G_t^F h_t^F \end{aligned}$$

where  $C_t^i$ ,  $C_t^{j,i}$ ,  $G_t^i$  and  $h_t^i$  are as defined before. Notice that the previous constraints already embed the optimal condition whereby the different good types in any given country should be produced and consumed in identical quantities.

Maximization with respect to consumption leads to the following optimality conditions:

$$\begin{aligned} \frac{1-\alpha}{\alpha} &= \frac{C_t^H}{C_t^{H,*}} = \frac{C_t^{F,*}}{C_t^F} \\ \frac{\nu_t}{\xi_t} &= \frac{\alpha}{1-\alpha} \frac{C_t^H}{C_t^F} = MRS^H = \frac{1-\alpha}{\alpha} \frac{C_t^{H,*}}{C_t^{F,*}} = MRS^F \end{aligned}$$

where  $\xi_t$  is the shadow value of an additional unit of the good produced at home and  $\nu_t$  is the shadow value of an additional unit of the foreign good.

Solving the social planner’s problem with respect to home employment, we obtain:

$$\begin{aligned} \chi_0 \frac{(N_t^H)^\phi C_t^{H,W}}{A_t^H} &\leq 1 - B(1+\varphi) (x_t^H)^\varphi \\ &+ B(1-\delta) E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} B \left[ \varphi (x_{t+1}^H)^\varphi + \right. \right. \\ &\left. \left. \left. \varphi (x_{t+1}^H)^\varphi (1-x_{t+1}^H) \right] \right\} \end{aligned} \tag{55}$$

which must hold with strict equality if  $N_t^H < 1$ .

The important point to note is that the above expression implies a *constant level of employment*. Note in fact that, world consumption of the home good is proportional to productivity

$$C_t^{H,W} \equiv C_t^H + C_t^{H,*} = A_t^H (N_t^H - B (x_t^H)^\varphi h_t^H) \quad (56)$$

It follows that the optimality condition does not depend on the productivity levels prevailing at home (or foreign). This invariance is the result of two main assumption:

1. The utility function is log in consumption: this implies that income and substitution effects offset each other on the labor supply.
2. Unit hiring costs varies one-for-one with productivity shocks.

The fact that employment is constant is a very useful result, since it allows us to say that any fluctuation in employment is inefficient<sup>64</sup>.

To determine the efficient level of employment we can proceed in two steps. First, the efficient level for the labor market tightness indicator,  $x_E^H$ , is implicitly determined as the solution to

$$\chi_0 \left( \frac{x^H}{\delta + (1-\delta)x^H} \right)^{1+\phi} \left( 1 - \delta B (x^H)^\varphi \right) \leq 1 - (1 - \beta(1 - \delta))(1 + \varphi)B (x^H)^\varphi - \beta(1 - \delta)\varphi B (x^H)^{1+\varphi} \quad (57)$$

Second, the optimal level of employment at home is given by

$$N_E^H = \frac{x_E^H}{\delta + (1-\delta)x_E^H} \quad (58)$$

The optimal employment level depends therefore on the separation rate  $\delta$ , on the hiring costs' scaling parameter  $B$ , on the sensitivity of hiring costs to labor market conditions  $\varphi$  and on parameters influencing the disutility of working ( $\phi$  and  $\chi_0$ ).

A constant employment level implies that output is proportional to home productivity ( $Y_t = A_t^H N_E^H$ ) while consumption depends on both home and foreign productivity ( $C_t = (A_t^H)^{1-\alpha} (A_t^F)^\alpha N_E^H (1 - \delta B (x_E^H)^\varphi)$ ).

Similar conditions and the same conclusions hold for the foreign country.

## C Equilibrium under Flexible Prices

In this section we derive the equilibrium under the assumption that prices are flexible. We first describe the optimal price setting of a firm, given the wage. We then characterize the equilibrium that emerges with Nash bargained wages. Finally, we introduce real wage rigidities in form of a Hall (2005) type wage norm.

We focus on the home country; the solution for the foreign country is completely symmetric.

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<sup>64</sup>Blanchard and Gali (2006, p. 9-11) get the same result in the context of a one-country model.



## C.1 Optimal Price Setting

Suppose that all firms adjust prices optimally each period to maximize the present discounted value of expected profits:

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s}^H \{ P_{t+s}^H Y_{t+s}(i) - P_{t+s}^H G_{t+s}^H h_{t+s}^H(i) - W_{t+s}^H N_{t+s}^H(i) \} \quad (59)$$

subject to the sequence of demand constraints  $Y_t(i) = \left( \frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon} Y_t$ , the production function and the employment evolution equation.  $Q_{t,t+s}^H = \beta^s \frac{C_t}{C_{t+s}} \frac{P_t}{P_{t+s}}$  is the relevant stochastic discount factor for nominal payoffs. Notice that the unit recruitment costs are expressed in units of *domestic* goods.

The optimal price setting rule takes the form of a markup  $\mu = \frac{\epsilon}{\epsilon-1}$  over the real marginal cost:

$$\frac{P_t^H(i)}{P_t^H} = \frac{\epsilon}{\epsilon-1} MC_t^H = \mu MC_t^H \quad (60)$$

where the firm's real marginal cost is (expressed in terms of domestic goods):

$$MC_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha + B (x_t^H)^\varphi - \beta(1-\delta) E_t \left\{ \frac{C_t (S_t)^\alpha}{C_{t+1} (S_{t+1})^\alpha} \frac{A_{t+1}^H}{A_t^H} B (x_{t+1}^H)^\varphi \right\} \quad (61)$$

and  $W_t^{H,R} = \frac{W_t^H}{P_t}$  is the real wage expressed in terms of the consumption good.

The key difference between the supply side in our model and in a standard New Keynesian model with a neoclassical labor market is the behaviour of the real marginal cost. In a model with a competitive labor market the real marginal cost is strictly related to the evolution of the real wage:

$$MC_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha \quad (62)$$

In our model, which embeds the NK model as a special case, the presence of hiring costs creates a wedge between the real wage and the marginal costs relevant for the firm, which in turn are essential to explain inflation dynamics.

In a symmetric equilibrium,  $P_t^H(i) = P_t^H$  for all  $i \in [0, 1]$ , and hence the optimal price setting implies:

$$MC_t^H = \frac{1}{\mu} \quad (63)$$

for all  $t$ . When shocks occur, each firm varies its prices and hiring decisions to keep the marginal cost constant. It follows that in equilibrium:

$$\frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha = \frac{1}{\mu} - B (x_t^H)^\varphi + \beta(1-\delta) E_t \left\{ \frac{C_t (S_t)^\alpha}{C_{t+1} (S_{t+1})^\alpha} \frac{A_{t+1}^H}{A_t^H} B (x_{t+1}^H)^\varphi \right\} \quad (64)$$

Similar conditions hold for the foreign country. To get a full characterization of the equilibrium, we now need to specify a mechanism of wage determination.

## C.2 Equilibrium with Nash Bargained Wages

In this model, the presence of hiring costs creates a positive rent for existing employment relationships. Following much of the literature, we assume wages are bargained to split this rent between the firm and the employee, according to their respective bargaining power.

Consider the generic firm  $j$  in the home country.

The value of a job for firm  $j$  "is simply given by the hiring costs  $G_t^H$ . Notice however that hiring costs are expressed in terms of the domestic goods, while wages are set in terms of the consumption goods. The relevant firm's surplus - expressed in terms of consumption goods - is therefore

$$\frac{P_t^H G_t^H}{P_t} \quad (65)$$

Turning to the problem of the worker, let  $W_t^E$  and  $W_t^U$  denote the value of being employed or unemployed, expressed in consumption units.

The marginal value of an employment relationship is given by:

$$W_t^E = W_t^{H,R} - \chi_0 C_t (N_t^H)^\phi + \beta E_t \left\{ \frac{C_t}{C_{t+1}} [(1 - \delta(1 - x_{t+1}^H)) W_{t+1}^E + \delta(1 - x_{t+1}^H) W_{t+1}^U] \right\} \quad (66)$$

The first term represents the worker's wage income; the second the disutility of work and the last the discounted expected continuation value.  $\delta(1 - x_{t+1}^H)$  is the probability of being unemployed at time  $t$  conditional on being employed at time  $t$ .

The corresponding value for a member who remains unemployed after hiring take place is:

$$W_t^U = \beta E_t \left\{ \frac{C_t}{C_{t+1}} [x_{t+1}^H W_{t+1}^E + (1 - x_{t+1}^H) W_{t+1}^U] \right\} \quad (67)$$

Combining both conditions we obtain the household's surplus from an established relationship:

$$W_t^E - W_t^U = W_t^R - \chi_0 C_t (N_t^H)^\phi + \beta(1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} [(1 - x_{t+1}^H)(W_{t+1}^E - W_{t+1}^U)] \right\} \quad (68)$$

Let  $\zeta$  denote the share of the surplus going to the worker. The bargaining solution is given by:

$$W_t^N - W_t^U = \frac{\zeta}{1 - \zeta} \frac{P_t^H G_t^H}{P_t} = \eta G_t^H (S_t)^{-\alpha} \quad (69)$$

where we make use of the fact that  $\frac{P_t^H}{P_t} = (S_t)^{-\alpha}$  and we define  $\eta = \frac{\zeta}{1 - \zeta}$  as the relative weight of workers in the Nash bargaining, which reflects workers' bargaining power.

Imposing this condition to (68) and rearranging, we get the *Nash wage schedule*:

$$\frac{W_t^{H,R}}{A_t^H} (S_t^H)^\alpha = \frac{\chi_0 C_t^{H,W} (N_t^H)^\phi}{A_t^H} + \eta B (x_t^H)^\varphi \quad (70)$$

$$- \beta(1 - \delta) E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}^H}{A_t^H} [(1 - x_{t+1}^H)(\eta B (x_{t+1}^H)^\varphi)] \right\} \quad (71)$$

where we use the fact that  $C_t(S_t)^\alpha = C_t^H + C_t^{H,*} = C_t^{H,W}$ .

Finally, substituting this wage rule in the (64), we obtain the *equilibrium under Nash bargaining*:

$$\frac{\chi_0 C_t^{H,W} (N_t^H)^\phi}{A_t^H} = \frac{1}{\mu} - (1 + \eta) B (x_t^H)^\varphi \quad (72)$$

$$+ \beta(1 - \delta) E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} \left[ (1 + \eta(1 - x_{t+1}^H)) B (x_{t+1}^H)^\varphi \right] \right\}$$

This condition determines the evolution of (un)employment under Nash bargaining.

It is easy to verify that the decentralised equilibrium with Nash bargained wages involves a constant job-finding rate and, hence, a constant level of unemployment.

Again, this crucial result derives from two assumptions: a utility function that is log in consumption and recruitment costs that vary one for one with productivity shocks.

Combining the equilibrium under Nash bargaining and the Nash wage rule, we can determine the actual behaviour of real wages:

$$\frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha = \frac{1}{\mu} - [1 - \beta(1 - \delta)] B (x_M^H)^\varphi \quad (73)$$

where  $x_M^H$  is the (constant) equilibrium job-finding rate, which is solution of (72).

Compare the equilibrium under the efficient allocation (55) and under the decentralised equilibrium (72). While the (un)employment level is constant in both cases, these levels generally differs. Mainly due to the monopolistic distortions, the unemployment level under the efficient allocation is higher than the one prevailing in the decentralised solution. It is easy to verify that the conditions under which the two equilibria correspond, are the following:

1. Perfect competition in the goods market, i.e.  $\mu = 1$ .
2.  $\varphi = \eta$ , i.e. the share of the surplus that goes to workers has to coincide with the elasticity of hiring costs with respect to the job-finding rate.

Similar conditions and exactly the same conclusions hold for the foreign country.

### C.3 Introducing Real Wage Rigidities

Accordingly, and following much of the recent literature, we introduce real wage rigidity by employing a version of Hall's (2005) notion of wage norm. Specifically, we assume the real wage is determined as follows:

$$W_t^{H,R} = (W_t^{H,n})^{1-\gamma} (\bar{W}^H)^\gamma \quad (74)$$

$$W_t^{F,R} = (W_t^{F,n})^{1-\gamma^*} (\bar{W}^F)^{\gamma^*} \quad (75)$$

For home and foreign respectively. The wage norm  $\bar{W}^H$  ( $\bar{W}^F$  for foreign) is simply the wage prevailing in steady state while  $\gamma$  ( $\gamma^*$ ) is an index of the real wage rigidities present in the economy, with  $0 \leq \gamma^i \leq 1$ .

The introduction of such a wage rule modifies the decentralised equilibrium solution. Consider for instance the home country. In equilibrium:

$$\frac{(W_t^{H,n})^{1-\gamma} (\bar{W}^H)^\gamma}{A_t^H} (S_t)^\alpha = \frac{1}{\mu} - B(x_t^H)^\varphi + \beta(1-\delta)E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} B(x_{t+1}^H)^\varphi \right\} \quad (76)$$

As shown before, the Nash bargained wage varies proportionally to  $A_t^H (S_t)^{-\alpha}$  and thus neutralizes the effect of productivity changes on employment. When real wage rigidities are present, instead, the wages do not move enough to absorb the impact of technology shocks. As a result, in a decentralised equilibrium with sticky wages, employment will *not* be constant.

## D Introducing Sticky Prices

We introduce nominal price rigidity using a model à la Calvo (1983). Each period, a firm faces a fixed probability  $(1-\theta)$  of adjusting its price, irrespective of the time elapsed since it last reset its price. The firm resets the price in order to maximize its present discounted value, while taking into consideration that the price it chooses will remain effective for a (random) number of periods. It can be shown that the optimal price setting rule for a home firm resetting prices in period  $t$  is given by:<sup>65</sup>

$$E_t \left\{ \sum_{s=0}^{\infty} \theta^s Q_{t,t+s}^H Y_{t+s/t} \left( \tilde{P}_t^H - \frac{\epsilon}{\epsilon-1} P_{t+s}^H MC_{t+s}^H \right) \right\} = 0 \quad (77)$$

where  $\tilde{P}_t^H$  denotes the price newly set at time  $t$ ,  $Y_{t+s/t}$  is the level of output in period  $t+s$  for a firm resetting its price in period  $t$  and  $\mu = \frac{\epsilon}{\epsilon-1}$  is the gross desired markup. The real marginal cost takes the usual form:

$$MC_t^H = \frac{W_t^{H,R}}{A_t^H} (S_t)^\alpha + B(x_t^H)^\varphi - \beta(1-\delta)E_t \left\{ \frac{C_t^{H,W}}{C_{t+1}^{H,W}} \frac{A_{t+1}^H}{A_t^H} B(x_{t+1}^H)^\varphi \right\} \quad (78)$$

As Blanchard and Galí (2006) note, the two previous equations embody the essence of the integration of hiring costs in a standard NK model. In fact:

1. Taking as given the path of marginal costs, the optimal price setting rule takes the same form as in the standard Calvo model.

2. The dynamics of the marginal costs are however deeply influenced by the introduction of hiring costs and real wage rigidities (which enters through  $W_t^{H,R} = (W_t^{H,n})^{1-\gamma} (\bar{W}^H)^\gamma$ ).

Log-linearizing around a zero inflation steady state the optimal price setting rule and the price index equation  $P_t^H = \left[ (1-\theta)(\tilde{P}_t^H)^{1-\epsilon} + \theta(P_{t-1}^H)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$ , we get the New Keynesian Phillips Curve:

$$\hat{\pi}_t^H = \beta E_t \{ \hat{\pi}_{t+1}^H \} + \lambda \widehat{mc}_t^H \quad (79)$$

<sup>65</sup>See Blanchard-Gali (2006) for a detailed description of the derivation of the New Keynesian Phillips Curve with Hiring Costs.

where  $\hat{\pi}_t^H$  is domestic (i.e. producer prices') inflation,  $\widehat{mc}_t^H$  represent the log deviation of real marginal cost from its steady state value and  $\lambda = (1 - \beta\theta)(1 - \theta)/\theta$ . Note that, while (79) looks like a standard New Keynesian Phillips Curve, the dynamics of the real marginal costs are now substantially different from the ones of a standard NK model. We defer a full discussion of this important point to later. The foreign New Keynesian Phillips Curve can be derived in analogy.

### D.1 Log-linearized Marginal Costs

Consider the home country. Log-linearization of the marginal cost around the steady state gives<sup>66</sup>:

$$\begin{aligned} \widehat{mc}_t^H = & \mu W^{H,R} (S)^\alpha \{ \hat{w}_t^H - \hat{a}_t^H + \alpha \hat{s}_t \} + \varphi g \mu \hat{x}_t^H \\ & - \beta(1 - \delta) g \mu E_t \left\{ \left( \hat{c}_t^{H,w} - \hat{a}_t^H \right) - \left( \hat{c}_{t+1}^{H,w} - \hat{a}_{t+1}^H \right) + \varphi \hat{x}_{t+1}^H \right\} \end{aligned} \quad (80)$$

Variables with ‘‘hat’’ denote log-deviations from steady state.  $\mu$  is the markup. Note that we have normalized the steady state value of productivity to unity ( $A^H = 1$ ).

The loglinear approximations for the labor market tightness  $x_t^H = \frac{h_t^H}{U_t^H}$  and for the world consumption of the home good  $C_t^{H,W} = A_t^H (N_t^H - B (x_t^H)^\varphi h_t^H)$  are given by:

$$\delta \hat{x}_t^H = \hat{n}_t^H - (1 - \delta)(1 - x) \hat{n}_{t-1}^H \quad (81)$$

$$\hat{c}_t^{H,w} = \hat{a}_t^H + \frac{1 - g}{1 - \delta g} \hat{n}_t^H + \frac{g(1 - \delta)}{1 - \delta g} \hat{n}_{t-1}^H - \frac{\varphi g}{1 - \delta g} \delta \hat{x}_t^H \quad (82)$$

Following Blanchard-Gali (2006), we introduce two approximations that considerably simplify the characterization of the equilibrium:

1. Hiring costs are small relative to output, so that we can approximate  $\hat{c}_t^{H,w}$  with  $\hat{c}_t^{H,w} = \hat{a}_t^H + \hat{n}_t^H$ . More precisely, we assume that  $\delta$  and  $g$  are of the same order of magnitude as  $\hat{n}_t^H$ , implying that terms involving  $g \hat{n}_t^H$  or  $\delta \hat{n}_t^H$  are of second order.

2. Fluctuations in  $\hat{x}_t^H$  are large relative to those in  $\hat{n}_t^H$ , an approximation that follows from the log-linearization of the labor tightness index (81) and the assumption of a low separation rate. This implies that terms involving  $g \hat{x}_t^H$  or  $\delta \hat{x}_t^H$  cannot be ignored.

We can therefore rewrite the expression for marginal cost as:

$$\widehat{mc}_t^H = \mu W^{H,R} (S)^\alpha \{ \hat{w}_t^H - \hat{a}_t^H + \alpha \hat{s}_t \} + \varphi g \mu \{ \hat{x}_t^H - \beta E_t \hat{x}_{t+1}^H \} \quad (83)$$

To fully determine the marginal costs, we need a characterisation of the processes for the real wages. Log-linearization of the home wage rule,  $W_t^{H,R} = \left( W_t^{H,n} \right)^{1-\gamma} (\bar{W}^H)^\gamma$ , gives

$$\hat{w}_t^H = (1 - \gamma) \hat{w}_t^{H,Nash} \quad (84)$$

<sup>66</sup>Notice that in steady state:  $MC^H = \frac{W^{H,R}}{A^H} (S^H)^\alpha + g(1 - \beta(1 - \delta)) = \frac{1}{\mu}$

where  $\hat{w}_t^{H,Nash}$  comes from the log-linearization of the Nash wage schedule and can be shown to be equal to

$$\hat{w}_t^{H,Nash} = \frac{1}{W^{H,R}(S)^\alpha} \left\{ \begin{array}{l} \chi_0 (N^H)^{1+\phi} (1+\phi) \hat{n}_t^H \\ + \eta g \varphi \hat{x}_t^H - \beta(1-x^H) \eta g \left( \varphi - \frac{x^H}{1-x^H} \right) E_t \hat{x}_{t+1}^H \end{array} \right\} + \hat{a}_t^H - \alpha \hat{s}_t \quad (85)$$

Using these results, we finally obtain the dynamics of the real marginal costs for the home country:

$$\begin{aligned} \widehat{m}c_t^H &= \Psi_2 \mu \hat{n}_t^H + g \mu \Psi_0 \hat{x}_t^H - g \mu \Psi_1 E_t \{ \hat{x}_{t+1}^H \} \\ &\quad - \gamma \mu W^{H,R}(S)^\alpha (\hat{a}_t^H - \alpha \hat{s}_t) \end{aligned} \quad (86)$$

Where the structural parameters  $\Psi_0$ ,  $\Psi_1$ , and  $\Psi_2$  depend on the bargaining power of workers, on labor market conditions and on the degree of real wage stickyness<sup>67</sup>. The steady state level of the terms of trade is  $S = \frac{P_t^F}{P_t^H} = \frac{A^H N^H (1-g\delta)}{A^F N^F (1-g^*\delta^*)}$ .

In order to express marginal costs in terms of unemployment, let  $\hat{u}_t^i = u_t^i - u^i$  denote the deviations of (after-hiring) unemployment from its steady state value  $u^i$ . Taking a first order Taylor expansion of  $u_t^i = 1 - N_t^i$ , it can be shown that

$$\hat{u}_t^i = - (1 - u^i) \hat{n}_t^i$$

Using this approximation and (81), we can rewrite the evolution of marginal costs as a function of unemployment and shocks:

$$\widehat{m}c_t^H = -\kappa_0 \hat{u}_t^H + \kappa_1 \hat{u}_{t-1}^H + \kappa_2 E_t \hat{u}_{t+1}^H - \gamma \kappa_3 \hat{a}_t^H + \alpha \gamma \kappa_3 \hat{s}_t \quad (87)$$

where the coefficients  $\kappa_i$  depends on the structural parameter of the model<sup>68</sup>.

<sup>67</sup>It can be shown that  $\Psi_0 = \varphi(1 + \eta(1 - \gamma))$ ,  $\Psi_1 = \beta \left[ \varphi + \eta(1 - x^H)(1 - \gamma) \left( \varphi - \frac{x^H}{1-x^H} \right) \right]$ , and  $\Psi_2 = \chi_0 (N^H)^{1+\phi} (1 + \phi) (1 - \gamma)$ .

<sup>68</sup>It can be shown that:

$$\begin{aligned} \kappa_0 &= \frac{1}{1-u} (g\Psi_0 b_0 + g\Psi_1 b_1 + \Psi_2) \mu \\ \kappa_1 &= \frac{1}{1-u} \mu g \Psi_0 b_1 \\ \kappa_2 &= \frac{1}{1-u} \mu g \Psi_0 b_0 \\ \kappa_3 &= \mu W^{H,R}(S)^\alpha \\ b_0 &= \frac{1}{\delta} \\ b_1 &= \frac{1}{\delta} (1 - \delta) (1 - x^H) \end{aligned}$$

where  $\Psi_0$ ,  $\Psi_1$  and  $\Psi_2$  have already been defined.

## E Equilibrium Fluctuations

### E.1 Constrained Efficient Allocation

The efficient constrained allocation is characterized by the following relationships (in log-linear form):

$$\begin{aligned}\bar{u}_t^H &= \bar{u}_t^F = 0 \\ \bar{y}_t^U &= \bar{c}_t^U = \frac{1}{2}(\hat{a}_t^H + \hat{a}_t^F) = \hat{a}_t^U \\ \bar{y}_t - \bar{y}_t^* &= \hat{a}_t^H - \hat{a}_t^F \\ \bar{c}_t - \bar{c}_t^* &= (1 - 2\alpha)(\hat{a}_t^H - \hat{a}_t^F)\end{aligned}$$

Where variables with bars denote the constrained efficient outcome and a union-wide variable is a simple average of the corresponding country-specific variables:  $X_t^U = \frac{X_t^H + X_t^F}{2}$ . As shown before, in the efficient equilibrium (un)employment is invariant to shocks and thus constant across time. Union-wide output and consumption depend only on the union's supply shocks. Asymmetric shocks influence the relative output of home and foreign. Finally notice that as long as there is home bias in consumption (i.e.  $\alpha < \frac{1}{2}$ ) the PPP does not hold and consumption is not equated in equilibrium.

### E.2 The Flexible Price Equilibrium

Given a variable  $X$ , we denote with  $\bar{X}$  deviation of the constrained efficient level from the steady state. We can characterize the flexible price equilibrium as follows:

$$\begin{aligned}\hat{u}_t^H &= \frac{1}{\kappa_0}(\kappa_1 \hat{u}_{t-1}^H + \kappa_2 E_t \hat{u}_{t+1}^H + \alpha \gamma \kappa_3 \hat{s}_t - \gamma \kappa_3 \hat{a}_t^H) \\ \hat{u}_t^F &= \frac{1}{\kappa_0^*}(\kappa_1^* \hat{u}_{t-1}^F + \kappa_2^* E_t \hat{u}_{t+1}^F - \alpha \gamma^* \kappa_3^* \hat{s}_t - \gamma^* \kappa_3^* \hat{a}_t^F) \\ \hat{y}_t^U - \bar{y}_t^U &= \hat{c}_t^U - \bar{c}_t^U = \hat{n}_t^U = -\hat{u}_t^{Ag} \\ \hat{y}_t - \bar{y}_t &= -\hat{u}_t^H; \quad \hat{y}_t^* - \bar{y}_t^* = -\hat{u}_t^F \\ \hat{s}_t - \bar{s}_t &= (\hat{y}_t - \bar{y}_t) - (\hat{y}_t^* - \bar{y}_t^*) \\ \hat{c}_t - \bar{c}_t &= (\hat{c}_t^* - \bar{c}_t^*) + (1 - 2\alpha)(\hat{s}_t - \bar{s}_t)\end{aligned}$$

where, to simplify the notation, we define aggregate unemployment as a weighted average of the unemployment rates prevailing in each country:  $\hat{u}_t^{Ag} = \frac{1}{2} \left( \frac{\hat{u}_t^H}{1-u^H} + \frac{\hat{u}_t^F}{1-u^F} \right)$ .

### E.3 Sticky Price equilibrium

The union-wide IS equation takes the form:

$$\hat{u}_t^{Ag} = E_t \hat{u}_{t+1}^{Ag} + (\hat{u}_t - E_t \hat{\pi}_{t+1}^U - E_t \hat{a}_{t+1}^U) \quad (88)$$

where  $\hat{\pi}_t^U$  and  $\hat{u}_t^{Ag}$  are the union-wide inflation. The IS equation - in terms of union-wide variables - takes the same form as in a standard closed economy model.

The terms of trade movements distribute production among the two countries and explain unemployment and consumption differentials:

$$\hat{s}_t - \bar{s}_t = \frac{1}{1 - u^F} \hat{u}_t^F - \frac{1}{1 - u^H} \hat{u}_t^H \quad (89)$$

and from the definition of the price level we get:

$$\Delta \hat{s}_t - \Delta \bar{s}_t = \Delta \hat{\pi}_t^H - \Delta \hat{\pi}_t^F \quad (90)$$

where

$$\bar{s}_t = \hat{a}_t^H - \hat{a}_t^F$$

The supply block of the model contains the aggregate supply equations for home:

$$\begin{aligned} \hat{\pi}_t^H &= \beta E_t \{ \hat{\pi}_{t+1}^H \} + \lambda \widehat{m}c_t^H \\ &= \beta E_t \{ \hat{\pi}_{t+1}^H \} - \lambda \kappa_0 \hat{u}_t^H + \lambda \kappa_1 \hat{u}_{t-1}^H + \lambda \kappa_2 E_t \hat{u}_{t+1}^H \\ &\quad - \gamma \lambda \kappa_3 \hat{a}_t^H + \alpha \gamma \lambda \kappa_3 \hat{s}_t \end{aligned} \quad (91)$$

and foreign:

$$\begin{aligned} \hat{\pi}_t^F &= \beta E_t \{ \hat{\pi}_{t+1}^F \} + \lambda^* \widehat{m}c_t^F \\ &= \beta E_t \{ \hat{\pi}_{t+1}^F \} - \lambda^* \kappa_0^* \hat{u}_t^F + \lambda^* \kappa_1^* \hat{u}_{t-1}^F + \lambda^* \kappa_2^* E_t \hat{u}_{t+1}^F \\ &\quad - \gamma^* \lambda^* \kappa_3^* \hat{a}_t^F - \alpha \lambda^* \gamma^* \kappa_3^* \hat{s}_t \end{aligned} \quad (92)$$

Equations (88)-(92) and a monetary policy rule define a sticky price equilibrium  $\{ \hat{u}_t, \hat{s}_t, \hat{\pi}_t^H, \hat{\pi}_t^F, \hat{u}_t^F, \hat{u}_t^H \}_{t=0}^{\infty}$ .