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PRESSURE DROP IN BLOWING GEOTHERMAL WELLS

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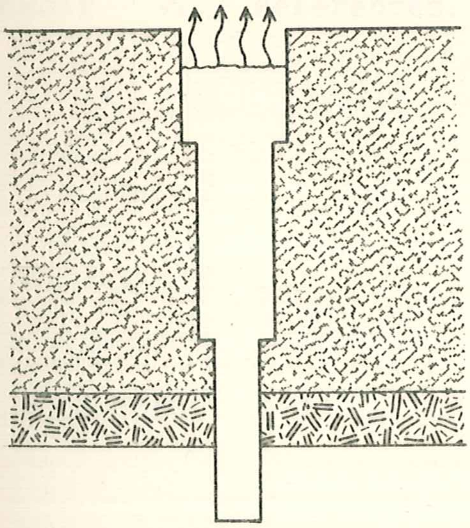

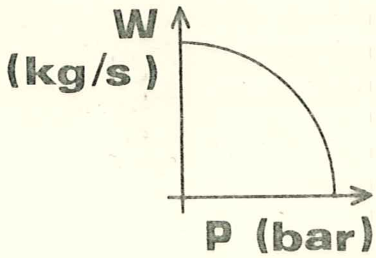

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CONTENTS:

CHAPTER 1.

INTRODUCTION.

The work presented here is my dissertation thesis for the degree of civil engineer at Danmarks Tekniske Højskole. The work has been carried out at the Department of Natural Heat at the National Energy Authority in Iceland. At the NEA there is a great interest in a model of two phase flow in blowing geothermal wells. With such a model one could calculate pressure drop in blowing wells and discharge flowrate against wellhead pressure relationships. A model makes it possible to find out what effect different parameters have on the borehole characteristics. For example, what effect does deposition in the boreholes have on the discharge-pressure relationship? Does the borehole yield increase much if the borehole is made wider? How does the pressure-discharge relationship change when the pressure in the aquifer decreases (because of drainage [redacted])? These questions and others about the pressure-discharge relation of boreholes is difficult to answer unless a model of the borehole discharge.

Professor Jónas Elíasson has been my supervisor in this work and that I thank him very much for.

The work is divided into three parts. The first is a literature study; the second model building for two phase flow and writing of a computer program; the third is a comparison of measurements of pressure and discharge relations by some models. Based on this comparison the best model is chosen.

The work started with a literature study of two phase flow in general. Two phase flow has been studied in the oil industry. In some oil wells oil and gas flow together and this has resulted in studies of two phase flow of oil and gas. In atomic power station water has been used for cooling

In the circulation the water boils and circulation becomes a two phase flow of water and steam. A great deal of material has been published in this field and I have used much of this literature to model flow in blowing geothermal wells.

I did not find in the literture any theoretical equations to describe frictional pressure gradient and void fraction or slip ratio but there were many empirical equations to determine these factors. In appendix II and III some of these empirical equations can be found.

Nicholas Hall who worked at NEA has written a computer program to describe flow in a blowing geothermal well. I have used this program and improved it. The program calculates the pressure drop in a flowing well and the pressure-dixcharge relation. The program can be found in appendix V and an example of the printout. In the program there are two subroutines. One is used to calculate frictional pressure gradient and the other to calculate void fraction. It is easy to change these subroutines and use different empirical equations to calculate pressure drop and void fraction. In this way it can be studied how calculated pressure-discharge relations change when different models are used.

The next step was to go through measured pressure-discharge relations and compare them to calculated relations using different empirical formular. In chapter 5 a comparison is made between measurements in boreholes and calculations by different models.

I have available temperature and pressure measurement in a blowing borehole. This measurement is very good because it shows how temperature and pressure vary with depth in the borehole and makes it easier to choose between the models. In chapter 6 the models are compared and that the model which fits the data best from the measurements is selected.

It is necessary to develop these models further. With a precise comparison of new measurements and computations it will be possible to improve the model. The work is therefore only the firs step in the direction to the making of a good usable model to calculate pressure-discharge relationships in blowing geothermal wells.

A number of symboles has been used in the literature to signify the various parameters in two phase flow. I have made a list in appendix I of all symbols that I use in these work but I will not decribe each symbol as it appears in equations in the text.

CHAPTER 2

Two phase flow in wells

In a blowing geothermal well the pressure falls as we go upwards and this causes boiling. Boiling means that the water is continuously changing to steam. The quality at the steam increase upwards and its watercontent will be reduced but the sum of the masses of water and steam is constant. In fig 1 there is a simplified model of a boiling well, or of two-phase flow in a well.
We will write down and later use equations for conservation of mass, momentum and energy between section 1 and section 2 in the model.

Between section 1 and 2 in fig 1 we can use following equations about conservation of mass, momentum and energy.

Conservation of mass

$$W_g + W_f = W \qquad 2.1$$

$$dW_g = - dW_f \qquad 2.2$$

$$W_g = A_g \cdot \rho_g \cdot u_g = W \cdot \chi \qquad 2.3$$

$$W_f = A_f \cdot \rho_f \cdot u_f = W\ (1-\chi) \qquad 2.4$$

$$\frac{d}{dz}\,(A_g \cdot \rho_g \cdot u_g) = W \cdot \frac{d\chi}{dz} \qquad 2.5$$

$$\frac{d}{dz}\,(A_f \cdot \rho_f \cdot u_f) = - W \cdot \frac{d\chi}{dz} \qquad 2.6$$

Conservation of momentum

$$- A \cdot dp - dF_g - dF_f - g \cdot \sin\theta \cdot dz\ (A_f \cdot \rho_f + A_g \cdot \rho_g) \qquad 2.7$$

$$= d(W_f \cdot u_f + W_g \cdot u_g) \qquad 2.8$$

Change of pressure because of potential

$$-\left(\frac{dp}{dz}Z\right) = g \cdot \sin\theta \cdot \left[\frac{Ag}{A} \cdot \rho_g + \frac{Af}{A} \cdot \rho_f\right] \qquad 2.9$$

$$= g \cdot \sin\theta \cdot [\alpha \cdot \rho_g + (1-\alpha) \cdot \rho_f] \qquad 2.10$$

Change of pressure because of acceleration

$$-\left(\frac{dp}{dz}a\right) = \frac{1}{A} \cdot \frac{d}{dz}\left(W_g \cdot u_g + W_f \cdot u_f\right) \qquad 2.11$$

$$= G^2 \frac{d}{dz}\left(\frac{\chi^2 \cdot V_g}{\alpha} + \frac{(1-\chi)^2 \cdot V_f}{1-\alpha}\right) \qquad 2.12$$

$V_g = V_g(P)$; $V_f = V_f(P)$; $\alpha = \alpha(\chi, P)$
Put this in equation 2.12 and differentiate

$$-\left(\frac{dp}{dz}a\right) = G^2\left(\frac{2 \cdot \chi \cdot V_g}{\alpha} - \frac{2(1-\chi) \cdot V_f}{1-\alpha}\right) \cdot \frac{d\chi}{dz}$$

$$+ G^2\left(\frac{\chi^2}{\alpha} \cdot \frac{dV_g}{dp} + \frac{(1-\chi)^2}{1-\alpha} \cdot \frac{dV_f}{dp} - \frac{\chi^2 \cdot V_g}{\alpha^2} \cdot \frac{d\alpha}{dp} + \frac{(1-\chi)^2 \cdot V_f}{(1-\alpha)^2} \cdot \frac{d\alpha}{dp}\right)\frac{dp}{dz}$$

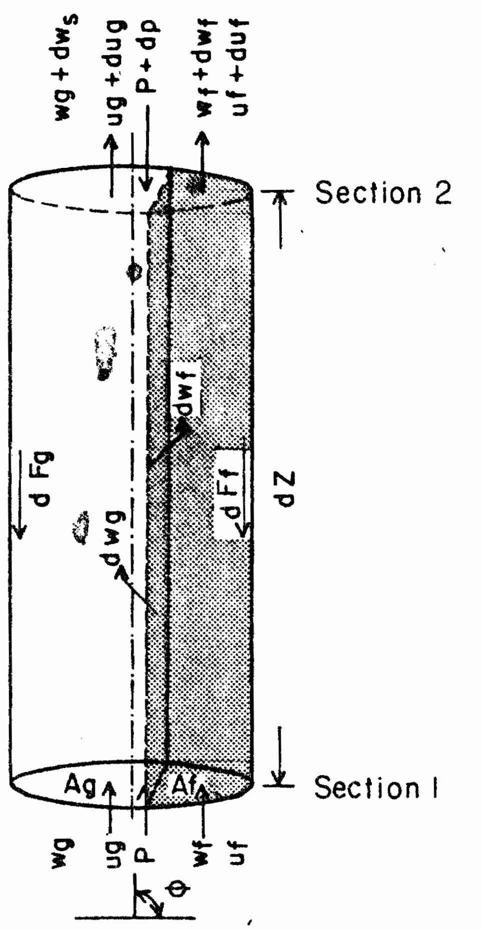
$$+ G^2\left(\frac{(1-\chi)^2 \cdot V_f}{(1-\alpha)^2} \cdot \frac{d\alpha}{d\chi} - \frac{\chi^2 \cdot V_g}{\alpha^2} \cdot \frac{d\alpha}{d\chi}\right) \cdot \frac{d\chi}{dz} \qquad 2.13$$

Conservation of energy

$$W \cdot \delta i + d\left[\frac{W_g \cdot u_g^2}{2} + \frac{W_f \cdot u_f^2}{2}\right] + W \cdot g \cdot \sin\theta \cdot dz = 0 \qquad 2.14$$

In most cases in wells is $\sin\theta$ equal to one. Put $\sin\theta$ equal one into equation 2.14 and divide by W and get,

$$di + d\left\{\frac{\chi \cdot u_g^2}{2} + \frac{(1-\chi) \cdot u_f^2}{2}\right\} + g \cdot dz = 0 \qquad 2.15$$



Simplified model for two-phase flow in wells.

Flow Patterns.

Bubble flow. Here there is a dispersion of bubbles in a continuum of liquid.

Slug flow. When the concentration of bubbles in bubble flow becomes high bubble coalesence occurs and progressively the bubble diameter approaches that of the tube. When this occurs the slug-flow regime is entered with the characteristic bullet-shaped bubbles as illustarted in figure 2.

Annular flow. Here the liquid flows on the wall of the tubes as a film and the gas phase flows in the centre. Usually some of the liquid phase is entrained as small droplets in the gas core.

Wispy annular flow-drop flow. As the liquid flow rate is increased the droplet concentration in the gas core of annular flow increases and ultimately droplet coalescence occurs leading to large lumps or streaks or 'wisps' of liquid occuring in the gas core. This regime is characteristic of high mass velocity flows.

Flow-pattern maps.

In these maps we can see flow-pattern by plotting velocities flow rates and densities. For vertical two-phase flow as in wells we can use the flow-pattern map by Griffith and Wallis (1961) Fig 3 or by Herwitt and Roberts (1969) Fig 4.

In the map by Griffith and Wallis we plot gas phase volumetric flow fraction $\beta = Q_g/Q$ against velocity parameter $= (\frac{Q}{A})^2/g \cdot D$. From this map we can read three flow patterns that is bubbly, slug and annular.

The map by Herwitt and Roberts is more complicated. We plot the parameter $\rho_g \cdot (u_g)^2$ against the parameter $\rho_f \cdot (u_f)^2$. In the map are five flow patterns. That is bubble plug churn annular and wispy annular. In my computation I have used the map by Griffith and Wallis to determine the flow pattern.

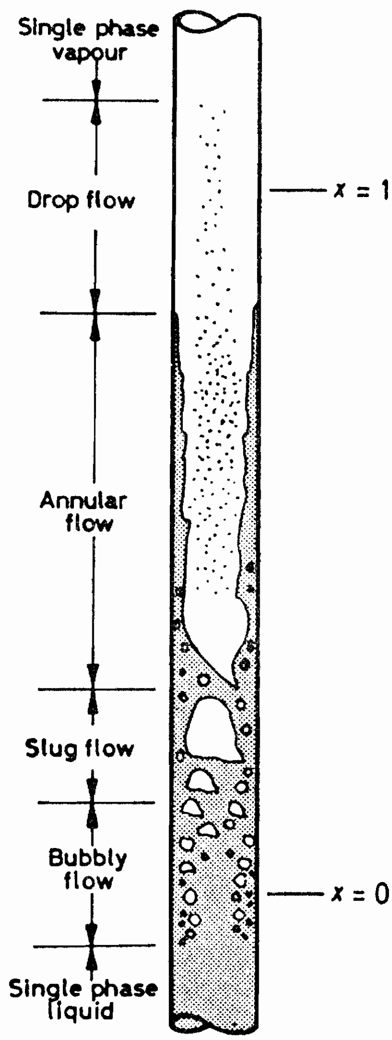


Fig 2 Flow Patterns in a Vertical Evaporator Tube

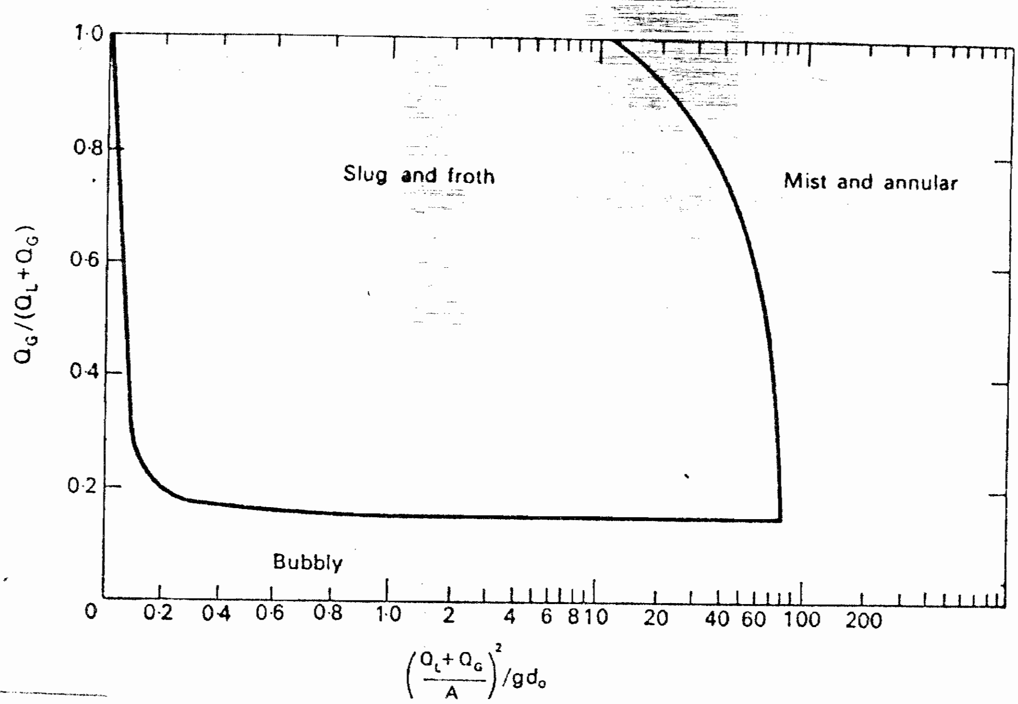


Fig 3 Flow pattern diagram suggested by Griffith and Wallis (1961) for vertical upwards flow.

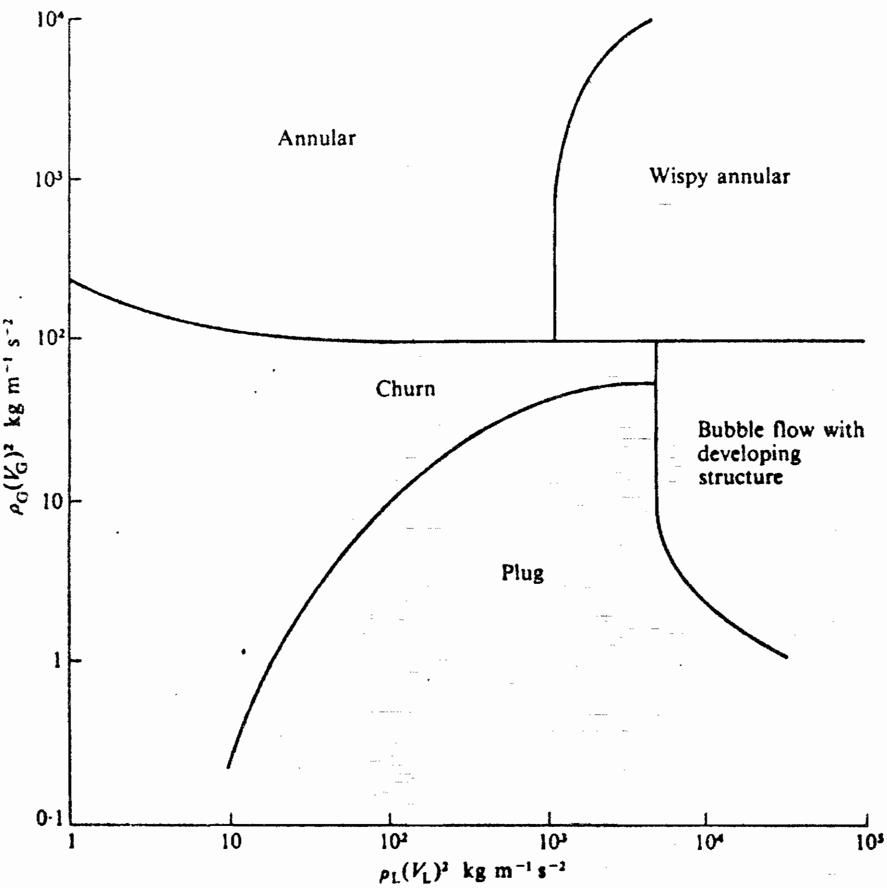


Fig 4

Map of Hewitt and Roberts (1969) for vertical two-phase flow.

Chapter 3. FRICTION FACTOR AND PRESSURE GRADIENT IN TWO PHASE FLOW

Friction factor for single phase flow

In turbulent flow the pressure drop is proportional to the velocity squared

$$\Delta p = C \cdot V^2$$

Pressure drop in pipes is often calculated by Daicy-Weisbach equation which is

$$\frac{\Delta P}{L} = \frac{f}{D} \cdot \frac{\rho \cdot V^2}{2} = -\left(\frac{dp}{dz}\,F\right) \qquad 3.1$$

where f is a friction factor. The friction f is a function of the Reynolds number

$$R_e = \frac{G \cdot D}{\mu} \doteq \frac{V \cdot D \cdot \rho}{\mu}$$, and relative roughness $\frac{K}{D}$.

In the Moody diagram in Fig. 5 the friction factor f is plotted as a function of Reynolds number R_e and relative roughness $\frac{K}{D}$.

In the table 1 there are some emperical equation to cumpute friction factor as a function of relative roughness.

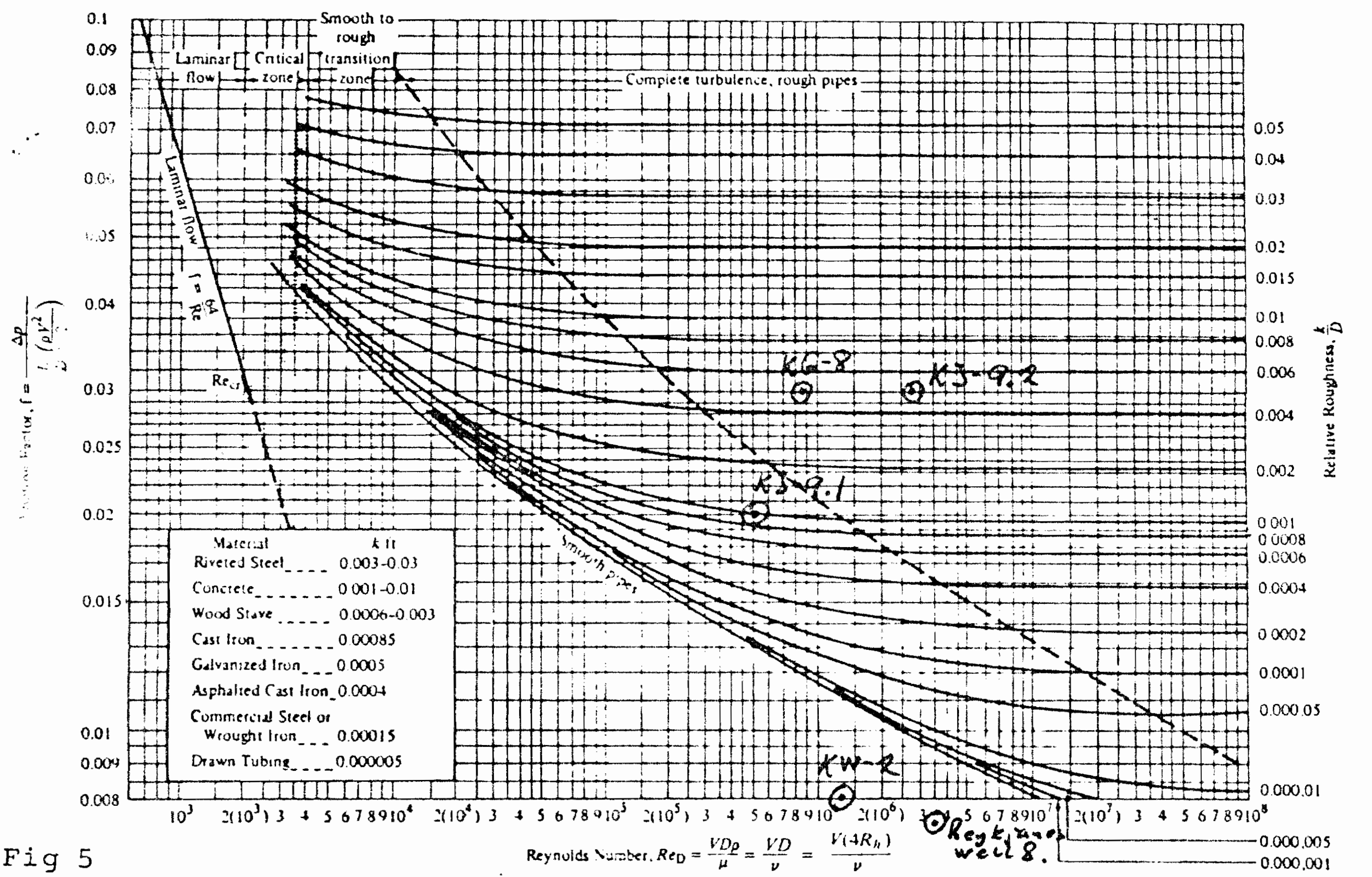


Fig 5

Friction factors for commercial pipe. (From "Friction Factors for Pipe Flow," by L. F. Moody, *Trans. ASME*, Vol. 66, 1944, with permission of the publishers, The American Society of Mechanical Engineers.) ~~This figure appears in larger~~ form on pages 626 and 627.

Equation to calculate friction factor	References
$f = \frac{64}{R_e}$	Laminar flow $R_e < 2000$
$\frac{1}{\sqrt{f}} = 2 \cdot \log \left(R_e \cdot \sqrt{f} \right) - 0{,}8$	Collier
$\frac{1}{\sqrt{f}} = 1{,}74 - 2 \cdot \log \left(\frac{2k}{D} + \frac{18{,}7}{R_e \cdot \sqrt{f}} \right)$	Colebrook and White
$f = 0{,}0056 + \frac{0{,}5}{R_e^{0{,}32}}$	Dukler and Mitarb
$f = \frac{C}{R_e^{\,n}}$ n = 1 laminer, n = 0 turbulence	Blasius
$f = \frac{0{,}198}{R_e^{0{,}202}}$	K.M. Becker et.al.

Table 1. Some equations to calculate friction factor f, in single phase flow.

Pressure drop in two-phase flow

The homogeneous flow model is a common simplified model of two-phase flow. In this model it is assumed that the two phases are flowing as a homogeneous mixture where the gas and liquid are at the same velocity and are in thermodynamic equlibrium.

In this model we can use the following equations.

Continuity: $W = A \cdot \bar{\rho} \cdot \bar{u}$ 3.2

Momentum: $- A \cdot d_p - d\bar{F} - A \cdot \bar{\rho} \cdot g \cdot \sin\theta \cdot dZ = W \cdot d\bar{u}$ 3.3

Energy: $di + d\left(\frac{\bar{u}^2}{2}\right) + g\cdot \sin \cdot dz = 0$ 3.4

In the homogeneous model we can use an equation which is similar to Darcy-Weisbach equation (3,1) for single phase flow.

$$-\left(\frac{dp}{dz}F\right) = \frac{f_{TP}\cdot\bar{\rho}\cdot\bar{u}^2}{D\cdot 2} = \frac{f_{TP}\cdot G^2}{2\cdot D\cdot\bar{\rho}} \quad 3.5$$

where f_{TP} is friction factor in homogeneous two phase flow. Equation 3.5 describes pressure gradient because of friction-loss. But total pressure gradient is friction term plus accereation plus gravitation. The following equation describes accereration

$$-\left(\frac{dp}{dz}a\right) = \frac{W}{A}\cdot\frac{d(\bar{u})}{dz} = G\frac{d(\bar{u})}{dz} \quad 3.6$$

But $\bar{u} = G\cdot\bar{v}$, and put that into eq. 3.6

$$-\left(\frac{dp}{dz}a\right) = G^2\frac{d(\bar{v})}{dz} \quad 3.7$$

$\bar{v}$ is specific volume for the homogeneous mixture.

$$\bar{v} = \frac{Q}{W} = \left(\chi\cdot v_g + (1-\chi)\cdot v_f\right) \quad 3.8$$

differentiate his equation

$$\frac{d\bar{v}}{dz} = \frac{\delta\chi}{\delta z}\cdot v_g + \chi\frac{\delta v_g}{\delta z} - \frac{\delta\chi}{\delta z}\cdot v_f + (1-\chi)\cdot\frac{\delta v_f}{\delta z} \quad 3.9$$

But $\frac{\delta v_f}{\delta z} = \frac{\delta v_f}{\delta p}\cdot\frac{\delta P}{\delta z} \approx 0$ 3.10

eq. 3.9 + 3.10

$$\frac{d\bar{v}}{dz} = \chi\cdot\frac{\delta v_g}{\delta z} + \frac{\delta\chi}{\delta z}\cdot(v_g - v_f) \quad 3.11$$

$$\frac{\delta v_g}{\delta z} = \frac{\delta v_g}{\delta p} \cdot \frac{\delta p}{\delta z} \quad \text{and} \quad v_g - v_f = v_{fg}$$

$$\frac{d\bar{v}}{dz} = \chi \cdot \frac{\delta v_g}{\delta p} \cdot \frac{\delta p}{\delta z} + v_{fg} \cdot \frac{\delta \chi}{\delta z} \qquad 3.12$$

Put together equations 3.7 and 3.12 and get

$$-\left(\frac{dp}{dz} \cdot a\right) = G^2\left(v_{fg} \cdot \frac{\delta \chi}{\delta z} + \chi \cdot \frac{\delta v_g}{\delta p} \cdot \frac{\delta p}{\delta z}\right) \qquad 3.13$$

We have now an equation to compute pressure gradient becouse of acceleration. Now we shall take the potential term, and get the following equation.

$$-\left(\frac{dp}{dz}\, z\right) = \bar{\rho} \cdot g \cdot \sin\theta = \frac{g \cdot \sin\theta}{\bar{v}} \qquad 3.14$$

Now we can compute the total pressure gradient

$$\frac{dp}{dz} = \left(\frac{dp}{dz}\, F\right) + \left(\frac{dp}{dz} \cdot a\right) + \left(\frac{dp}{dz}\, z\right) \qquad 3.15$$

pressure gradient = friction + acceleration + potential.

We now add equations 3.5., 3.13 and 3.14 together and get total pressure gradient.

$$-\left(\frac{dp}{dz}\right) = \frac{f_{TP} \cdot G^2 \cdot \bar{v}}{2 \cdot D} + G^2\left(v_{fg} \cdot \frac{\delta \chi}{\delta z} + \chi \frac{dv_g}{dp} \frac{dp}{dz}\right) + \frac{g \cdot \sin\theta}{\bar{v}} \qquad 3.16$$

or

$$-\frac{dp}{dz} - G^2 \cdot \chi \cdot \frac{dv_g}{dp} \cdot \frac{dp}{dz} = \frac{f_{TP} \cdot G^2 \cdot \bar{v}}{2 \cdot D} + G^2 \cdot v_{fg} \cdot \frac{\delta \chi}{\delta z} + \frac{g \cdot \sin\theta}{\bar{v}} \qquad 3.17$$

But $\bar{v} = \chi \cdot v_g + (1-\chi) \cdot v_f \quad = v_f\left(1 + \chi\left(\frac{v_{fg}}{v_f}\right)\right)$, and put this into

equation 317.

$$-\left(\frac{dp}{dz}\right)\left(1+G^2\cdot\chi\cdot\frac{dp}{dz}\right)=$$

$$\frac{f_{TP}\cdot G^2\cdot v_f\cdot\left(1+\chi\cdot\left(\frac{v_{fg}}{v_f}\right)\right)}{2\cdot D}+G^2\cdot v_{fg}\cdot\frac{\delta\chi}{\delta z}+\frac{g\cdot\sin\theta}{v_f\cdot\left(1+\chi\cdot\left(\frac{v_{fg}}{v_f}\right)\right)} \qquad 3.18$$

Divide eq. 3.18 by $\left(1+G^2\cdot\chi\cdot\frac{dp}{dz}\right)$

$$-\frac{dp}{dz}=\frac{\frac{f_{TP}\cdot G^2\cdot v_f\cdot\left(1+\chi\cdot\left(\frac{v_{fg}}{v_f}\right)\right)}{2\cdot D}+G^2\cdot v_{fg}\cdot\frac{\delta\chi}{\delta z}+\frac{g\cdot\sin\theta}{v_f\cdot\left(1+\chi\cdot\left(\frac{v_{fg}}{v_f}\right)\right)}}{1+G^2\cdot\chi\cdot\frac{dp}{dz}} \qquad 3.19$$

This equation gives total pressure gradient $\left(\frac{dp}{dz}\right)$ as a function of other parameters.

Friction factor in homogeneous two phase-flow

Equation 3.5 describes pressure gradient because of friction loss.

$$-\left(\frac{dp}{dz}F\right)_{TP}=\frac{f_{TP}\cdot G^2}{2\cdot D\cdot\bar{\rho}}=\frac{f_{TP}\cdot G^2\cdot v_f\left(1+\chi\left(\frac{v_{fg}}{v_f}\right)\right)}{2\cdot D} \qquad 3.20$$

where f_{TP} is friction factor in homogeneous two phase flow. We have discussed how the friction factor in single phase flow, is a function of Reynolds number R_e. Similar we can write f_{TP} as a funktion of Reynolds number for the mixture in the two phase flow.

$$R_e=\frac{G\cdot D}{\bar{\mu}}$$

where $\bar{\mu}$ is dynamic viscosity for the mixture.

Mc Adams et al 1942 define viscosity of the mixture as following:

$$\frac{1}{\bar{\mu}} = \frac{\chi}{\mu_g} + \frac{1-\chi}{\mu_f} \qquad 3.22$$

Combine equations 3.21 and 3.22

$$R_e = G \cdot D \left(\frac{\chi}{\mu_g} + \frac{1-\chi}{\mu_f} \right) \qquad 3.23$$

$$R_e = \frac{G \cdot D}{\mu_f} \cdot \left(\chi \cdot \frac{\mu_f}{\mu_g} + 1-\chi \right) = \frac{G \cdot D}{\mu_f} \left(1+\chi \left(\frac{\mu_{fg}}{\mu_g} \right) \right) \qquad 3.24$$

We can use Blasius equation to compute friction factor as a function of Reynolds number.

$$f = \frac{C}{R_e^{\,n}} \qquad 3.25$$

Combine 3.24 and 3.25

$$f_{TP} = C \cdot \left(\frac{G \cdot D}{\mu_f} \right)^{-n} \cdot \left(1+\chi \cdot \left(\frac{\mu_{fg}}{\mu_g} \right) \right)^{-n} \qquad 3.26$$

$$f_{TP} = f_{fD} \cdot \left(1+\chi \cdot \left(\frac{\mu_{fg}}{\mu_g} \right) \right)^{-n} \qquad 3.27$$

where f_{fo} is friction factor if total flow is assumed liquid. Combine eq. 3.20 and 3.27 .

$$-\left(\frac{dp}{dz} F \right) = \frac{f_{fo} \cdot G^2 \cdot v_f}{2 \cdot D} \cdot \left(1+\chi \cdot \left(\frac{v_{fg}}{v_f} \right) \right) \cdot \left(1+\chi \left(\frac{\mu_{fs}}{\mu_g} \right) \right)^{-n} \qquad 3.28$$

$$-\left(\frac{dp}{dz} F \right)_{TP} = -\left(\frac{dp}{dz} \right)_{fo} \left(1+\chi \cdot \left(\frac{v_{fg}}{v_f} \right) \right) \cdot \left(1+\chi \cdot \left(\frac{\mu_{fg}}{\mu_g} \right) \right)^{-n} \qquad 3.29$$

where $\left(\frac{dp}{dz} F\right)_{fo}$ is friction pressure gradient if total flow is assumed liquid. The two-phase frictional pressure gradient is usually correlated in terms of factors which multiply single-phase gradients.

For example, we have

$$\left(\frac{dp}{dz} F\right)_{TP} = \Phi^2_{fo} \cdot \left(\frac{dp}{dz} F\right)_{fo} \qquad 3.30$$

where $\left(\frac{dp}{dz} F\right)_{TP}$ is frictional pressure gradient for two-phase flow,

and $\left(\frac{dp}{dz} F\right)_{fo}$ is frictional pressure gradient if total flow is assumed liquid. In eq. 3.29 this multiplyer was

$$\Phi^2_{fo} = \left(1+\chi\left(\frac{v_{fg}}{v_f}\right)\right) \cdot \left(1+\chi\left(\frac{\mu_{fg}}{\mu_g}\right)\right)^{-n} \qquad 3.31$$

Pressure gradient in two-phase separated flow

We have found equations to compute the pressure gradient in homogeneous two-phase flow, actually the two-phase flow is not homogeneous. The velocity of the gas is greater than the velocity of the liquid. Therefore we have to find new equations to comput pressure gradient. We can use equation 3.15 for pressure gradient.

$$-\left(\frac{dp}{dz}\right) = -\left(\frac{dp}{dz} F\right) - \left(\frac{dp}{dz} a\right) - \left(\frac{dp}{dz} z\right) \qquad 3.15$$

Total pressure gradient	=	friction term	+	acceleration term	+	potential term

We can use eq. 2.10 to compute the potential term

$$-\left(\frac{dp}{dz} z\right) = g \cdot \sin\theta \cdot \left(\alpha \cdot \rho_g + (1-\alpha) \cdot \rho_f\right) \qquad 2.10$$

We can use eq. 2.12 to compute the acceleration term.

$$-\left(\frac{dp}{dz}\,a\right) = G^2 \frac{d}{dz}\left(\frac{\chi^2 \cdot v_g}{\alpha} + \frac{(1-\chi)^2 \cdot v_f}{1-\alpha}\right) \qquad 2.12$$

It is helpful to use correlating parameters to compute the friction term. Martinetti - Nelson (1948) introduced multipliers which are defined

$$\left(\frac{dp}{dz}\,F\right)_{TP} = \Phi^2_{L\text{ or }G}\left(\frac{dp}{dz}\,F\right)_{L\text{ or }G} \qquad 3.32$$

where $\left(\frac{dp}{dz}\,F\right)_{TP}$ is the two-phase frictional pressure gradient, and

for $\left(\frac{dp}{dz}\,F\right)_L$ and $\left(\frac{dp}{dz}\,F\right)_G$ are the frictional pressure gradient

for the liquid or gas respectively if they are flowing alone in the same tube. The multipliers Φ^2_L and Φ^2_G are determinal empirically

Other correlating parameters have been defined if the total mass was flowing with the physical properties of one of the phases.

$$\left(\frac{dp}{dz}\,F\right)_{TP} = \Phi_{LO\text{ or }GO} \cdot \left(\frac{dp}{dz}\,F\right)_{LO\text{ or }GO} \qquad 3.33$$

where $\left(\frac{dp}{dz}\,F\right)_{LO}$ and $\left(\frac{dp}{dz}\,F\right)_{GO}$ are the pressure gradients

for the total flow of fluid having the liquid or gas physical properties respectively, and Φ^2_{LO} amd Φ^2_{GO} are the corresponding multipliers.

In Fig. 6 are plots of Φ_L and $1-\alpha$ versus the parameter X, but X is defined in eq. 6.5 and APPENDIX II. In Fig. 7 is a plot of Φ^2_{LO} as a function of mass vapour quality χ and pressure. Fig. 6 and 7 are by Martinelli-Nelson (1948).

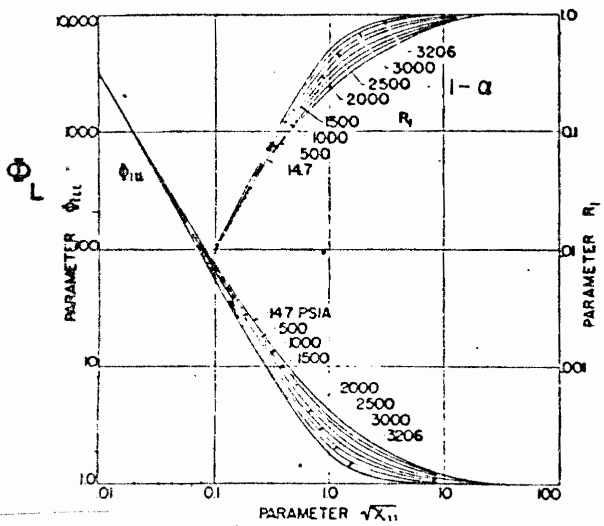


Fig 6

Plot of Parameters Φ_{ttt} and R_l Versus Parameter $\sqrt{X_{tt}}$ for Various Pressures From 1 Atm Abs Pressure to Critical Pressure for Water and Water Vapor

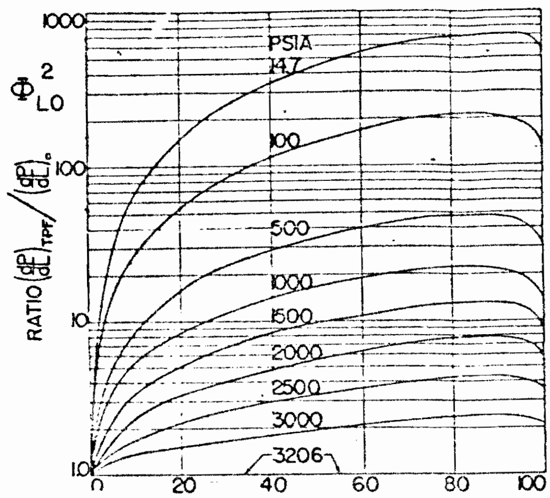


Fig 7 QUALITY-X -% VAPOR BY WEIGHT FLOW

Ratio of Local Two-Phase Pressure Gradient to Pressure Gradient for 100 Per Cent Liquid Flow as a Function of Quality and Pressure

In APPENDIX II there are some emperical equations to comput the multipliers Φ^2.

CHAPTER 4

Computation of blowing well and flow diagram.

We compute stepwise trom the pressure in the aquifer to the wellhead pressure.

Part 1:

From aquifer to the well , Input data is pressure in the aquifer. From the aquifer to the well there may be pressure loss, if it is turbulence flow. See fig 8.

$$PA = PAA - C \cdot W^2 \qquad 4.1$$

Where

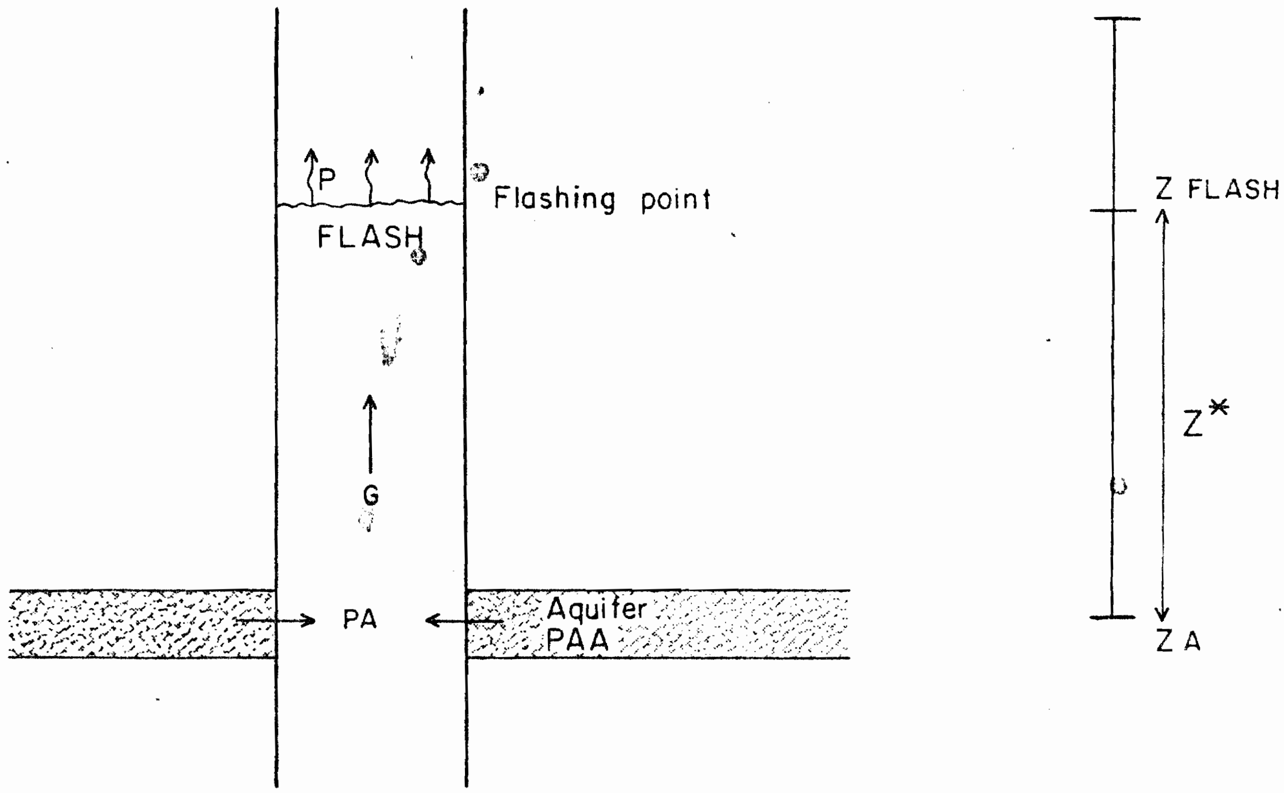
PAA : Pressure in the aquifer (bar).

W : Mass rate flow (Kg/s).

C : Turbulent pressure loss $(bar/(kg/s)^2)$.

PA : Pressure in the well (bar).

Fig. 8



Aquifer level and flash point level.

PART 2:

From ~~equation~~ aquifer level in well to flashing point. When PA is ~~less~~ greater than flashing pressure, we shall compute flashing point level. We compute the pressure gradient in the water.

$$-\left(\frac{dp}{dz}\right) = -\left(\frac{dp}{dz}\,Z\right) - \left(\frac{dp}{dz}\,F\right) - \left(\frac{dp}{dz}\,a\right) \qquad 4.2$$

Where

$$-\left(\frac{dp}{dz}\,Z\right) = \rho \cdot g \qquad 4.3$$

$$-\left(\frac{dp}{dz}\,F\right) = \frac{f \cdot G^2}{2 \cdot D \cdot \rho} \qquad 4.4$$

$-\left(\frac{dp}{dz}\,a\right) = 0$, if diameter is constant.

$$-\left(\frac{dp}{dz}\right) = \rho \cdot g + \frac{f \cdot G^2}{2 \cdot D \cdot \rho} \qquad 4.5$$

Flashing pressure is noted by P_f, and distanse from aquifer to flashing point is Z^*, see fig 8.

$$P_f = PA - \left(\rho \cdot g + \frac{f \cdot G^2}{2 \cdot D \cdot \rho}\right) \cdot Z^* \qquad 4.6$$

or

$$Z^* = \frac{PA - P_f}{\rho \cdot g + \frac{f \cdot G^2}{2 \cdot D \cdot \rho}} \qquad 4.7$$

We can now compute flashing point level, see fig 8.

$$Z_{flash} = ZA - Z^* \qquad 4.8$$

ZA: Aquifer level

Z_{flash}: Flashing point level

PART 3

From flashing point to wellhead. We have to compute stepwise from flashing point to wellhead. In each step the total energy per unit mass is constant. In each section we have to compute pressure, pressure gradient, enthalpy of water and steam, mass vapour quality, void fraction, and density of water and steam.

First we use equation which describes contant energy per unit mass in each section.

$$E = \chi \cdot h_g + (1-\chi) \cdot h_f + Z \cdot g + \frac{1}{2} \cdot \chi \cdot u_g^2 + \frac{1}{2}(1-\chi) \cdot u_f^2 \qquad 4.9$$

E: Total energy per unit mass, constant.

χ: Mass vapour quality.

h_g: Enthalpy of steam.

h_f: Enthalpy of water.

$Z \cdot g$: Potential energy.

$\frac{1}{2} \cdot \chi \cdot u_g^2$: Kinetic energy of steam.

$\frac{1}{2}(1-\chi) \cdot u_f^2$: Kinetic energy of water.

We can use following equations to compute pressure gradient in each section.

$$-\left(\frac{dp}{dz}\right) = -\left(\frac{dp}{dz} Z\right) - \left(\frac{dp}{dz} F\right) - \left(\frac{dp}{dz} a\right) \qquad 4.10$$

Total pressure gradient = potential term + friction term + acceleration term.

$$-\left(\frac{dp}{dz} Z\right) = g(\alpha \cdot \rho_g + (1-\alpha) \cdot \rho_f) \qquad 4.11$$

$$-\left(\frac{dp}{dz} F\right)_{TP} = -\left(\frac{dp}{dz} F\right)_{LO} \cdot \Phi^2_{LO} = \frac{f \cdot G^2}{2 \cdot D \cdot \rho_f} \cdot \Phi^2_{LO} \qquad 4.12$$

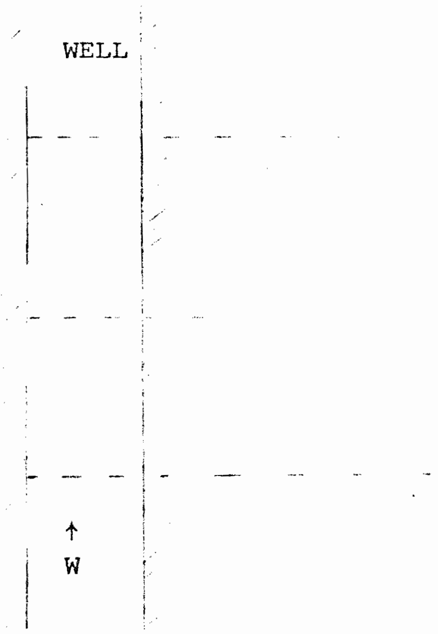
$$-\left(\frac{dp}{dz}\, a\right) = G^2 \frac{d}{dz}\left(\frac{\chi^2}{\alpha \cdot \rho_g} + \frac{(1-\chi)^2}{(1-\alpha)\cdot \rho_f}\right) \qquad 4.13$$

Enthalpy and density of water and steam will we get from steam tables.

The multiplier Φ^2 are computed by emperical equations, which are described in APPENDIX II.

Void fraction α is computed by emperical equations, which are described in APPENDIX III.

We have to use itaration method to compute all parameters in each section.



Section i+1:	z_{i+1},	p_{i+1},	χ_{i+1},	α_{i+1}
	ρ_{gi+1},	ρ_{fi+1},	μ_{gi+1},	μ_{fi+1}
Section i:	z_i,	p_i,	χ_i,	α_i
	ρ_{gi},	ρ_{fi},	μ_{gi},	μ_{fi}
Section i-1:	z_{i-1},	p_{i-1},	χ_{i-1},	α_{i-1}
	ρ_{gi-1},	ρ_{fi-1},	μ_{gi-1},	μ_{fi-1}

The following flowdiagram describes this iteration method.
The index "i" is the number of the section, and the index "j" is the number of iteration in each section.

FLOW DIAGRAM

1	$i = i + 1$	Number of next section
2	$j = 0$	Number of iteration
3	$\Delta P_i^0 = \Delta P_{i-1}$	Start value of ΔP; $P_i^1 = P_{i-1} + (P_{i-1} - P_{i-2})$
4	$Z_i = Z_{i-1} + dZ$	Level of section "i"
5	$j = 1$	Iteration number one
6	$\rho_{g_i}', \rho_{f_i}', h_{g_i}', h_{f_i}' \leftarrow P_{i-1} + \Delta P_i'$	Get this from steam tables
7	$x_i' = \dfrac{Ork - h_{f_i}' - Z_i g - \frac{1}{2}(u_{f_{i-1}})^2}{h_{g_i}' - h_{f_i}' + \frac{1}{2}(u_{g_{i-1}})^2 - \frac{1}{2}(u_{f_{i-1}})^2}$	Start value of vapour quality x
8	Go to 12	Now we go to the iteration loop

9 $j = j + 1$ — Number of iteration

↓

10 $\rho_{g_i}^{j}, \rho_{f_i}^{j}, h_{g_i}^{j}, h_{f_i}^{j} \leftarrow P_{i-1} + \Delta P_i^{j-1}$ — From steam tables

↓

11 $$x_i^j = \frac{Ork - h_{f_i}^{j} - Z_i \cdot g - \frac{1}{2}\left(u_{f_i}^{j-1}\right)^2}{h_{g_i}^{j} - h_{f_i}^{j} + \frac{1}{2}\cdot\left(u_{g_i}^{j-1}\right)^2 - \frac{1}{2}\left(u_{f_i}^{j-1}\right)^2}$$ — New value of x

↓

12 $\alpha_i^j \leftarrow x_i^j,\ P_{i-1} + \Delta P_i^{j-1}$ — Emperical relation

↓

13 $$-\left(\frac{dP}{dz}z\right)_i^j = g \cdot \left[\alpha_i^j \cdot \rho_{g_i}^j + (1-\alpha_i^j)\cdot \rho_{f_i}^j\right]$$ — Potential gradient

↓

14 $$-\left(\frac{dP}{dz}a\right) = \frac{G^2}{dz}\left\{\frac{(x_i^j)^2}{\alpha_i^j \cdot \rho_{g_i}^j} + \frac{(1-x_i^j)^2}{(1-\alpha_i^j)\cdot\rho_{f_i}^j} - \frac{(x_{i-1})^2}{(\alpha_{i-1})\,\rho_{g_{i-1}}} - \frac{(1-x_{i-1})^2}{(1-\alpha_{i-1})\,\rho_{f_{i-1}}}\right\}$$ — Acceleration term

↓

15 $$u_{g_i}^j = \frac{x_i^j \cdot G}{(\alpha_i^j)\cdot \rho_{g_i}^j}$$ — Steam velocity

↓

16 $$u_{f_i}^j = \frac{(1-x_i^j)\cdot G}{(1-\alpha_i^j)\cdot \rho_{f_i}^j}$$ — Water velocity

17 $$\left(\frac{dP}{dZ}F\right)^{j}_{i\ \text{two-phase}} = \Phi^{2\,j}_{i} \cdot \left(\frac{dP}{dZ}F\right)_{\text{Liquid}}$$ Friction term. Emperical relation

↓

18 $$\Delta P_i^{j} = \left\{ \left(\frac{dP}{dZ}F\right)^{j}_{i\ \text{two-phase}} + \left(\frac{dP}{dZ}Z\right)^{j}_{i} + \left(\frac{dP}{dZ}a\right)^{j}_{i} \right\} dZ$$ New value of ΔP

↓

19 ◇ j even — No → Go to 9. Accelerate iteration method then j is even

↓ Yes

20 $$COR \leftarrow \Delta P_i^{j},\ \Delta P_i^{j-1},\ \Delta P_i^{j-2}$$ Method to accelerate iteration.

↓

21 $$\Delta P_i^{j} = \Delta P_i^{j} + COR$$

↓

22 $$\frac{\Delta P_i^{j} - \Delta P_i^{j-1}}{\Delta P_i^{j-1}} < 0.0025$$ Yes → 23; No → 25

23 $$P_i = P_{i-1} + \Delta P_i^{j}$$

24 Step completed. Take next section. Go to 1

25 Itereration again. Go to 9

CHAPTER 5

Mass flow from wells against wellhead pressure, comparison of measurement and computation.

In this chapter some measurements of mass flow as a function of wellhead pressure in some high temperature wells are discussed. I will compare these measurements to my computations in chapter 6.

In my model I need one emperical equation to compute multiplier Φ^2, and another emperical equation to determine void fraction α. In APPENDIX II are some equations to compure Φ^2 and in APPENDIX III there are equations to determine void fraction α. In my computations I used the following models.

I: MODELS TO DETERMINE $-(\frac{dp}{dz}F)$

Becker model

$$\Phi^2_{LO} = 1 + 2547 \cdot (\frac{\chi}{p})^{0.96} \qquad 5.1$$

$$-(\frac{dp}{dz}F)_{TP} = \frac{f \cdot G^2}{2 \cdot D \cdot \rho_f} \cdot (1 + 2547 \cdot (\frac{\chi}{p})^{0,96}) \qquad 5.2$$

Chisholm model

$$\Phi^2_L = 1 + \frac{C}{X} + \frac{1}{X^2} \qquad 5.3$$

C is determained by J. Thom equation.

$$C = 1 + \frac{\chi/\rho_f}{\chi/\rho_g + (1-\chi)/\rho_f} - \alpha \qquad 5.4$$

X is defined in APPENDIX II

$$X = (\frac{1-\chi}{\chi}) \cdot (\frac{\rho_g}{\rho_f})^{1/2} \qquad 5.5$$

$$-(\frac{dp}{dz}F)_{TP} = \frac{f \cdot G^2}{2 \cdot D \cdot \rho_f} \cdot (1-\chi)^2 \cdot \Phi^2_L \qquad 5.6$$

II: Models to determine void fraction, α.

Armand and Treahcher model + Kowalczewski model.

Named ATK - model.

Armand and Treahcher model:

$$\alpha = \frac{0.833+0,05\cdot \log P}{1+\frac{1-\chi}{\chi}\cdot\frac{\rho_g}{\rho_f}} \tag{5.7}$$

Kowalczewski: model

$$\alpha = \beta - 0,71\cdot\beta\cdot(1-\beta)^{0,5}\cdot\left(1-\frac{P}{P_{crit}}\right)\cdot\left(\frac{v^2}{g\cdot d}\right)^{-0.045} \tag{5.8}$$

In fig 26 measurements and equations to compute void fraction are compared. It seems that Armand and Treahcher model fit the data best when χ is less than 0,02, but for greater χ Kowalczewski model fits better. In my ATK- model, I use equation 5.7 to compute α if χ is less than 0,02, and I use eq. 5.8 if χ is greater than 0,02.

Moody - model

The void fraction is a function of the velocity ratio-slip ratio K

$$\alpha = \frac{\rho_f\cdot\chi}{(1-\chi)\cdot\rho_g\cdot K+\chi\cdot\rho_f} \tag{5.9}$$

One emperical equation which gives slip ratio K is named Moody-model, see APPENDIX III.

This equation is,

$$K = \left(\frac{\rho_f}{\rho_g}\right)^{1/3} \tag{5.10}$$

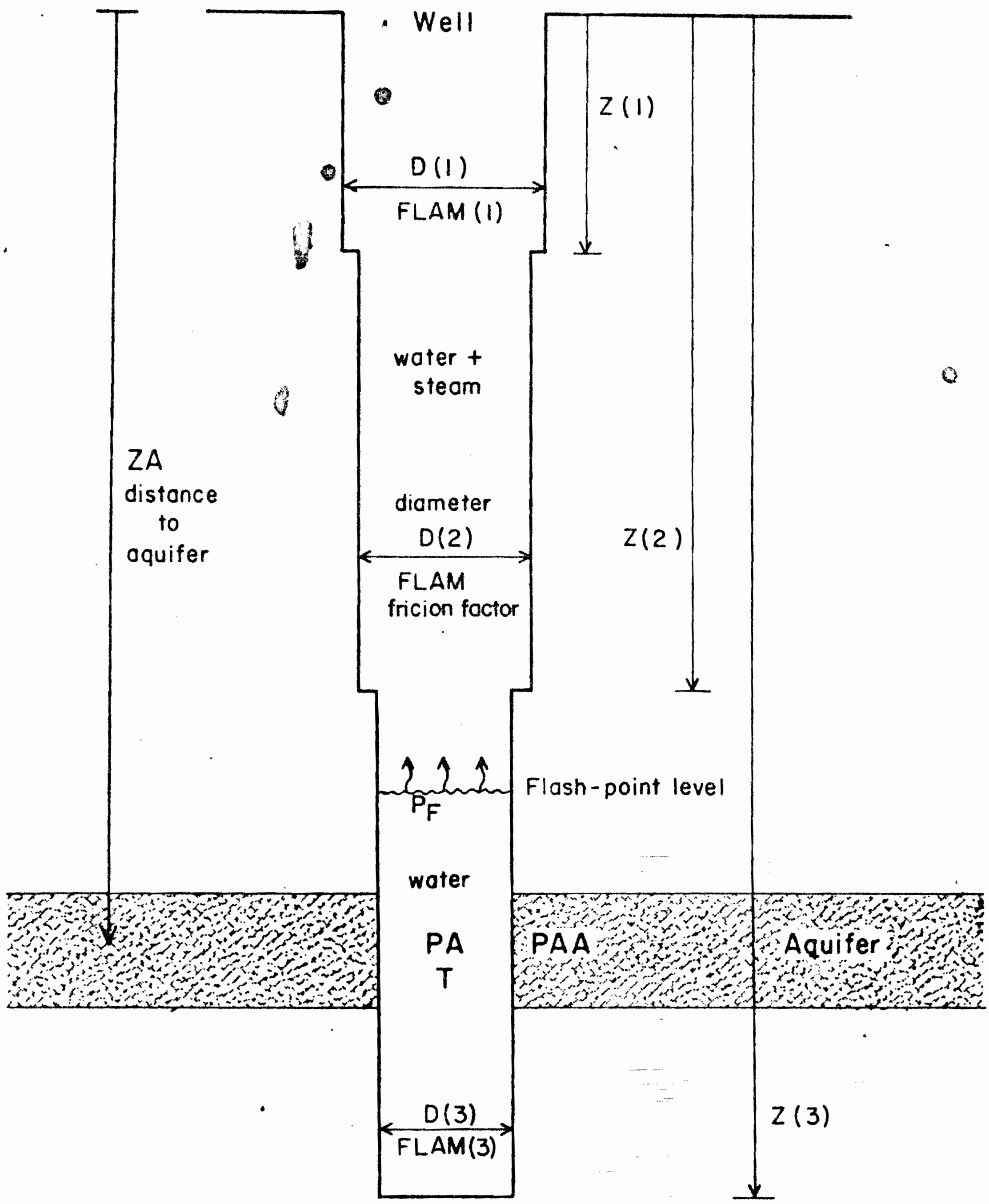
Put eq. 5.9 and 5.10 together

$$\alpha = \frac{\chi\cdot\rho_f}{(1-\chi)\cdot\rho_g\cdot\left(\frac{\rho_f}{\rho_g}\right)^{1/3}+\chi\cdot\rho_f} \tag{5.11}$$

This is Moody-model to compute void fraction α.

Fig. 9

Input data for each well and notation



In following pages are data for 9 geothermal wells in Iceland, and in figs 10-19 are plots of measured and compared mass flow agains wellhead pressure in these wells. By composition of computation of some models, I will try to find the best one.

DATA

WELL KW - 2 KRAFLA

plot on fig 10.

$T_{measuret}$ = 194°C → P_f = 13,69 bar

T_{SiO_2} = 220°C → P_f = 23,20 bar

ZA = 1.000 m

PAA = 77,8 bar

Z(1) = 296 m , D(1) = 8 3/4 "

Z(2) = 1138 m , D(2) = 7 3/8 "

WELL KJ - 6

plot on fig 11

Data

$T_{measuret}$ = 270°C → P_f = 55,1 → H_T = 1185 KJ/Kg

T_h = 340°C ← P_f = 146 bar ← $H_{measured}$ = 1595 KJ/Kg

ZA = 1400 m

PAA = 73,1 Kg/sm^2

Z(1) = 142 m D(1) = 13 3/8 "

Z(2) = 491 m D(2) = 9 5/8 "

Z(3) = 2000 m D(3) = 7 5/8 "

KJ - 7 KRAFLA

plot on fig 12

Data

$T_{measuret}$ = 342°C → P_p = 149,8 bar → H_t = 1610 KJ/Kg

Measuret enthalpy is 1592 - 2263 KJ/Kg

ZA = 1900 m

PAA = 152 bar

Z(1) = 276 m	D(1) = 13 3/8 "
Z(2) = 716 m	D(2) = 9 5/8 "
Z(3) =2101 m	D(3) = 7 5/8 "

KG - 8 KRAFLA

plot on fig 13

Data

T = 206°C ↔ P_f = 17,6 bar ↔ H = 880 KJ/Kg

ZA = 1200 m

PAA = 94,3 bar

Z(1) = 142 m	D(1) = 13 3/8 "
Z(2) = 517 m	D(2) = 9 5/8 "
Z(3) =1646 m	D(3) = 7 5/8 "

KJ - 9.1 KRAFLA

Measurement in the blowing well, Q = 18 Kg/s

Plot on fig 14 and 15

$T = 195^{\circ}C \leftrightarrow P_f = 14$ bar $\leftrightarrow H = 830$ KJ/Kg

ZA = 400 m

PAA = 22,3 bar

Z(1) = 275 m D(1) = 13 3/8 "

Z(2) = 1101 m D(2) = 7 5/8 "

KJ - 9,2 KRAFLA

Plot on fig 16

$T = 276^{\circ}C \leftrightarrow P_f$ 60,4 bar $\leftrightarrow H = 1215$ KJ/Kg

ZA = 1225 m

PAA = 95 bar

Z(1) = 252 m D(1) = 13 3/8 "

Z(2) = 1094 m D(2) = 8 5/8 "

Z(3) = 1259 m D(3) = 7 5/8 "

WELL NO 3, SVARTSENGI

Plot on fig 17

Data

$T_{measuret} = 229^{\circ}C \leftrightarrow P_f = 27,0$ bar

$T_{SiO_2} = 232 - 242^{\circ}C \leftrightarrow P_f = 29-35$ bar

ZA = 358 m

PAA ≈ 27,5 bar

Z(1) = 480 m D(1) = 6 "

In thes well, the flash-point is below aquifer level. That meens mixture of water and steam came to the well from the aquifer. But in my computer-model, the flash-point have to be inside the well

WELL NO 4, SVARTSENGI

Plot on fig 18

$T = 242 - 244^{\circ}C \leftrightarrow P_p = 34,7 - 35,9$ bar

ZA = 1024 m

PAA = 88 bar

Z(1) = 350 D(1) = 9 5/8 "

Z(2) = 1650 D(2) = 7 5/8 "

WELL NO 8, REYKJANES

Plot on fig 19

$T = 287 - 291^{\circ}C \leftrightarrow P_p = 71,3 - 75,5$ bar

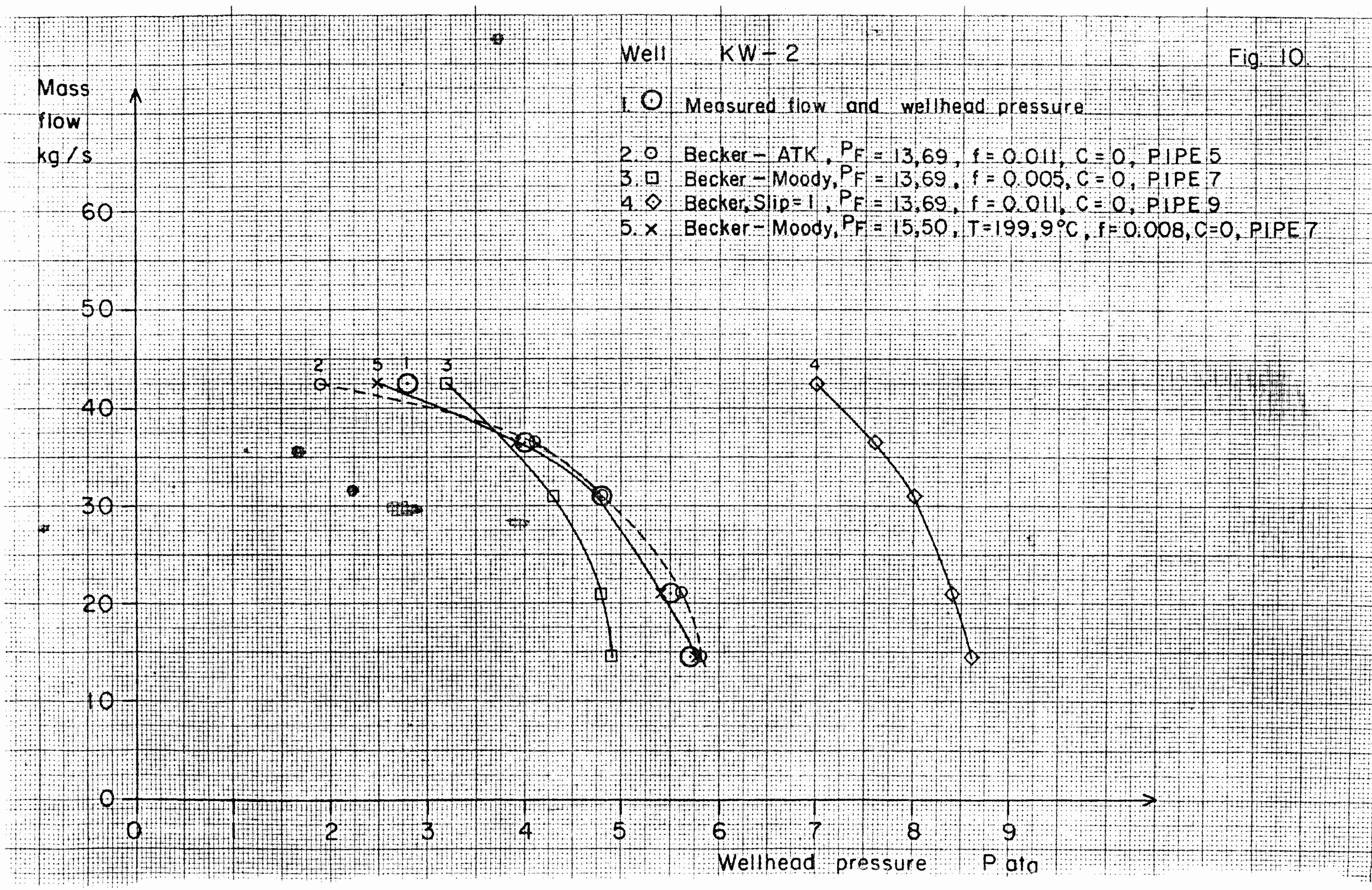
ZA = 1300 m

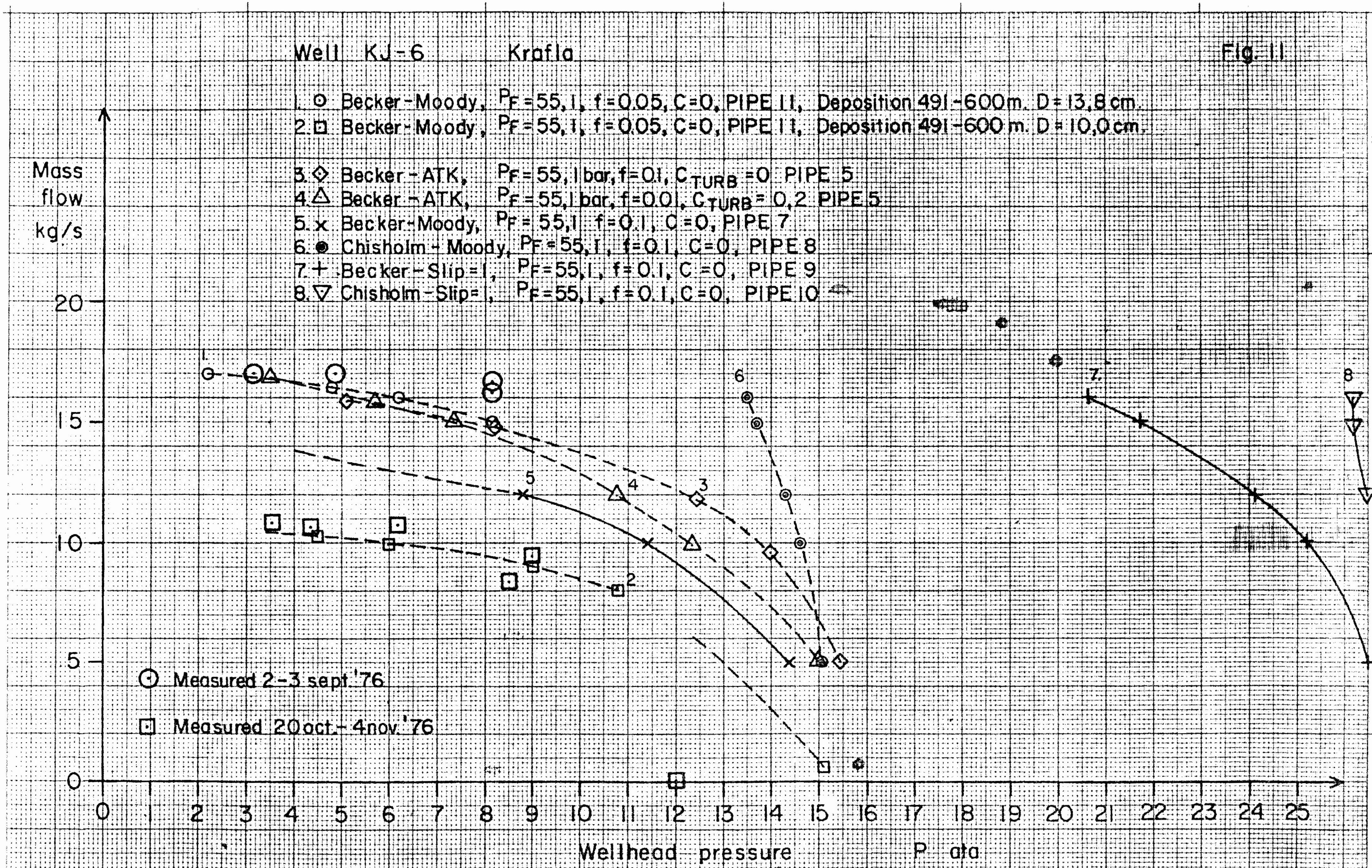
PAA = 74-90 bar

Z(1) = 88,5 m D(1) = 13 3/8 "

Z(2) = 260 m D(2) = 9 5/8 "

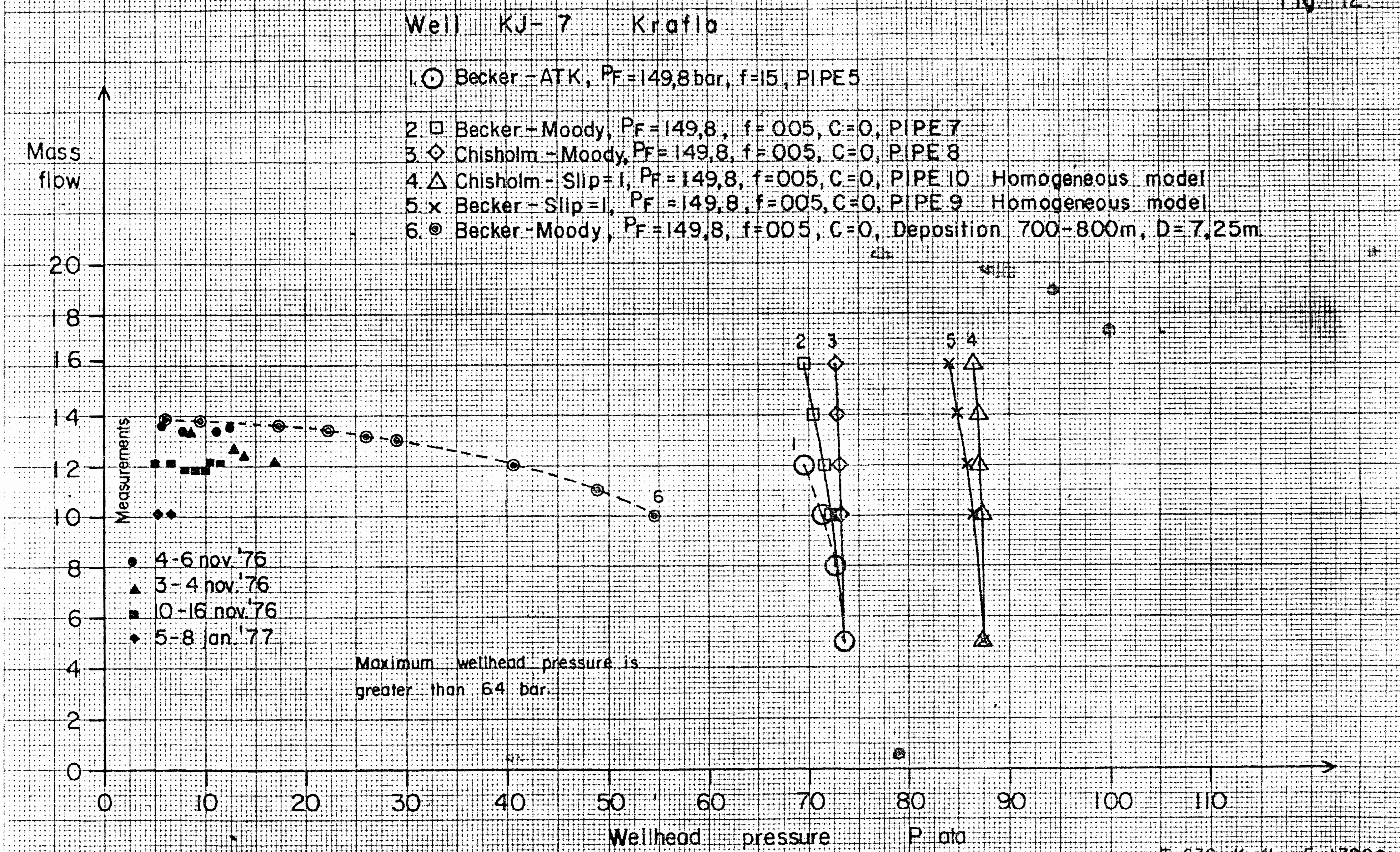
Z(3) = 1752 m D(3) = 7 5/8 "


Well KW-2
Fig. 10.
1. ⊙ Measured flow and wellhead pressure
2. ○ Becker - ATK, P_F = 13,69, f = 0.011, C = 0, PIPE 5
3. □ Becker - Moody, P_F = 13,69, f = 0.005, C = 0, PIPE 7
4. ◇ Becker, Slip = 1, P_F = 13,69, f = 0.011, C = 0, PIPE 9
5. × Becker - Moody, P_F = 15,50, T = 199,9°C, f = 0.008, C = 0, PIPE 7
Mass flow kg/s
0
10
20
30
40
50
60
0
2
3
4
5
6
7
8
9
Wellhead pressure P ata
2
5
1
3
4



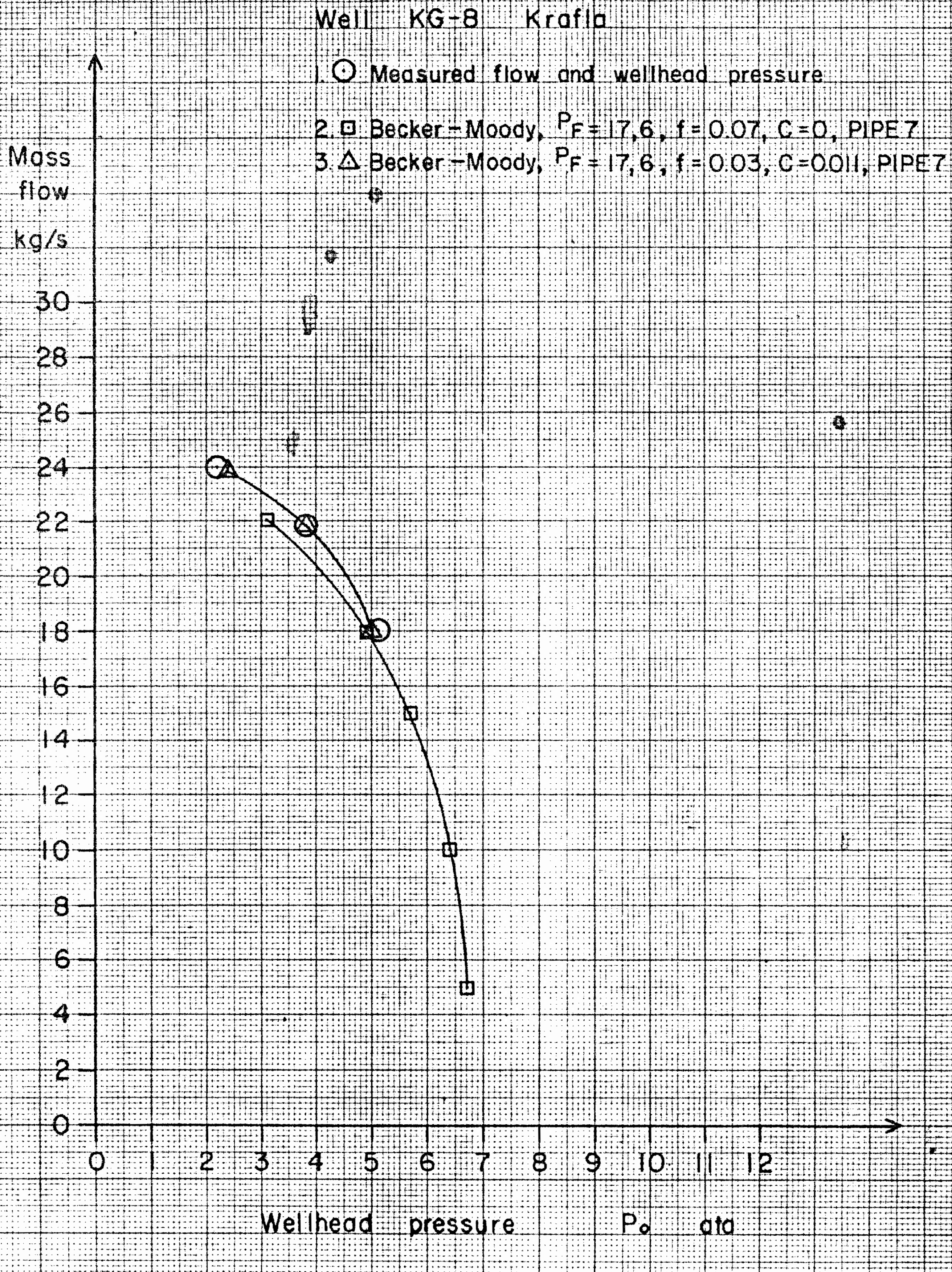
T-637 Krafla F-17205

Fig. 12.

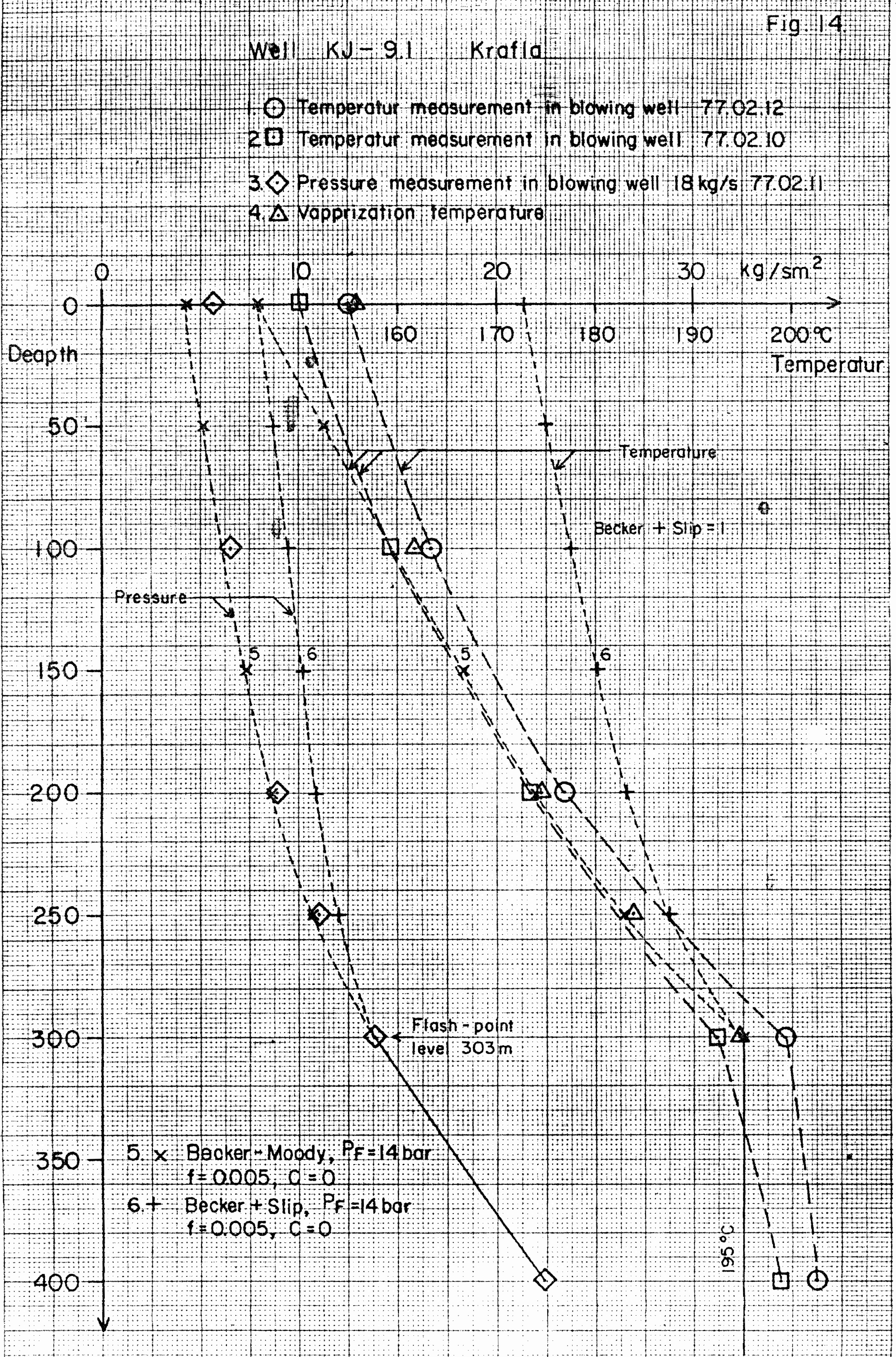


T-638 Krafla F-17206

Fig. 13.



T-639 Krafla F-17207



T-643 Krafla F-17211

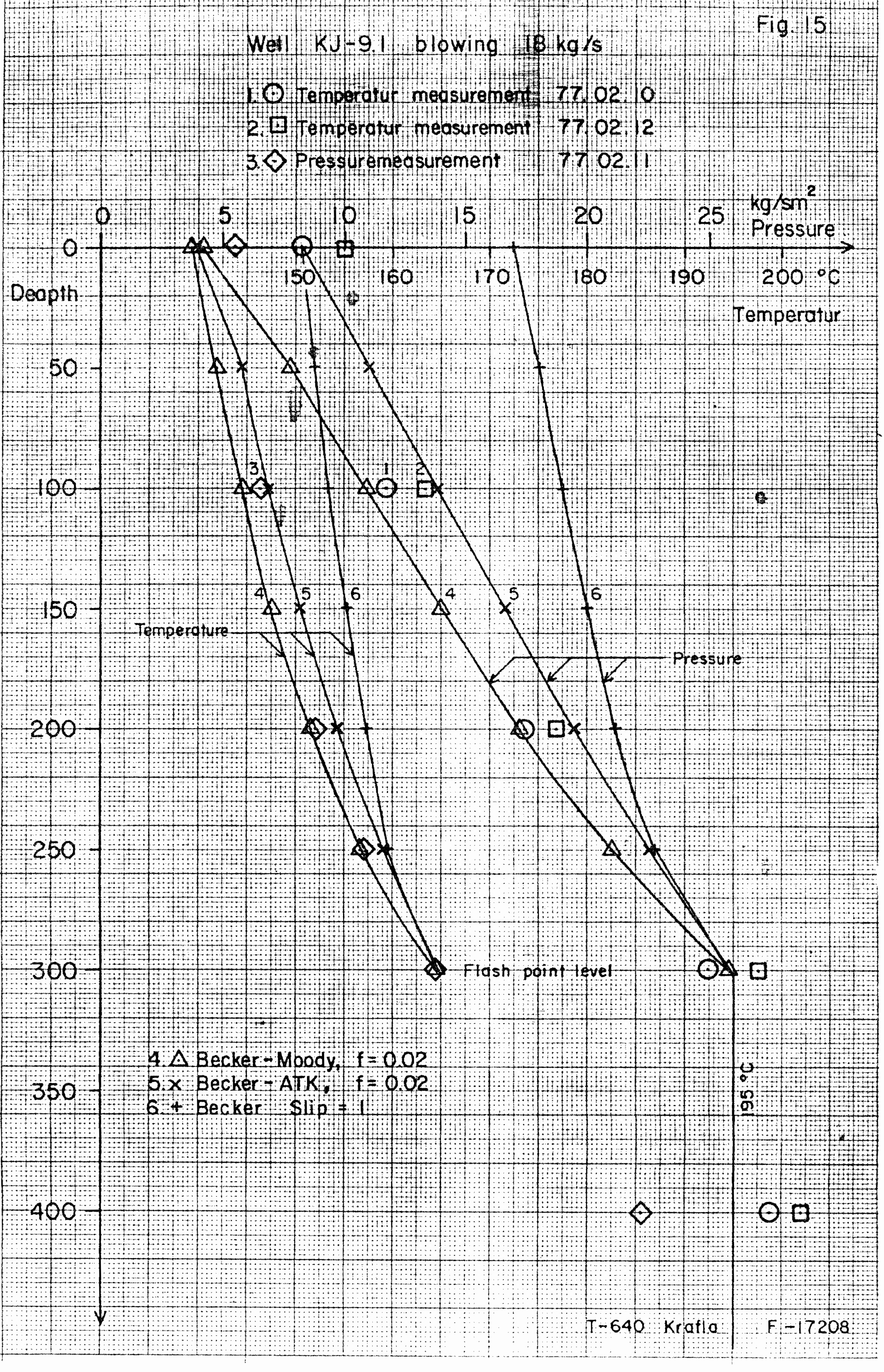
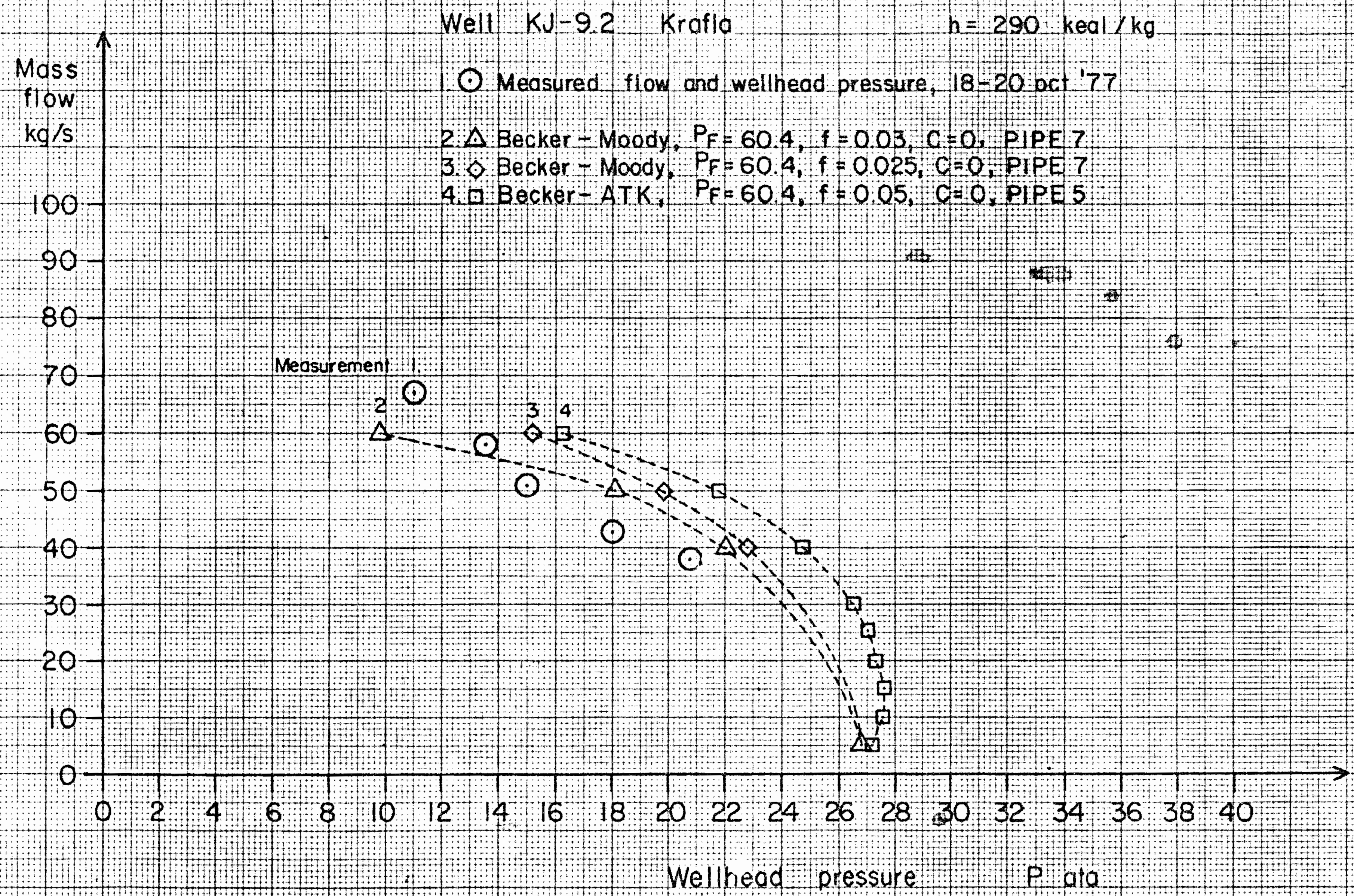


Fig. 15
Well KJ-9.1 blowing 18 kg/s
1. ⊙ Temperatur measurement 77.02.10
2. ⊡ Temperatur measurement 77.02.12
3. ◇ Pressuremeasurement 77.02.11
0 5 10 15 20 25
kg/sm²
Pressure
150 160 170 180 190 200 °C
Temperatur
Deapth
0
50
100
150
200
250
300
350
400
Temperature
Pressure
Flash point level
195 °C
4. △ Becker-Moody, f= 0.02
5. x Becker-ATK, f= 0.02
6. + Becker Slip = 1
T-640 Krafla F-17208

Fig. 16.



T-642 Krafla F-17210

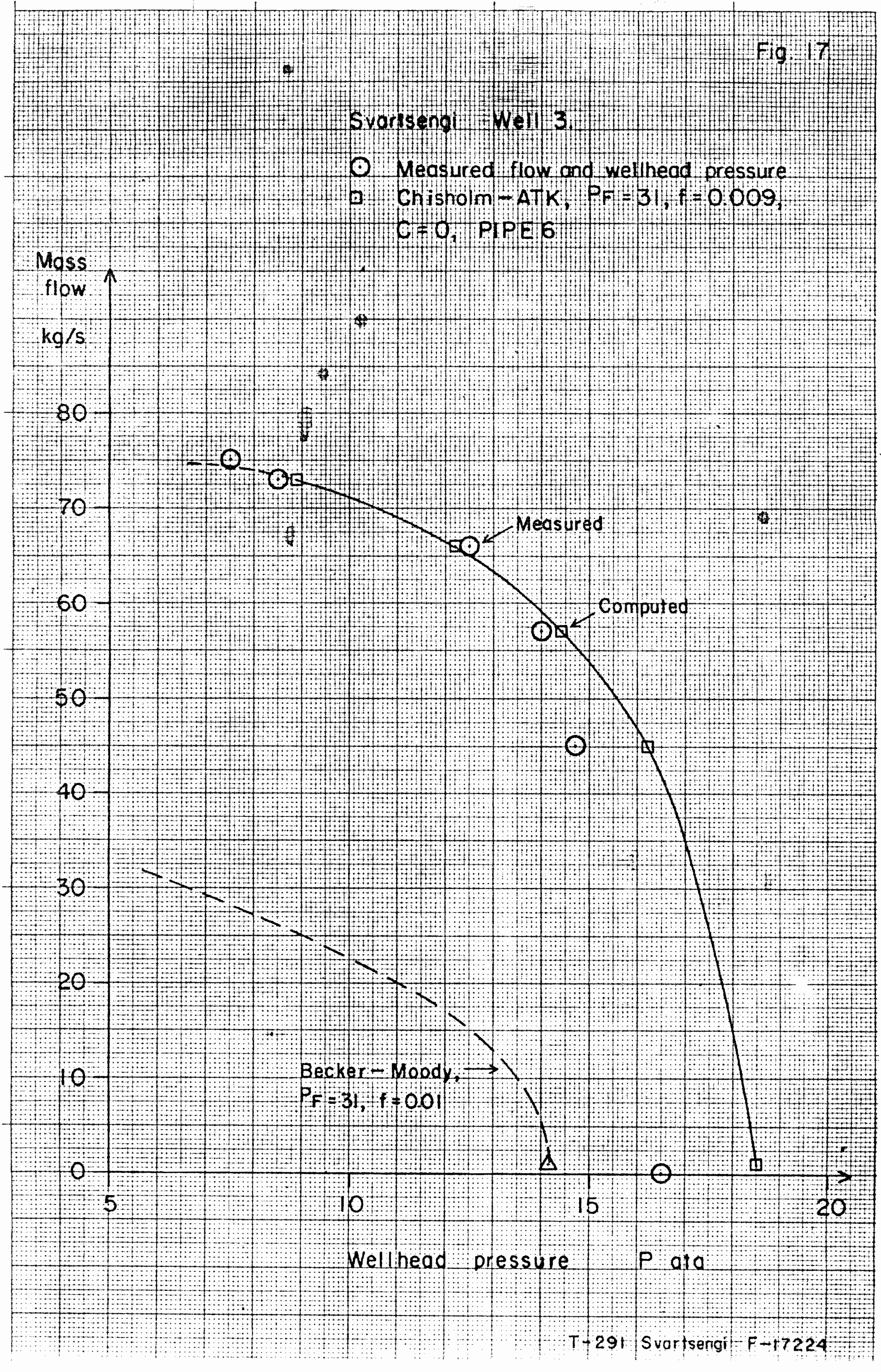


Fig. 17
Svartsengi Well 3.
Measured flow and wellhead pressure
Chisholm – ATK, P_F = 31, f = 0.009, C = 0, PIPE 6
Mass flow kg/s
80
70
60
50
40
30
20
10
0
5
10
15
20
Measured
Computed
Becker – Moody, P_F = 31, f = 0.01
Wellhead pressure P ata
T-291 Svartsengi F-17224

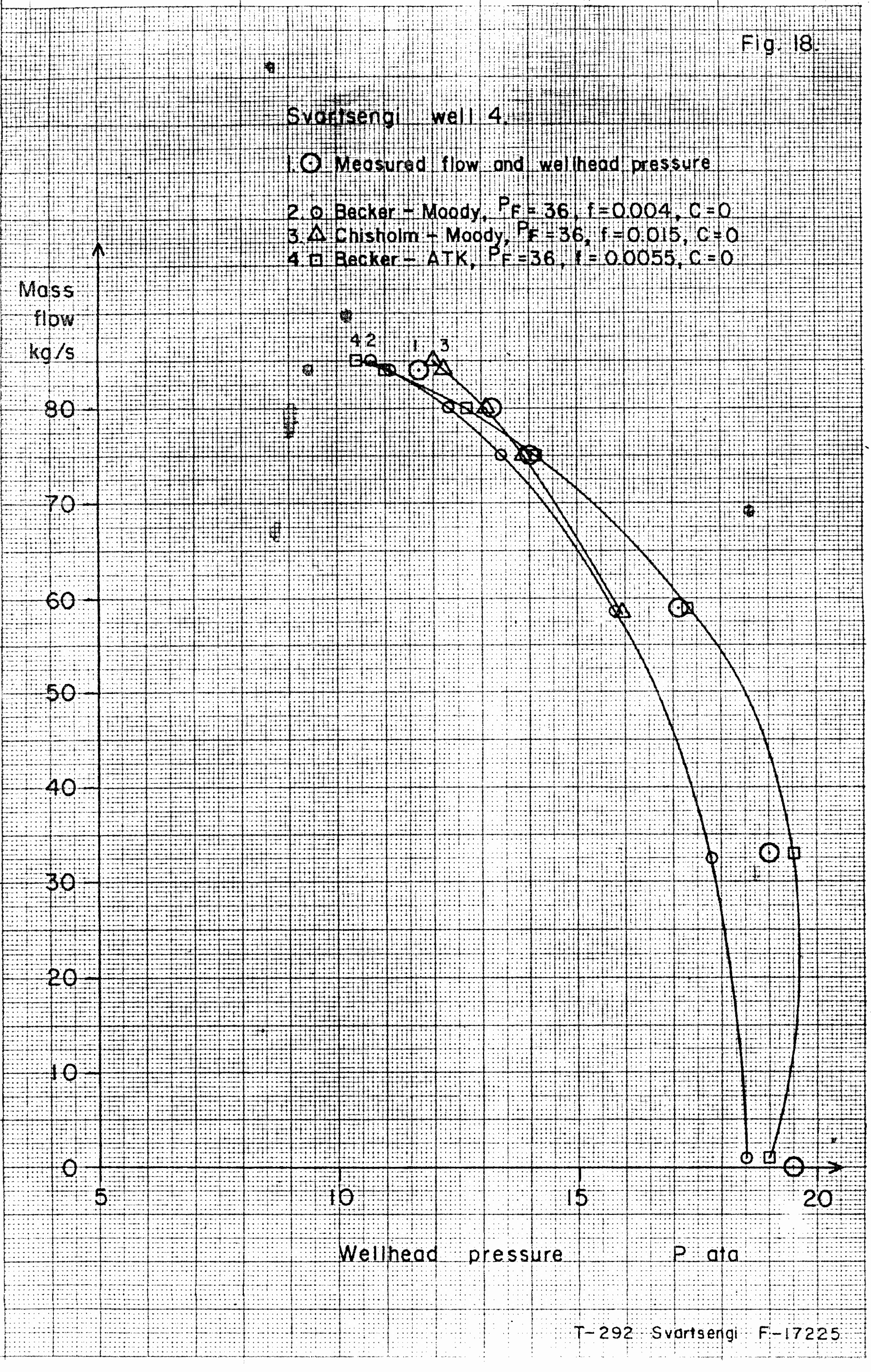
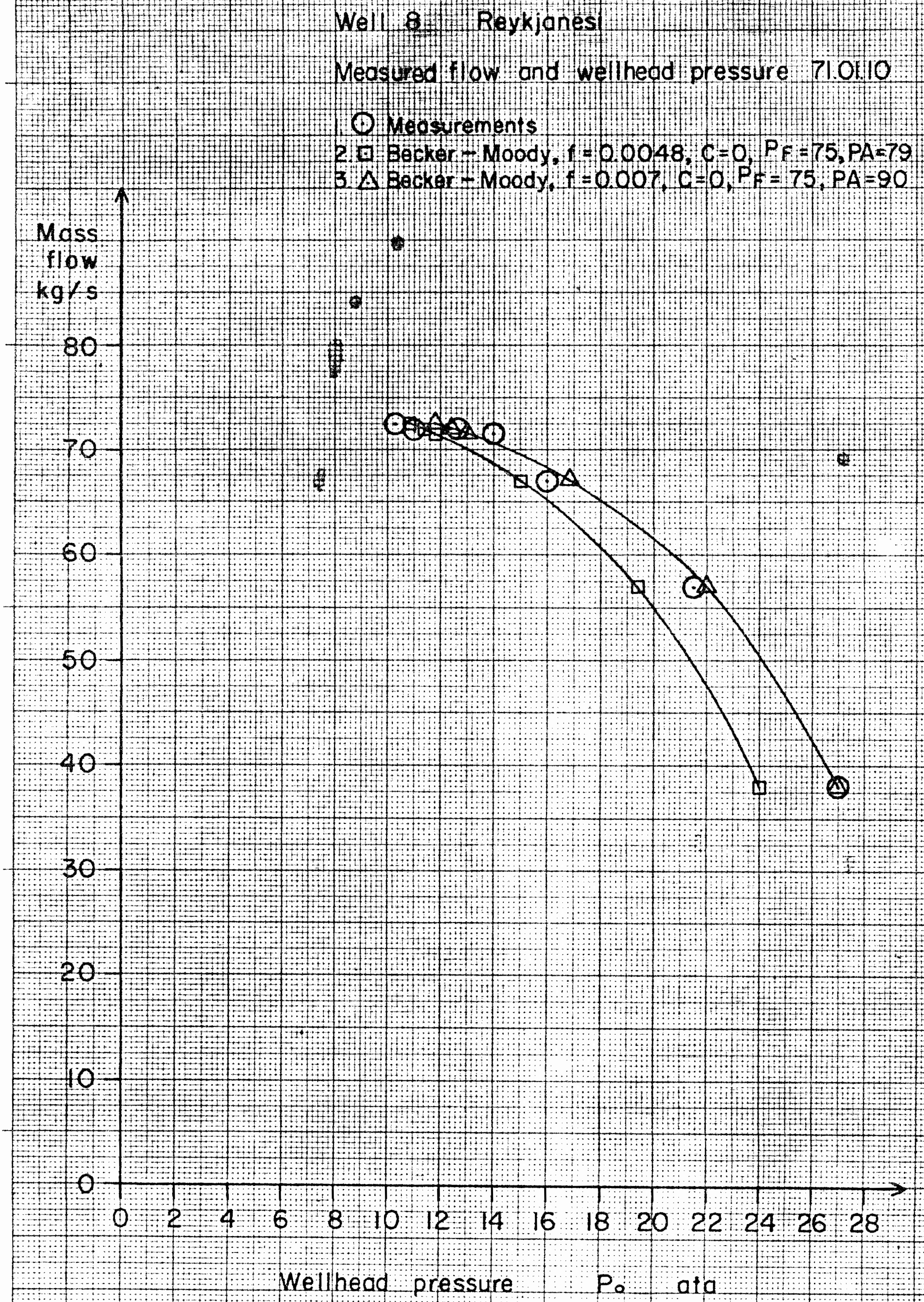


Fig. 18.
Svartsengi well 4.
1. Measured flow and wellhead pressure
2. Becker - Moody, P_F = 36, f = 0.004, C = 0
3. Chisholm - Moody, P_F = 36, f = 0.015, C = 0
4. Becker - ATK, P_F = 36, f = 0.0055, C = 0
Mass flow kg/s
80
70
60
50
40
30
20
10
0
5
10
15
20
Wellhead pressure P ata
4 2
1
3
T-292 Svartsengi F-17225

Fig. 19.



T-293 Svartsengi F=17226

CHAPTER 6.

CONCLUSIONS.

The main conclusion is that the Becker-Moody model for computing pressure drop and slip ratio confirms well to pressure-discharge measurements of boreholes. Friction factor in single phase flow f is selected in such a way that calculations and measurements confirm.

In my calculations friction factor f ranges from 0.007 to 0.03 when Becker-Moody model is used. The Chisholm model for Φ^2 seems not to fit as well as the Becker model. It is very clear that the Homogeneous model is no good. One cannot expect water and steam to flow with the same velocity. In two of the boreholes from which I have measurements of pressure and discharge deposition occurs. This has the effect that the borehole diameter becomes smaller in certain parts. I have taken this into account in the calculations on boreholes KJ-7 and KJ-6. In borehole KJ-7 I expected the deposition to be at 700-800 m depth and the physical real diameter to be 7.2 cm. The measurements of pressure and discharge confirm well to calculations when the deposition was taken into account but not if other models are used as can be seen in figure 12. In borehole KJ-6 I have two measurements of pressure and discharge that do not councide in time. These two measurements were done with about 55 days interval and it is known that deposition was taking place during that time. In the calculations I use the Becker-Moody model and expect deposition at 491-600 m depth. I assume the physical diameter to be 13.8 cm for the first measurement, on september 2 to 3 1976. In figure 11, measurements and calculations are compared. I expect that the deposition has increased when the later measurement is done 20 oct.-4.nov. 1976. In the calculations I use the same fundamental data as for the measurement for 2-3 sept. 1976 expect the borehole diameter is now 10 cm at 491-600 m depth. Calculation and measurements fit well as can be seen in figure 11.

Flash pressure and aquifer pressure are measured. But somtime these parameters can vary within a certain range. In well KW-2 is flash pressure of the range 13,69-23,2 bar. In well Reykjanes no. 8 the aquifer pressure is of the range 79-9o bar. The turbulence factor C is zero for all wells other than KG-8 where C= o.o11 bar/(ks/s)^2. If we use Becker-Moody model the friction factor f ranges between o.oo7-o.o3 except when there is deposition in the borehole. We can say that the friction factor is rather small. In Fig 5 is a plot of fricton factor f for each well as a function of Reynolds number Re.

In the following table are given values of the parameters used in the models for computations and a score for goodness of fit to measurements.

WELL Fig no.	MODEL Plot no.	PARAMETERS Friction factor f	Turbulence factor C bar/(kg	Flash pressure P_F	Aquifer pressure PAA	Goodnes of fit
KW-2 Fig 10	Becker-Moody Plot no 5	0.008	0	15.5	77.8	++++
	Becker-ATK Plot no 2	0.011	0	13.69	77.8	++++
	Becker-Moody Plot no 3	0.005	0	13.69	77.8	+
	Becker-slip 1	0.011	0	13.69	77.8	÷
KJ-6 Fig 11	Becker-Moody Deposition 491-600 m Diameter 13.8cm 2 Sept 1976 Plot no 1	0.05	0	55.1	73.1	++++
	Becker-Moody Deposition 491-600 m Diameter 10 sm 20 Oct 1976 Plot no 2	0.05	0	55.1	73.1	++++
	Becker-ATK Plot no 3	0.1	0	55.1	73.1	(+)
	Becker-ATK Plot no 4	0.01	0.2	55.1	73.1	+
	Becker-Moody Plot no 5	0.1	0	55.1	73.1	(+)
	Chisholm-Moody Plot no 6	0.1	0	55.1	73.1	÷
	Becker-slip 1 Plot no 7	0.1	0	55.1	73.1	÷÷
	Chisholm-slip 1	0.1	0	55.1	73.1	÷÷÷

WELL Fig no	MODEL Plot no	PARAMETERS Friction factor f	Turbulence factor C	Flash pressure P_F	Aquifer pressure PAA	Goodness of fit
KJ-7 Fig 12	Becker-ATK Plot no 1	0.15	0	149.8	152	÷ ÷
	Becker-Moody Plot no2	0.05	0	149.8	152	÷ ÷
	Chisholm-Moody Plot no 3	0.05	0	149.8	152	÷ ÷
	Chisholm-slip Plot no 4	0.05	0	149.8	152	÷ ÷ ÷
	Becker-slip Plot no 5	0.05	0	149.8	153	÷ ÷ ÷
	Becker-Moody Deposition 700-800 m Diameter 7.2cm Plot no 6	0.05	0	149.8	152	+ + + +
KG-8 Fig 13	Becker-Moody Plot no 2	0.07	0	17.6	94.3	+ + +
	Becker-Moody Plot no 3	0.03	0.011	17.6	94.3	+ + + +
KJ-9.1 Fig 14and15	Becker-Moody Plot no 4	0.02	0	14	22.3	+ + +
	Becker-ATK Plot no 5	0.02	0	14	22.3	+ + +
	Becker-slip	0.02	0	14	22.3	÷ ÷
KJ-9.2 Fig 16	Becker-Moody Plot no 2.03	0.03	0	60.4	95	+ + +
	Becker-Moody Plot no 3	0.025	0	60.4	95	+ +
	Becker-ATK Plot no4	0.05	0	60.4	95	+
Svartsengi Well no 3: I have no model for this well. Flash point below aquifer level						
Svartsengi Well 4	Becker-Moody Plot no 2	0.004	0	36	88	+
	Chisholm--Moody, Plot 3	0.015	0	36	88	+
	Becker-ATK Plot no 4	0.0055	0	36	88	+ +
Reykjanes Well 8 Fig 19	Becker-Moody Plot no 2	0.0048	0	75	79	+ +
	Becker-Moody Plot no 3	0.007	0	75	90	+ + +

APPENDIX I

LIST OF SYMBOLS

Ag	Flow area occupied by gaseous phase	m^2
Af	Flow area occupied by liquid phase	m^2
A	Flow area	m^2
C	Parameter used in Chisholm correlation	-
C_1	Constant in Blasius equation	-
C_2	Parameter used in Chisholm correlation	-
D	Pipe diameter	m
E	Dissipation of mechanical energy into heat	J/kg
f_L	Frictional factor for the liquid flowing along in the pipe	-
f	" " "	-
f_{Tp}	Two-phase friction factor	
F_g	Force exerted by vapour phase in overcoming friction	N
F_f	Force exerted by liquid phase in overcoming friction	N
F_r^1	Froude - number	-
G	Mass velocity $= \frac{W}{A}$	Kg/m^2s
G^*	Reference mass velocity in Chisholm correlation	Kg/m^2S
H	Enthalpy	J/kg
i	Enthalpy of fluid	J/kg
K	Slip ratio, =	-
M	Mack ~~tala rennslisins~~ number for the flow	-
n	Index in Blasius equation	-
P	Wetted perimeter	m
P	Static pressure	N/m^2 (bar)
PA	Pressure in well in aquifer level	bar
PAA	Pressure in the aquifer	bar
P_{crit} =	Critical pressure = 221.2 bar	bar
P_F	— Flash pressure	bar
Q	Volumetric rate of flow	m^3/s
Q_f	Volumetric rate of liquid phase	m^3/s
Q_g	Volumetric rate of gas phase	m^3/s
q	Heat absorbed from surroundings	J/kg

Re	Reynolds number $= \frac{G \cdot D}{\mu}$	-
S	Ratio A_1/A_2	-
T	Temperature	°C
U_f	Actual velocity of liquid phase	m/s
U_g	Actual velocity of gasous phase	m/s
V	Arearge velocity	m/s
V_a	$= \frac{\chi^2}{\alpha \cdot \rho_g} + \frac{(1-\chi)^2}{(1-\alpha)\,\rho_f}$	M^3/kg
V_g	Specific volume of gas $= \frac{1}{\rho_g}$	m^3/kg
V_f	Specific volume of liquid $= \frac{1}{\rho_f}$	m^3/kg
V_{fg}	Diffierence in specific volumes of saturated liquid and vapour $= V_g - V_f$	m^3/kg
V_H	$= \bar{V}$ average specific volume of homogenous fluid	m^3/kg
w	Work done on surroundings	J/kg
W_g	Gas-phase mass flow rate	kg/s
W_f	Mass rate of flow of liquid phase	kg/s
W	Mass ~~rate~~ flow	kg/s
X^2	$= (\frac{dp}{dz}F)_L \,/\, (\frac{dp}{dz}F)_G$	-
Z	Axial co-ordinate	m
ZA	Deapth to aquifer	m
α	Void fraction $= \frac{Ag}{A}$	-
β	Gas phase volumetric flow fraction $= \frac{\chi \cdot V_g}{\chi \cdot V_g + (1-\chi) \cdot V_f} = \frac{Q_g}{Q}$	-
Γ^2	$= (\frac{dp}{dz})_{Go} \,/\, (\frac{dp}{dz})_{Lo}$	-
	index Go total flow assumed gas	
	" Lo " " " liquid	-
ε	Pipe roughness	m
λ	Parameter used in Chisholm cirrelation	-
μ	Viscosity	Ns/m^2
μ_f	Viscosity of liquid	Ns/m^2
μ_g	Viscosity of gas	Ns/m^2
μ_{fs}	Difference in viscosity between liquid and gas phases	Ns/m^2
ρ_H	Homogeneous density $\frac{1}{\rho M} = \frac{\chi}{\rho_g} + \frac{1-\chi}{\rho_f}$	kg/m^3

ρ_g	Gas density	kg/m^3
ρ_f	Liquid density	kg/m^3
σ	Surface tension	$kg/s^2 = N/m$
τ_w	Wall shear stress	N/m^2
Φ_L^2	Two-phase frictional multiplier, if the liquid ~~are~~ is flowing alone in the pipe	-
Φ_G^2	Two-phase frictional multiplier, if the gas ~~are~~ is flowing alone in the pipe	-
$\Phi_{f_o}^2$	Two-phase frictional multipler based on pressure gradient for total flow assumed liquid	-
$\Phi_{L_o}^2$	$= \Phi_{f_o}^2$	-
$\Phi_{G_o}^2$	$= \Phi_{g_o}^2$ Two-phase frictional multipler based on pressure gradient for total flow assumed gas	-
χ	Mass vapour quality $= W_g/W$	-
θ	Angle to horizontal plane	deg
$(\frac{dp}{dz}a)$	Pressure gradient due to acceleration	$N/m^2 m$
$(\frac{dp}{dz}z)$	Pressure gradient due to static head	$N/m^2 m$
$(\frac{dp}{dz}F)$	Pressure gradient due to friction	$N/m^2 m$
$(\frac{dp}{dz}F)_{fo}$	Frictional pressure gradient assuming total flow to be liquid	$N/m^2 \cdot m$
$(\frac{dp}{dz}F)_L$	Frictional pressure gradient, if the liquid ~~are~~ is flowing alone in the pipe	$N/m^2 m$
$(\frac{dp}{dz}F)_G$	Frictional pressure gradient, if the gas ~~are~~ is flowing alone in the pipe	$N/m^2 m$

APPENDIX II

Emperical equations to determine pressure gradient in two-phase flow.

Martinelli and Nelson (1948) introduced multipliers to determine pressure gradient in two-phase flow. This multipliers are

$$\left(\frac{dp}{dz}F\right)_{T\rho} = \Phi^2_{L\ og\ G} \cdot \left(\frac{dp}{dz}F\right)_{L\ og\ G} \qquad A.1$$

where $\left(\frac{dp}{dz}F\right)_{T\rho}$ is the two-phase frictional pressure gradient, and $\left(\frac{dp}{dz}F\right)_{G}$ are the frictional pressure gradient for the liquid or gas respectively if they are flowing alone in the same tube.

Lockhart and Martinelli (1949) introduced a graph to determine the multipliers Φ_L and Φ_G as a function of the parameter X

$$X^2 = \left(\frac{dp}{dz}F\right)_L \Big/ \left(\frac{dp}{dz}F\right)_G \qquad A.2$$

where $\left(\frac{dp}{dz}F\right)_L$ and $\left(\frac{dp}{dz}F\right)_G$ are the frictional pressure gradient if the liquid or gas are flowing alone in the same tube

$$-\left(\frac{dp}{dz}F\right)_L = \frac{f_L \cdot G^2 \cdot (1-\chi)^2}{2 \cdot D \cdot \rho_f} \qquad A.3$$

f_L is friction factor for the liquid

$$-\left(\frac{dp}{dz}F\right)_G = \frac{f_G \cdot G^2 \cdot \chi^2}{2 \cdot D \cdot \rho_g} \qquad A.4$$

f_G is friction factor for the gas.

Put together eq. A.2, A.3 and A.4

$$X^2 = \frac{f_L}{f_G} \cdot \frac{(1-\chi)^2}{\chi^2} \cdot \frac{\rho_g}{\rho_f} \qquad A.5$$

We can use Blasius equation (Chapter 3) to determine the friction factors f_L and f_G

$$f_L = \frac{C_L}{Re^n} = C_L \left(\frac{\mu_f}{G \cdot D \cdot (1-x)} \right)^n \qquad A.6$$

$$f_G = \frac{C_G}{Re^m} = C_G \left(\frac{\mu_g}{G \cdot D \cdot x} \right)^m \qquad A.7$$

The parameters n and m in Blasius equation depend on flow pattern. If it is turbulent flow in rough pipe, when $n = m = 0$. In laminar flow $n = m = 1$. In our case, we can use $n = m = 0.2$, and $C_L = C_G$. Put that into eq. A.6 and A.7, and put them into eq. A5.

$$X^2 = \left(\frac{1-x}{x} \right)^{1.8} \cdot \frac{\rho_g}{\rho_f} \cdot \left(\frac{\mu_f}{\mu_g} \right)^{0.2} \qquad A.8$$

TABLE 2

Independent variables used in frictional pressure-gradient and void-fraction correlations.

	X^2	Γ^2
Full definition	$(\frac{dp_F}{dz})_L / (\frac{dp_F}{dz})_G$ Lockhart and Martinelli (1949)	$(\frac{dp_F}{dz})_{GO} / (\frac{dp_F}{dz})_{LO}$ Chisholm and Sutherland (1969-70)
$f \propto Re^{-n}$ in both cases	$\frac{\rho_G}{\rho_L} (\frac{1-x}{x})^{2-n} (\frac{\mu_L}{\mu_G})^n$	$\frac{\rho_L}{\rho_G} (\frac{\mu_G}{\mu_L})^n$
$n = 1$ in both cases	$\frac{\rho_G}{\rho_L} (\frac{1-x}{x}) (\frac{\mu_L}{\mu_G})$	$\frac{\rho_L}{\rho_G} (\frac{\mu_G}{\mu_L})$
$n = 0{\cdot}2$ in both cases	$\frac{\rho_G}{\rho_L} (\frac{1-x}{x})^{1{\cdot}8} (\frac{\mu_L}{\mu_G})^{0{\cdot}2}$	$\frac{\rho_L}{\rho_G} (\frac{\mu_G}{\mu_L})^{0{\cdot}2}$ Riciprocal of this parameter introduced by Baroczy (1966)
$n = 0$ in both phases	$\frac{\rho_G}{\rho_L} (\frac{1-x}{x})^2$	$\frac{\rho_L}{\rho_G}$

In table 2 is shown how the variable X^2 depends on the parameter n in Blasius equation.

In fig 20 is the graph who Lochart and Martinelli introduced 1949.

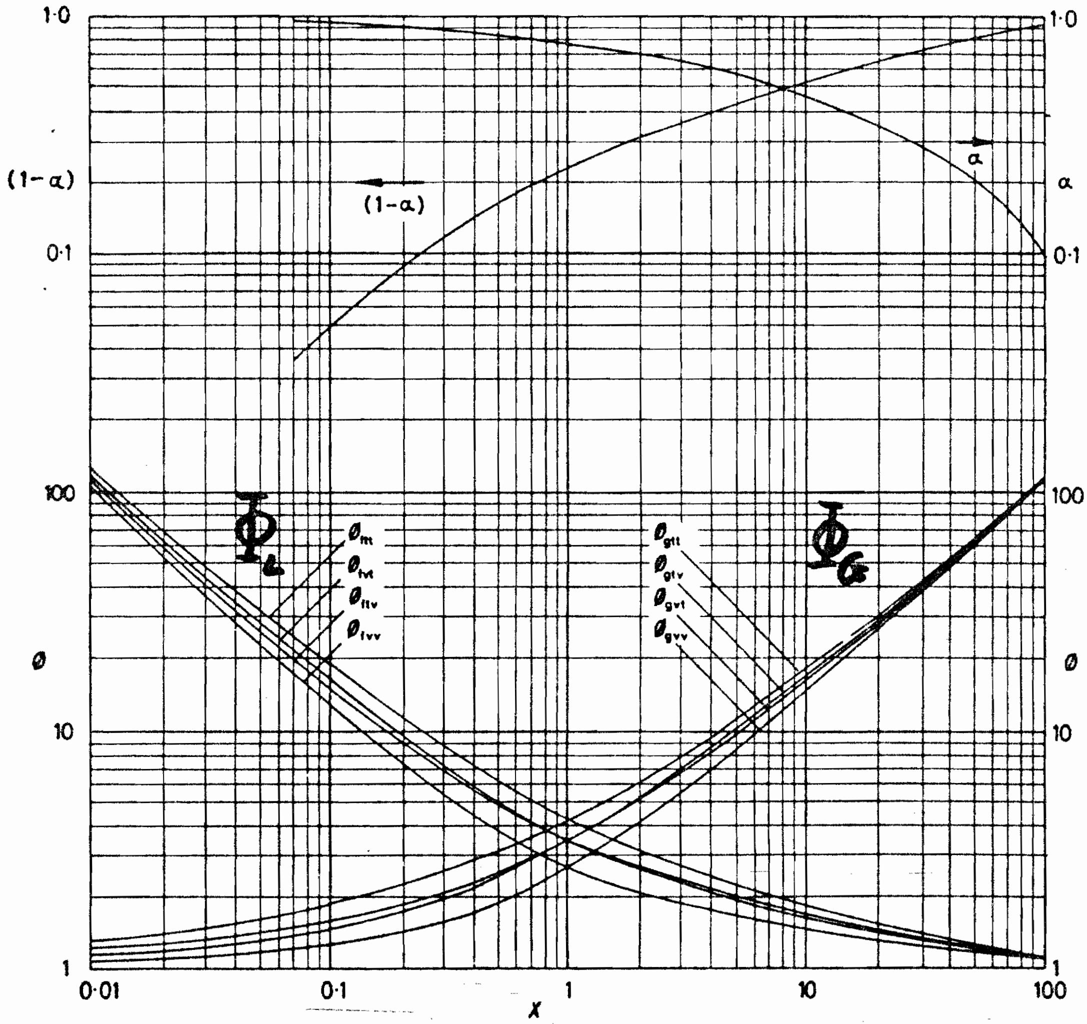


Fig 20 Lockhart–Martinelli correlation

Kurt M. Becker, Gunnar Hernborg and Manfied Bode (1962)

They made many measurement on pressure gradients for flow of boiling water in a vertical round duct. A correlation to the measurments give equations for friction factor and the multiplier Φ_{LO}^2.

$$f = 0.198 \cdot R_e^{-0.202} \qquad A.9$$

where f is friction factor, if the liquid is flowing alone in the pipe.

$$\Phi_{LO}^2 = 1 + 2547 \cdot \left(\frac{x}{p}\right)^{0.96} \qquad A.10$$

where: x = steam quality

p = Pressure (kg/cm^2)

$$-\left(\frac{dp}{dz}F\right)_{TP} = \frac{f \cdot G^2}{2 \cdot D \cdot \rho_f} \cdot \left(1 + 2547 \cdot \left(\frac{x}{p}\right)^{0.96}\right) \qquad A.11$$

where $\left(\frac{dp_F}{dz}\right)_{TP}$ is the frictional pressure gradient for the two-phase flow.

Casagrande et al (1962)

They got emperical equation which is

$$-\left(\frac{dP_F}{dz}\right)_{TP} = \frac{0.43}{D^{1.2}} \cdot \left(\frac{\sigma}{73}\right)^{0.4} \cdot \left(\frac{\mu_f}{0.016}\right)^{0.04} \cdot \left(\frac{G^2}{\rho_H}\right)^{0.75} \quad A.12$$

(c.g.s. units)

Where: σ: Surface tension

d_o: tube diameter

μ_ℓ: Liquid viscosity

G: Mass flux

ρ_H: Homogeneous density

$$\frac{1}{\rho_H} = \frac{x}{\rho_g} + \frac{1-x}{\rho_f} \quad A.13$$

$\left(\frac{dp_F}{dz}\right)_{TP}$ is the frictional pressure gradient in two phase flow.

This has been shown to correspond reasonably closely to the variation observed by Martinelli and Nelson (Cravarolo and Hassid, 1963).

A.E. Dukler et al. (1964), measured pressure drop in two-phase flow. With correlation to the measurements, they got following equation.

$$-\left(\frac{dp}{dz}F\right)_{TP} = \frac{f_f \cdot G^2}{2 \cdot D} \cdot \alpha(z) \cdot \frac{\left(\rho_f \cdot \frac{(1-\beta)^2}{1-\alpha} + \rho_g \cdot \frac{\beta^2}{\alpha}\right)}{\left(\rho_f \cdot (1-\beta) + \rho_g \cdot \beta\right)^2} \quad A.$$

where

$$\alpha(z) = 1 + \frac{z}{1.281 - 0.478 \cdot z + 0.444 \cdot z^2 - 0.094 \cdot z^3 + 0.00843 \cdot z^4} \quad A.15$$

$$z = -\ln \beta = -\ln \frac{Q_g}{Q} \quad A.16$$

Baroczy - correlation

Baroczy measured pressure gradient, and made correlation of the multiplier Φ_{LO}^2 as a function of vapour quality x, and Γ^2 where.

$$x = W_g / W$$

$$\Gamma^2 = \left(\frac{dp}{dz} F\right)_{GO} \Big/ \left(\frac{dp}{dz} F\right)_{LO} \qquad A.17$$

$$\Gamma^2 = \frac{\rho_f}{\rho_g} \cdot \left(\frac{\mu_g}{\mu_f}\right)^{0.2} \qquad A.18$$

Baroczy presentered his resultad as a graph, where Φ_{LO}^2 is plot as a function of x and $\frac{1}{\Gamma^2}$

In tablo 2 are compared the parameter X^2 and Γ^2, where the definition of X^2 is in eq. A.2 and the definition of Γ^2 is in eq. A.17.

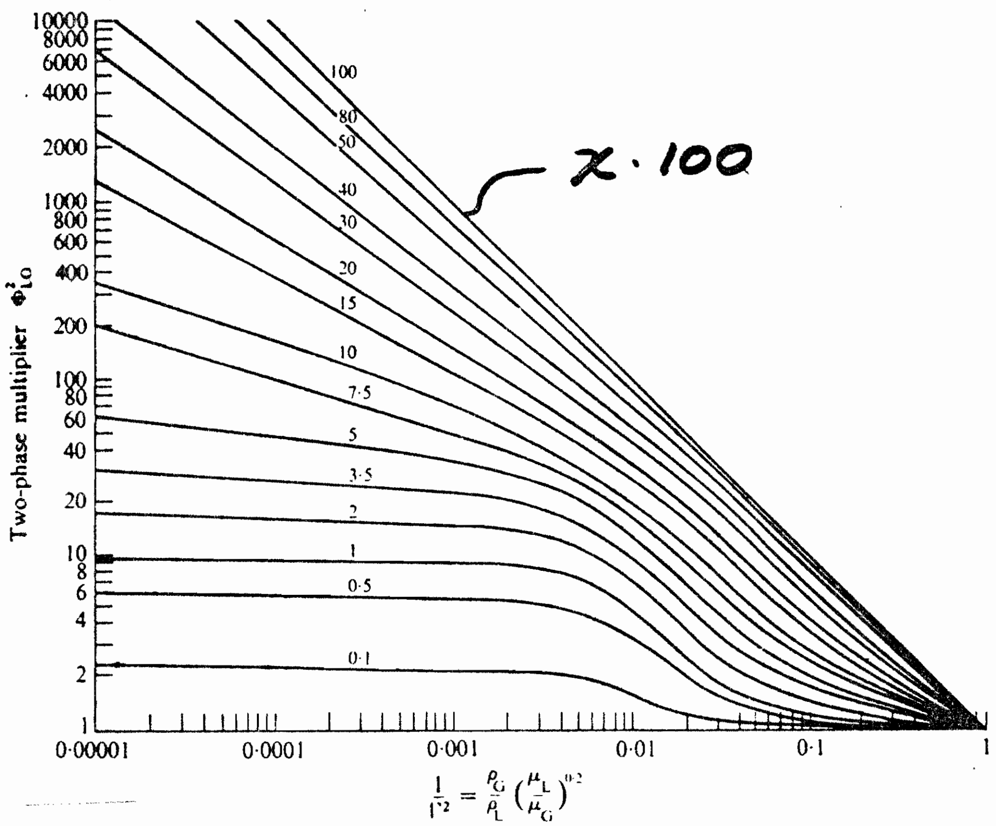


Fig 21 Baroczy (1966) frictional pressure drop correlation. The numbers on the curves give the quality in per cent. It should be noted that values are for mass velocities of 1356 kg s^{-1} m^{-2}.

D. Chisholm - correlation

D. Chisholm (1972) introduced following equation to compute friction pressure gradient for two-phase flow.

$$-\left(\frac{dp}{dZ}F\right)_{TP} = \frac{f \cdot G^2}{2 \cdot D}\left(\frac{\alpha \cdot x^2}{\rho_g} + \frac{(1-\alpha)\cdot(1-x)^2}{\rho_f}\right) \qquad A.19$$

D. Chisholm computed void fraction α and put that into eq. A.19.

We can use following equation for mass continuity of water

$$(1-x)\cdot W = (1-\alpha)\cdot A \cdot u_f \cdot \rho_f \qquad A.20$$

And for mass continuity of the steam

$$x \cdot W = \alpha \cdot A \cdot u_g \cdot \rho_g \qquad A.21$$

List of symbols is in APPENDIX I.

Put eq. A.20 and A.21 together

$$(1-x)\cdot\frac{\alpha \cdot A \cdot u_g \cdot \rho_g}{x} = (1-\alpha)\cdot A \cdot u_f \cdot \rho_f \qquad A.22$$

Elimenate the void fraction α.

$$\frac{1}{\alpha} = \frac{1-x}{x}\cdot\frac{\rho_g}{\rho_f}\cdot\frac{u_g}{u_f} + 1 \qquad A.23$$

The slip, K, is the velocity ratio of gas and liquid.

$$K = \frac{u_g}{u_f} \qquad A.24$$

Put that into eq. A.23

$$\frac{1}{\alpha} = \frac{1-x}{x} \cdot \frac{\rho_g}{\rho_f} \cdot K + 1 \qquad A.25$$

Equation A.3 describes frictional pressure gradient if the liquid is flowing alone in the pipe.
Divide eq. A.3 to eq. A.19 and get.

$$\frac{-\left(\frac{dp}{dz}F\right)_{TP}}{-\left(\frac{dp}{dz}F\right)_{L}} = \frac{x^2 \cdot \rho_f}{\alpha \cdot (1-x)^2 \cdot \rho_g} + \frac{1}{1-\alpha} \qquad A.26$$

We can use the definition of X^2 in table 2 and suppouse turbulence flow, $n = 0$.

$$X^2 = \frac{\rho_g}{\rho_f} \cdot \left(\frac{1-x}{x}\right)^2 \qquad A.27$$

Put that into eq. A.26

$$\frac{\left(\frac{dp}{dz}F\right)_{TP}}{-\left(\frac{dp}{dz}F\right)_{L}} = \frac{1}{\alpha}\cdot\frac{1}{X^2} + \frac{1}{1-\alpha} \qquad A.28$$

In eq. A.23 we computed void fraction α, put that into eq. A.28 and get.

$$\frac{-\left(\frac{dp}{dz}F\right)_{TP}}{-\left(\frac{dp}{dz}F\right)_{L}} = \frac{1}{X^2} + \frac{K\cdot\sqrt{\frac{\rho_g}{\rho_f}} + \frac{1}{K}\sqrt{\frac{\rho_f}{\rho_g}}}{X} + 1 \qquad A.2$$

or.

$$\Phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \qquad A.30$$

Where

$$C = K\sqrt{\frac{\rho_g}{\rho_f}} + \frac{1}{K}\cdot\sqrt{\frac{\rho_f}{\rho_g}} \qquad A.31$$

In same way, we can get equati[on] to compute Φ_G^2,

$$\Phi_G^2 = 1 + C\cdot X + X^2 \qquad A.32$$

In this way Chisholm got a simple equations to compute Φ_L and Φ_G as a function of X. If we us $C = 21$ when we have equation which approximate Lockhart-Martinelli graphic very well, as we can see in fig 22. In fig 20 are plot of equations A.30 and A.32 when C change as following.

Liquid	Gas	index	C	Φ_L	Φ_G
turbulent	turbulent	tt	20	Φ_{ftt}	Φ_{gtt}
viscous	turbulent	vt	12	Φ_{fvt}	Φ_{gvt}
turbulent	viscous	tv	10	Φ_{ftv}	Φ_{gtv}
viscous	viscous	vv	5	Φ_{fvv}	Φ_{gvv}

Table 3

The parameter C, in eq A.30 and A.32

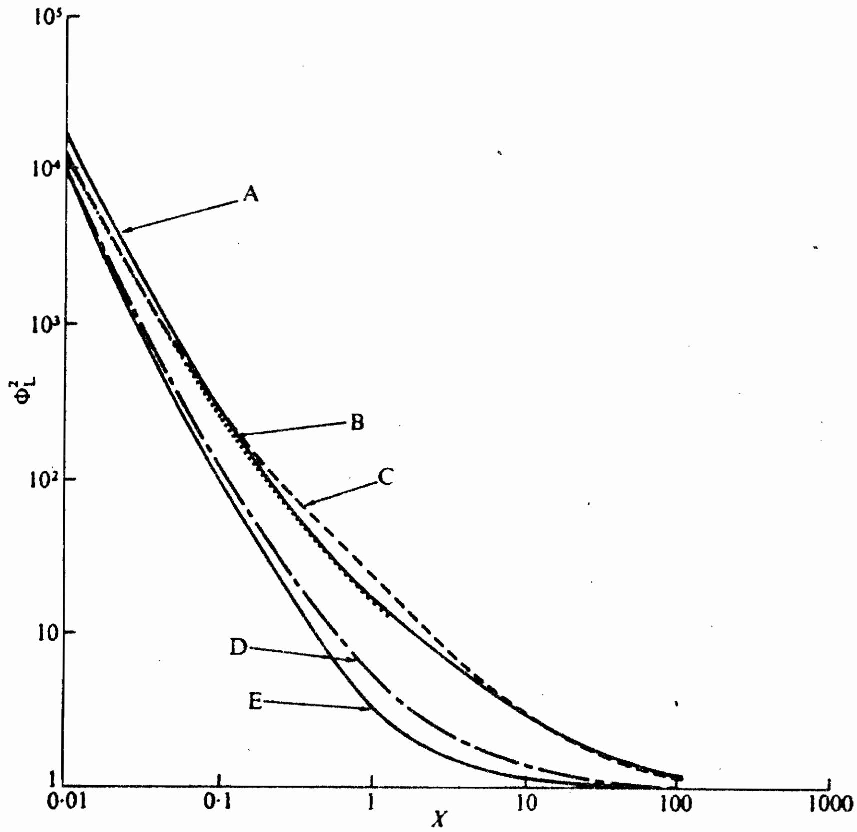


Fig 22. Comparison of empirical and theoretical pressure - gradient relationships: A, Lockhart and Martinelli (1949); B, homogeneous eqn (4.31); C, Chisholm and Sutherland (1969-70) with $C = 21$; D, separate cylinders, eqn (4.39) with $n = 0{\cdot}02$; E, critical pressure, eqn (4.42) with $n = 0{\cdot}2$.

J.R.S. Thom introduced two empirical equations to determine the parameter C in eq. A.30 and A.32.

First (1969) he got

$$C = 1.1 \cdot \left(\sqrt{\frac{\rho_f}{\rho_g}} + \sqrt{\frac{\rho_g}{\rho_f}} \right) - 0.2 \qquad A.33$$

And later (1970) he introduced following equation, which I have used in "Chisholm model".

$$C = 1 + \frac{x/\rho_g}{x/\rho_g + (1-x)/\rho_f} - \alpha \qquad A.34$$

In table 4 is a summary of equations to determine the multiplier $\bar{\Phi}^2$.

Table 4

Multiplier $\bar{\Phi}^2$ or Frictional pressure gradient	Equation	Litteratur or Editor.
$\bar{\Phi}_{Lo}^2 = \left(1 + x \cdot \left(\frac{v_{fg}}{v_f}\right)\right) \cdot \left(1 + x \left(\frac{\mu_{fg}}{\mu_g}\right)\right)^{-1}$	4.30	Homogen model
$\bar{\Phi}_{Lo}^2$, $\bar{\Phi}_L^2$ and $\bar{\Phi}_G^2$	Graphical Figs 6, 7, 20	Martinelli - Nelson
$\bar{\Phi}_{Lo}^2$	Graphical Fig. 21	Baroczy
$\bar{\Phi}_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$ $C = 5 - 20$ $\bar{\Phi}_G^2 = 1 + C \cdot X + X^2$	A.30 A.32	D. Chisholm
$-\left(\frac{\partial p}{\partial z} F\right) = \frac{0.43}{D^{1.2}} \left(\frac{\sigma}{73}\right)^{0.4} \left(\frac{\mu_f}{0.016}\right)^{0.04} \left(\frac{G^2}{\rho_H}\right)^{0.75}$ c.g.s units	A.12	Casagrande
$-\left(\frac{\partial p}{\partial z} F\right) = \frac{f_f \cdot G^2}{2 \cdot D} \cdot \alpha(z) \frac{\left(\rho_f \frac{(1-\beta)^2}{1-\alpha} + \rho_g \cdot \frac{\beta^2}{\alpha}\right)}{\left(\rho_f (1-\beta) + \rho_g \cdot \beta\right)^2}$	A.14	A.E. Dukler
$\bar{\Phi}_{Lo}^2 = 1 + 2547 \cdot \left(\frac{x}{P}\right)^{0.96}$	A.11	K.M. Becker
$\bar{\Phi}_{Lo}^2 = \frac{(1-x)^{1.75}}{(1-\alpha)^2}$		Levy
$\bar{\Phi}_{Lo}^2 = \left(1 - \alpha \left(1 - \frac{\rho_g}{\rho_f}\right)\right)^{3/4} \cdot \left(1 - x \cdot \left(1 - \frac{\rho_f}{\rho_g}\right)\right)^{7/4}$		Bankoff
$\bar{\Phi}_L^2 = \frac{A}{(1-\alpha)^n}$; $A = 1$ og $n = 1,2$ if $\alpha \leq 0.5$; $A = 0.48$ og $n = 1.9 + 1.51 \cdot 10^{-3} \cdot P$ of $\alpha > 0.5$		Armand og Treshchev

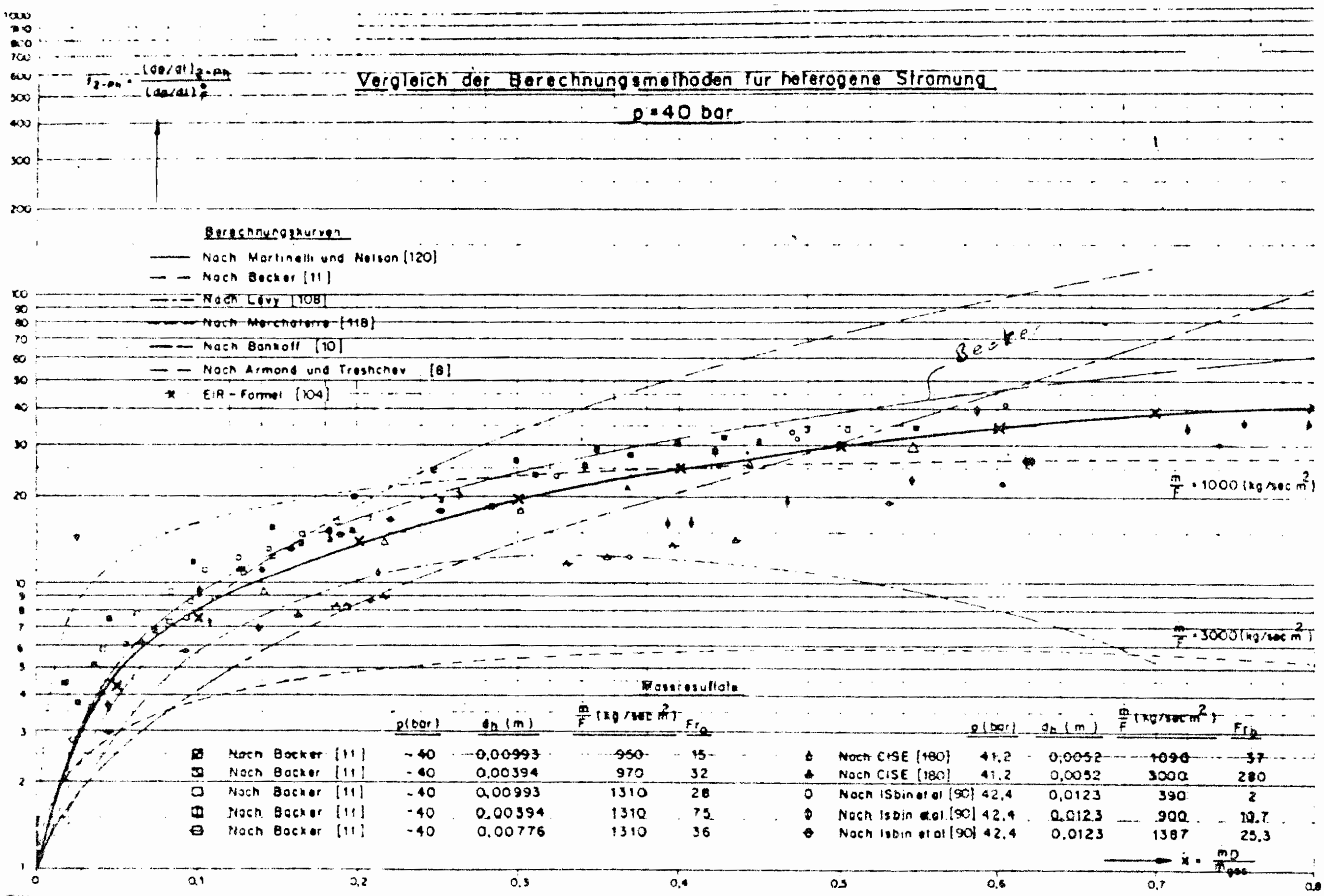


Fig 23 Vergleich der Berechnungsmethoden für heterogene Strömung

In fig 23 are compared measurements and som emperical equations to compute frictional pressure gradient, or the multiplier Φ_{LO}^2

APPENDIX III

Some equations to compute slip ratio and void fraction.

Slip ratio K is velocity ratio of gas and liquid.

$$K = \frac{u_g}{u_f} \qquad \text{A.35}$$

Where

u_g = velocity of the gas (steam).

u_f = velocity of the liquid (water).

Void fraction is the ratio between the areal of the gas in a section and the total area.

$$\alpha = \frac{A_g}{A} \qquad \text{A.36}$$

There is a strong relation between slip ratio and void fraction. We can get this relation by following equations.

$$G = \frac{W}{A} = u \cdot \rho \qquad \text{A.37}$$

$$\frac{W_g}{A_g} = \frac{\chi \cdot W}{\alpha \cdot A} = \frac{\chi}{\alpha} \cdot G = u_g \cdot \rho_g \qquad A.38$$

$$\frac{W_f}{A_f} = \frac{(1-\chi) \cdot W}{(1-\alpha) \cdot A} = \frac{1-\chi}{1-\alpha} \cdot G = u_f \cdot \rho_f \qquad A.39$$

From eq. A.38 and A.39 we get.

$$u_g = \frac{\chi \cdot G}{\alpha \cdot \rho_g} \quad A.40 \quad \text{and} \quad u_f = \frac{(1-\chi) \cdot G}{(1-\alpha) \cdot \rho_f} = \qquad A.41$$

Put eq. A.40 and A.41 into eq. A.35

$$K = \frac{u_g}{u_f} = \frac{\chi}{1-\chi} \cdot \frac{1-\alpha}{\alpha} \cdot \frac{\rho_f}{\rho_g} \qquad A.42$$

or

$$\alpha = \frac{\rho_f \cdot \chi}{(1-\chi) \cdot \rho_g \cdot K + \rho_f \cdot \chi} \qquad A.43$$

Slip ratio correlation.

Ryley (1952), Zivi (1964), Moody (1965).

$$K = \left(\frac{\rho_f}{\rho_g}\right)^{1/3} \quad \text{Moody-model} \qquad A.44$$

Prenoli, Francesco and Prina (1971)

which correlation to measurements they got

$$K = 1+a \sqrt{\frac{Y}{1+b \cdot Y} - b \cdot Y} = 1+a\sqrt{\frac{Y}{1+b \cdot Y} - b \cdot Y} \qquad A.45$$

where

$$Y = \frac{x}{1-x} \cdot \frac{\rho_f}{\rho_g} \qquad A.46$$

$$a = 1.578 \cdot Re_{L_o}^{-0.19} \, (\rho_f/\rho_g)^{0.22} \qquad A.47$$

$$b = 0.0273 \cdot We \cdot Re_{L_O}^{-0.51} \cdot (\rho_g/\rho_f)^{0.08} \qquad \text{A.48}$$

$$Re_{L_O} = \frac{G \cdot D}{\mu_L} \qquad \text{A.49}$$

$$We = \frac{G^2 \cdot D}{\sigma \cdot \rho_L} \qquad \text{A.50}$$

<u>S. Levy (1960)</u> used following equation.

$$K = \frac{u_g}{u_f} = \sqrt{\frac{\rho_f}{\rho_g}} \cdot \sqrt{2 \cdot \alpha} = \left(\frac{\rho_f}{\rho_g}\right)^{1/2} \cdot (2 \cdot \alpha)^{\frac{1}{2}} \qquad \text{A.51}$$

<u>Void fraction - correlation</u>

<u>Martinell, and Nelson (1948)</u> ~~interdict~~ introduced a graph, which show void fraction α, as a function of mass vapour quality χ and the pressure, see fig 24.

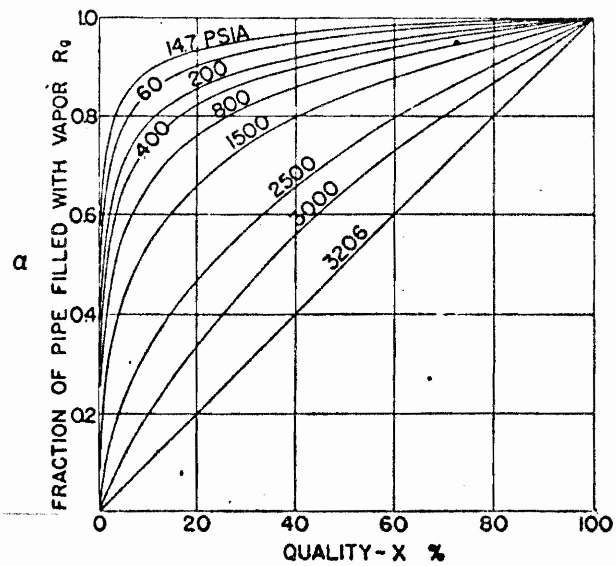


Fig 24 PER CENT OF PIPE VOLUME FILLED WITH LIQUID AS A FUNCTION OF QUALITY AND ABSOLUTE PRESSURE

D. Chisholm and L.A. Sutherland

Introduced following equation to determine void fraction α.

$$\alpha = 1 - \frac{1}{\left(1 + \frac{C}{X} + \frac{1}{X^2}\right)^{1/r}} \qquad \text{A.52}$$

Where

$$r = 1.8 - \frac{2.5 \cdot 10^{-5}}{\Gamma^2 \; (X + 0.06)} \qquad \text{A.53}$$

$$C = \frac{1}{\Gamma} + 0.5 \qquad \text{A.54}$$

X and Γ are defined in table 2, and in eq. A.2 and A.17

N. Zuber and Findlag (1964)

got following equation, for the relation between α and β.

$$\frac{\beta}{\alpha} = 1{,}13 + 0.435 \; (F_r^{1})^{-0.5} \qquad \text{A.55}$$

where

$$F_r^{1} = \frac{V^2}{g \cdot d} \cdot \frac{\rho_f}{\rho_f - \rho_g} \qquad \text{A.56}$$

$$\beta = \frac{Q_g}{Q} = \frac{1}{1 + \left(\frac{1-\chi}{\chi}\right) \cdot \frac{\rho_g}{\rho_f}} \qquad \text{A.57}$$

Baker used following emperical equation of the relation between the quality χ and void fraction α.

$$\chi = \frac{\alpha^2 \; (Y^{1/2} - 1) + \alpha}{Y - \alpha \cdot (Y - Y^{1/3})} \qquad \text{A.58}$$

Where

$$Y = 0.021 \; \left(\frac{\rho_f}{\rho_g}\right) \cdot G^{0.686} \qquad \text{A.59}$$

This equation can be used when $7{,}5 < Y < 300$ and $G < 950$ kg/m²s

In fig 25 are compared measurments and some emperical equations of void fraction α, when the pressure is 40 bar.

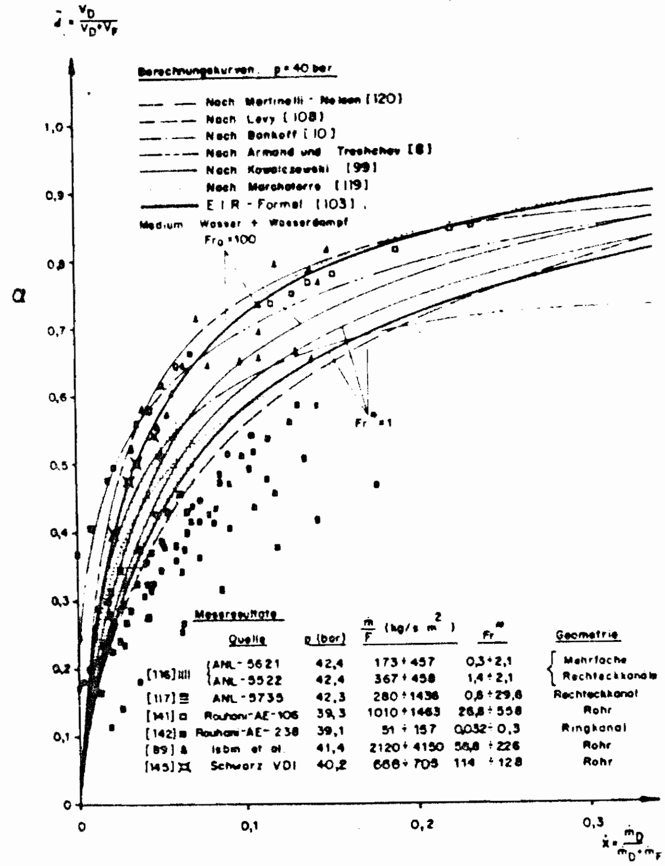


Fig 25 Dampfvolumenanteil bei 40 bar

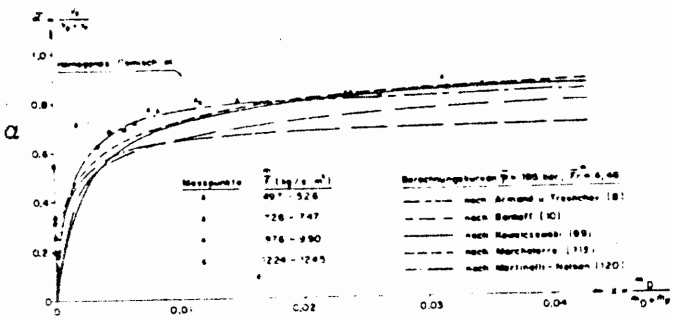


Fig 26 Mittlerer Dampfvolumenanteil Horizontale, unbeheizte Teststrecke, d = 18 mm Ø.

In fig 26 are compared measurments and void fraction α, when the pressure is 1.85 bar

Table 5 shows some equations to determain void fraction α.

Table 5

Void fraction α	Equation	Literature
$\alpha = f(p, x)$	Graphical Fig 24	Martinelli-Nelson
$\alpha = \dfrac{0.71 + 0.00145 \cdot p}{1 + \frac{1-x}{x} \cdot \frac{\rho_g}{\rho_f}}$		Bankoff
$\alpha = \dfrac{0,833 + 0.05 \cdot \log p}{1 + \frac{1-x}{x} \cdot \frac{\rho_s}{\rho_f}}$	5.7	Armand og Treahcher
$x = \dfrac{\alpha(1-2\cdot\alpha) + \alpha\left\{(1-2\alpha)^2 + \alpha\frac{1}{2}\cdot\frac{\rho_f}{\rho_s}(1-\alpha)^2 + \alpha\cdot(1-2\alpha)\right\}^{1/2}}{2\cdot\frac{\rho_f}{\rho_s}\cdot(1-\alpha)^2 + \alpha(1-2\alpha)}$		Levy
$\alpha = \beta - 0.71\cdot\beta(1-\beta)^{0.5}\left(1-\frac{p}{p_{crit}}\right)\left(\frac{V^2}{g\cdot d}\right)^{-0.045}$	5.8	Kowalczewski
$\alpha = 1 - \left(1 + \frac{C}{X} + \frac{1}{X^2}\right)^{-\frac{1}{r}}$	A.52	D. Chisholm og L.A. Sutherland
$\alpha = \left(1 + \left(\frac{1-x}{x}\right)\frac{\rho_s}{\rho_f}\right)^{-1}\left(1{,}13 + \frac{0.435\cdot\sqrt{gd}}{V}\left(\frac{\rho_f-\rho_s}{\rho_f}\right)^{1/2}\right)^{-1}$	A.55	N. Zuber et al.
$\alpha = \dfrac{-1 - x\cdot Y + x\cdot Y^{\frac{1}{3}} + \left(\left(1+x\cdot Y - x\cdot Y^{\frac{1}{3}}\right)^2 + 4\left(Y^{\frac{1}{3}}-1\right)x\,Y\right)^{\frac{1}{2}}}{2\cdot\left(Y^{\frac{1}{3}}-1\right)}$	A.58	Baker

APPENDIX IV

Pressure change through enlargement and effect of deposition in wells

In most wells, there are some enlargements, as we can see in Fig. 9. Therefore we need some equations to compute pressure change of two-phase flow through enlargement.

Following emperical equations are to comput this pressure change

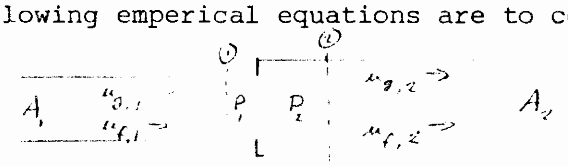


Fig. 27. Enlargement in a pipe.

D. Butterworth introduced following equation to compute pressure change through enlargement.

$$p_1 - p_2 = -G_1^2 \frac{A_1}{A_2} \left(v_{a,1} - \frac{A_1}{A_2} \cdot v_{a,2} \right) \qquad \text{A.60}$$

where

$$v_a = \frac{x^2}{\alpha \rho} + \frac{(1-x)^2}{(1-\alpha) \cdot \rho_f} \qquad \text{A.61}$$

F. Romie used following equation

$$p_2 - p_1 = G_1^2 \cdot s \cdot v_f \left(\left(\frac{(1-x)^2}{(1-\alpha_1)} + \left(\frac{v_g}{v_f} \right) \frac{x^2}{\alpha_1} \right) - s \left(\frac{(1-x)^2}{(1-\alpha_2)} + \left(\frac{v_g}{v_f} \right) \frac{x^2}{\alpha_2} \right) \right) \qquad \text{A.62}$$

where $s = A_1/A_2$ A-63

put $\alpha_1 = \alpha_2 = \alpha$

and we get

$$p_2-p_1 = G_1^2\cdot s(1-s)\cdot v_f\left(\frac{(1-x)^2}{(1-\alpha)}+\left(\frac{v_g}{v_f}\right)\cdot\frac{x^2}{\alpha}\right) \qquad \text{A.64}$$

For homogeneous flow this equation reduces to

$$p_2-p_1 = G_1^2\cdot s(1-s)v_f\left(1+\left(\frac{v_{fg}}{v_f}\right)\cdot x\right) \qquad \text{A.65}$$

$$= G_1^2\cdot s(1-s)\cdot v_f\left(1+x\cdot\left(\frac{v_g}{v_f}-1\right)\right) \qquad \text{A.66}$$

<u>Chisholm</u> method to compute pressure change through enlargement.

$$p_2-p_1 = G_1^2\cdot s(1-s)v_f(1-x)^2\left(1+\frac{C}{X}+\frac{1}{X^2}\right) \qquad \text{A.67}$$

C is defined in eq. A.31, and we can use I.Thom equation A.34 for determine C. <u>The effect of deposition in well, to the pressure gradient.</u> If it is deposition in well, then it will reduce the diameter of the casing. A change of diameter will have very great effect on the pressure drop. We can use eq. 5.12 to determine frictions pressure drop.

$$-\left(\frac{dp}{dz}F\right)_{TP} = \frac{f\cdot G^2}{2\cdot D\cdot\rho_f}\cdot\Phi_{LO}^2 \qquad 5.12$$

or

$$\left(\frac{dp}{dz}F\right)_{TP} = \frac{f}{2\cdot D\cdot\rho_f}\cdot\left(\frac{W}{A}\right)^2\Phi_{LO} = \frac{f\cdot W^2\cdot 8}{\rho_f\cdot\pi^2}\cdot\frac{1}{D^5}\cdot\Phi_{LO}^2 \qquad \text{A.68}$$

Therefore frictional pressure gradient is propotional to D^{-5} and f. But the effect of deposition is that it will reduce the diameter and increase roughness and the friction factor f. The effect of deposition on frictions pressure gradient is therefore very powerful.

APPENDIX V

The program and typical output.

This is the main program to read data and call subroutine U07SN5

```
      COMMON X,V,B,HSTAR,VSTAR,V[illegible]G,HW,HG,G
      DIMENSION ZZ(10),DD(10),FLAM(10)
      CALL ASSIGN(1,'PIPE.DAT')
      G=980.67
10    READ(1,701,END=20)ITYPE,DZ1,TYPEZ,ZA,ZT
      READ(1,702) N,(ZZ(I),DD(I),FLAM(I),I=1,N)
      READ(1,703)PSTAR,VSTAR
      READ(1,704)PAA,FLOW
      READ(1,705)DZ3,VAT1,VAT2
          READ(1,706) CTURB
15    CONTINUE
          TURBUL=CTURB*FLOW**2
          PA=PAA-TURBUL
          TYPE 803,PAA,TURBUL,PA
 803  FORMAT('PAA=',-6PF8.3,'TURB=',-6PF8.3,'PA=',-6PF8.3)
 706       FORMAT(F8.0)
      CALL U07SN5(ZA,ZT,N,ZZ,DD,FLAM,PSTAR,PA,FLOW,
      1        PT,ITYPE,TYPEZ,DZ1,DZ3,VAT1,VAT2)
      TYPE 801,PT
      READ(1,910,END=20) J,FLAM(J),JI,FLAM(JI),FLOW
      GO TO 15
20    STOP
701   FORMAT(I2,-2P4F8.0)
702   FORMAT(I2,(-2PF8.0,0P2F8.0))
703   FORMAT(-6PF8.0,0PF8.0)
704   FORMAT(-6PF8.0,-3PF8.0)
801   FORMAT(' ',-6PF10.2)
705   FORMAT(-2PF8.0,0P2F8.0)
910   FORMAT(I2,F8.0,I2,F8.0,-3PF8.0)
      END
```

This subroutine computes values from the steam tables.

```
     FUNCTION U07ST1(NI,VALUE,NO)
     DIMENSION ST(55,6),W(10),C(4),ST1(55),ST2(55),ST3(55),ST4(55),
    1          ST5(55),ST6(55)
     EQUIVALENCE(ST1(1),ST(1,1)),(ST2(1),ST(1,2)),(ST3(1),ST(1,3)),
    1           (ST5(1),ST(1,5)),(ST6(1),ST(1,6))
    2           ,(ST4(1),ST(1,4))
     DATA N/55/
     DATA ST1/
    17.000000E+01,7.500000E+01,8.000000E+01,8.500000E+01,9.000000E+01,
    19.500000E+01,1.000000E+02,1.050000E+02,1.100000E+02,1.150000E+02,
    11.200000E+02,1.250000E+02,1.300000E+02,1.350000E+02,1.400000E+02,
    11.450000E+02,1.500000E+02,1.550000E+02,1.600000E+02,1.650000E+02,
```

```
[illegible]0E+02,2.000000E+02,2[illegible]000E+02,2.100000E+02,2.1500[illegible]02,
12.200000E+02,2.250000E+02,2.300000E+02,2.350000E+02,2.400000E+02,
12.450000E+02,2.500000E+02,2.550000E+02,2.600000E+02,2.650000E+02,
12.700000E+02,2.750000E+02,2.800000E+02,2.850000E+02,2.900000E+02,
12.950000E+02,3.000000E+02,3.050000E+02,3.100000E+02,3.150000E+02,
13.200000E+02,3.300000E+02,3.400000E+02,3.500000E+02,3.600000E+02/
 DATA ST2/
13.119000E+05,3.858000E+05,4.739000E+05,5.783000E+05,7.014000E+05,
18.455000E+05,1.013500E+06,1.208200E+06,1.432700E+06,1.690600E+06,
11.985300E+06,2.321000E+06,2.701000E+06,3.130000E+06,3.613000E+06,
14.154000E+06,4.758000E+06,5.431000E+06,6.178000E+06,7.005000E+06,
17.917000E+06,8.920000E+06,1.002100E+07,1.122700E+07,1.254400E+07,
11.397800E+07,1.553800E+07,1.723000E+07,1.906200E+07,2.104000E+07,
12.318000E+07,2.548000E+07,2.795000E+07,3.060000E+07,3.344000E+07,
13.648000E+07,3.973000E+07,4.319000E+07,4.688000E+07,5.081000E+07,
15.499000E+07,5.942000E+07,6.412000E+07,6.909000E+07,7.436000E+07,
17.993000E+07,8.581000E+07,9.202000E+07,9.856000E+07,1.054700E+08,
11.127400E+08,1.284500E+08,1.458600E+08,1.651300E+08,1.865100E+08/
 DATA ST3/
12.929800E+09,3.139300E+09,3.349100E+09,3.559000E+09,3.769200E+09,
13.979600E+09,4.190400E+09,4.401500E+09,4.613000E+09,4.824800E+09,
15.037100E+09,5.249900E+09,5.463100E+09,5.676900E+09,5.891300E+09,
16.106300E+09,6.322000E+09,6.538400E+09,6.755500E+09,6.973400E+09,
17.192100E+09,7.411700E+09,7.632200E+09,7.853700E+09,8.076200E+09,
18.299800E+09,8.524500E+09,8.750400E+09,8.977600E+09,9.206200E+09,
19.436200E+09,9.667800E+09,9.901200E+09,1.013620E+10,1.037320E+10,
11.061230E+10,1.085360E+10,1.109730E+10,1.134370E+10,1.159280E+10,
11.184510E+10,1.210070E+10,1.235990E+10,1.262310E+10,1.289070E+10,
11.316300E+10,1.344000E+10,1.372400E+10,1.401300E+10,1.431000E+10,
11.461500E+10,1.525300E+10,1.594200E+10,1.670600E+10,1.760500E+10/
 DATA ST4/
12.626800E+10,2.635300E+10,2.643700E+10,2.651900E+10,2.660100E+10,
12.668100E+10,2.676100E+10,2.683800E+10,2.691500E+10,2.699000E+10,
12.706300E+10,2.713500E+10,2.720500E+10,2.727300E+10,2.733900E+10,
12.740300E+10,2.746500E+10,2.752400E+10,2.758100E+10,2.763500E+10,
12.768700E+10,2.773600E+10,2.778200E+10,2.782400E+10,2.786400E+10,
12.790000E+10,2.793200E+10,2.796000E+10,2.798500E+10,2.800500E+10,
12.802100E+10,2.803300E+10,2.804000E+10,2.804200E+10,2.803800E+10,
12.803000E+10,2.801500E+10,2.799500E+10,2.796900E+10,2.793600E+10,
12.789700E+10,2.785000E+10,2.779600E+10,2.773300E+10,2.766200E+10,
12.758100E+10,2.749000E+10,2.738700E+10,2.727300E+10,2.714500E+10,
12.700100E+10,2.665900E+10,2.622000E+10,2.563900E+10,2.481000E+10/
 DATA ST5/
11.022800E-00,1.025900E-00,1.029100E-00,1.032500E-00,1.036000E-00,
```

```
[illegible]
11.060300E-00,1.064900E-00,1.0[illegible]700E-00,1.074600E-00,1.079700[illegible]00,
11.085000E-00,1.090500E-00,1.096100E-00,1.102000E-00,1.108000E-00,
11.114300E-00,1.120700E-00,1.127400E-00,1.134300E-00,1.141400E-00,
11.148800E-00,1.156500E-00,1.164400E-00,1.172600E-00,1.181200E-00,
11.190000E-00,1.199200E-00,1.208800E-00,1.218700E-00,1.229100E-00,
11.239900E-00,1.251200E-00,1.263100E-00,1.275500E-00,1.288600E-00,
11.302300E-00,1.316800E-00,1.332100E-00,1.348300E-00,1.365600E-00,
11.383900E-00,1.403600E-00,1.424700E-00,1.447400E-00,1.472000E-00,
11.498800E-00,1.560700E-00,1.637900E-00,1.740300E-00,1.892500E-00/
 DATA ST6/
15.042000E+03,4.131000E+03,3.407000E+03,2.828000E+03,2.361000E+03,
11.982000E+03,1.672900E+03,1.419400E+03,1.210200E+03,1.036600E+03,
18.919000E+02,7.703000E+02,6.685000E+02,5.822000E+02,5.089000E+02,
14.463000E+02,3.928000E+02,3.468000E+02,3.071000E+02,2.727000E+02,
12.428000E+02,2.168000E+02,1.940500E+02,1.740900E+02,1.565400E+02,
11.410500E+02,1.273600E+02,1.152100E+02,1.044100E+02,9.479000E+01,
18.619000E+01,7.849000E+01,7.158000E+01,6.537000E+01,5.976000E+01,
15.471000E+01,5.013000E+01,4.598000E+01,4.221000E+01,3.877000E+01,
13.564000E+01,3.279000E+01,3.017000E+01,2.777000E+01,2.557000E+01,
12.354000E+01,2.167000E+01,1.994800E+01,1.835000E+01,1.686700E+01,
11.548800E+01,1.299600E+01,1.079700E+01,8.813000E-00,6.945000E-00/
 IF(NI.EQ.NIOLD.AND.VALUE.EQ.VOLD) GO TO 200
 IF(VALUE.LT.ST(1,NI).OR.VALUE.GT.ST(55,NI)) GO TO 301
 IF(NI.EQ.4.OR.NI.EQ.6) GO TO 302
 NIOLD=NI
 VOLD=VALUE
 IEXACT=0
 IUP=N
 ILO=1
 MID=1
10 IDIF=IUP-ILO
 IF(IDIF.GT.2) GO TO 20
 IF(IDIF.EQ.0)GO TO 110
 MID=ILO+1
 GO TO 21
20 MID=(IUP+ILO+1)/2
21 IF(VALUE.EQ.ST(MID,NI)) GO TO 100
 IF(VALUE.GT.ST(MID,NI)) GO TO 40
 IF(VALUE.EQ.ST(MID-1,NI)) GO TO 30
 IF(VALUE.GT.ST(MID-1,NI)) GO TO 110
 IUP=MID
 GO TO 10
30 MID=MID-1
 GO TO 100
```

```
 40 ILO=MID
    GO TO 10
100 U07ST1=ST(MID,NO)
    IEXACT=1
    RETURN
110 K=MID-2
    IF(K.LT.1) K=1
    IF(K+3.GT.N) K=N-3
    W( 1)=VALUE-ST(K  ,NI)
    W( 2)=VALUE-ST(K+1,NI)
    W( 3)=VALUE-ST(K+2,NI)
    W(4 )=VALUE-ST(K+3,NI)
    W( 5)=ST(K  ,NI)-ST(K+1,NI)
    W( 6)=ST(K  ,NI)-ST(K+2,NI)
    W( 7)=ST(K  ,NI)-ST(K+3,NI)
    W( 8)=ST(K+1,NI)-ST(K+2,NI)
    W( 9)=ST(K+1,NI)-ST(K+3,NI)
    W(10)=ST(K+2,NI)-ST(K+3,NI)
    C(1)= W(2)*W(3)*W(4)/W(5)/W(6)/W(7)
    C(2)=-W(1)*W(3)*W(4)/W(5)/W(8)/W(9)
    C(3)= W(1)*W(2)*W(4)/W(6)/W(8)/W(10)
    C(4)=-W(1)*W(2)*W(3)/W(7)/W(9)/W(10)
200 IF(IEXACT.EQ.1) GO TO 100
    U07ST1=ST(K,NO)*C(1)+ST(K+1,NO)*C(2)+ST(K+2,NO)*C(3)
   1                   +ST(K+3,NO)*C(4)
    RETURN
301 TYPE 501,VALUE,ST(1,NI),ST(55,NI)
    STOP
302 TYPE 502
    STOP
500 FORMAT(F6.2,F8.4,F7.2,F6.1,F6.4,F8.3)
501 FORMAT(' VALUE NOT WITHIN RANGE OF STEAM TABLE'/
   1       ' VALUE=',E12.4,' ST(1)=',E12.4,' ST(N)=',E12.4)
502 FORMAT(' COLUMNS 4 AND 6 CANNOT BE INPUT')
    END
```

```
      SUBROUTINE U07SN5(ZA,ZT,IN,ZZ D,FLAM,PSTAR,PA,FLOW,
     1                  PT,ITYPE,TYPEZ,DZ2,DZ3,VAT1,VAT2)
      COMMON X,V,B,HSTAR,VSTAR,VL,VG,HL,HG,G
      REAL*8 FLKIND,BUBB,SLUG,ANNU
      DATA IBLANK,ISTAR/' ','*'/
      DATA BUBB,SLUG,ANNU/' BUBBLY ',' SLUG ',' ANNULAR'/
C       VSTAR,B AND G ARE OBTAINED FROM COMMON
C       VSTAR IS THE SPECIFIC VOLUME OF THE WATER BE366 OHE FLASHING POINT
C       IT MAY BE DIFFERENT FROM THE STEAM TABLE VALUE
C       ONLY THE HYDROSTATIC TERM IS INCLUDED BELOW THE FLASHING POINT
      DIMENSION ZZ(1),DD(1),FLAM(1)
C       THESE ARRAYS GIVE THE PIPE CHARACTERISTICS AS A FUNCTION OF DISTANCE
C      ORDERED FROM THE WELL HEAD DOWN TO THE WELL BOTTOM
C       DISTANCES DOWNWARDS ARE POSITIVE
       LOGICAL LTYPE
      LTYPE=(ITYPE.NE.0)
C       INITIALIZATION
      FRIC=0.
      POT=0.
      III=IBLANK
      DZ1=DZ2
      NINT=0
      N=IN
      HSTAR=U07ST1(2,PSTAR,3)
      HL=U07ST1(2,PSTAR,3)
      HG=U07ST1(2,PSTAR,4)
      VL=U07ST1(2,PSTAR,5)
      VG=U07ST1(2,PSTAR,6)
      ZSTAR=U07VA1(ZA,PA,PSTAR,ZZ,DD,FLAM,FLOW,DZ3,VAT1,VAT2,IN,VL)
      Q=HSTAR-ZSTAR*G
      Z=ZSTAR
      P=PSTAR
       PACC=0.0025
      TEMPZ=AINT(Z/TYPEZ+1.)*TYPEZ
      DZ=-(Z-AINT((Z-1.)/DZ1)*DZ1)
      IF(Z.GT.ZZ(N)) TYPE 610,PA,DD(N)
   10 IF(N.LE.1) GO TO 20
      IF(Z.GT.ZZ (N-1)) GO TO 20
      N=N-1
      GO TO 10
   20 D=DD(N)
      FLAMDA=FLAM(N)
      FLOWA=FLOW/(D*D*0.78539816)
      FLOWA2=FLOWA*FLOWA
```

This is the main subrotine, to compute pressure in each section by iteration

```
      B=0.5*FLAMDA*FLOWA2/D
      U=Q+G*ZSTAR
      EKIN=0.5*FLOWA2*VL*VL
      PFLUX=FLOWA2*VSTAR
      VBAR1=VL
      VBAR2=VL
      VBAR3=VL
      VBAR4=VL
      VEFF=VL
      F=G/VSTAR
      X=0
      ALPHA=0.
      IF(.NOT.LTYPE) GO TO 30
      TYPE 620,FLOW,PA,HSTAR,PSTAR
      TYPE 630
      TYPE 640,D,FLAMDA,      ZZ(N)
      TYPE 670
C       START THE INTEGRATION LOOP
C          MODIFY THE PIPE CHARACTERISTICS
   30 IF(N.LE.1) GO TO 40
      IF(Z.GT.ZZ(N-1)) GO TO 40
      N=N-1
      IGO=1
      GO TO 300
   35 FLOWAO=FLOWA
      FLAMDA=FLAM(N)
      D=DD(N)
      FLOWA=FLOW/(D*D*0.78539816)
      FLOWA2=FLOWA*FLOWA
      B=0.5*FLAMDA*FLOWA2/D
      DZ=0.
      PFLUXO=PFLUX*FLOWA/FLOWAO
      GO TO 70
   40 FLOWAO=FLOWA
      IGO=2
      IF(-DZ.EQ.0.) GO TO 300
      IF(Z.GT.TEMPZ) GO TO 45
      GO TO 300
   45 PFLUXO=PFLUX
C                   DETERMINE THE Z-INCREMENT DZ
   50 DZ=-(Z-AINT((Z-1.)/DZ1)*DZ1)
      IF(N.LE.1) GO TO 65
      IF(Z+DZ.GT.ZZ(N-1)) GO TO 70
      DZ=-(Z-ZZ(N-1))
   65 IF(Z+DZ.GT.ZT) GO TO 70
```

```
      DZ=-(Z-ZT)
C           CALCULATE ONE INTEGRATIO STEP USING AN EXPLICIT SOLUTI( METHOD
   70 U=Q+(Z+DZ)*G
      DP=F*DZ+FMOM*DZ
       IF(DZ.EQ.0) DP=(X**2*VG/ALPHA+
     1   (1-X)**2*VG/(1-ALPHA))*(FLOWAO**2-FLOWA**2)
      NINT=0
      DX=X
      XLAST=X
      DDP=DP
      ISTOP=0
      FOLD=F
  210       NINT=NINT+1
            PPDP=P+DP
            IF(PPDP.GE.0.3119E6) GO TO 211
            DZ1=DZ1/2.
            IF( DZ1.LT.10.) GO TO 90
            X=XLAST
            GO TO 50
  211       HL=U07ST1(2,PPDP,3)
            HG=U07ST1(2,PPDP,4)
            VL=U07ST1(2,PPDP,5)
            VG=U07ST1(2,PPDP,6)
            IF(DP.EQ.0.0) GO TO 230
            IF(ISTOP.EQ.1.AND.ABS(DDP/DP).LE.PACC) GO TO 230
            EKIN=0.5*FLOWA2*VBAR4*VBAR4
            X=(U-EKIN-HL)/(HG-HL)
            IF(X.GT.0.) GO TO 214
            ALPHA=0.
            VBAR1=VL
            VBAR2=VL
            VBAR3=VL
            VBAR4=VL
            GO TO 216
  214       ALPHA=U07TP5(PPDP,X,VL,VG,FLOWA,D,ALPHA)
            C1=ALPHA
            C2=1.-ALPHA
            B1=X/C1
            B2=(1.-X)/C2
               VBAR1=1./(C1/VG+C2/VL)
            C1=C1*B1
            C2=C2*B2
               VBAR2=C1*VG+C2*VL
            CC1=C1*B1
            CC2=C2*B2
```

```
            VBAR3=CC1*VG+CC2*VL
          CC1=CC1*B1
          CC2=CC2*B2
             VBAR4=SQRT(CC1*VG*VG+CC2*VL*VL)
       VEFF  =U07TP4(X,ALPHA,PPDP,VL,VG)
 216      POT=G/VBAR1
          FRIC=B*VEFF
          F=POT+FRIC
          PFLUX=FLOWA2*VBAR3
          DPMOM=PFLUXO-PFLUX
          DPOLD=DP
          DP=(F+FOLD)*0.5*DZ+DPMOM
          DDPOLD=DDP
          DDP=DP-DPOLD
          IF(DDP.NE.0.0) GO TO 217
          ISTOP=1
          GO TO 220
 217           IF(NINT/2*2.NE.NINT) GO TO 210
           IF(ABS((DDPOLD-DDP)/P).GT.1E-8) GO TO 218
          COR=3*DDP
          GO TO 219
 218      COR=-DDP*DDP/(DDP-DDPOLD)
          DP=DP+COR
 219      IF(ABS(COR/DDP).LT.1.) ISTOP=1
          DDPOLD=DDP
          DDP=COR
 220      GO TO 210
 230      P=P+DP
          IF(DZ.NE.0) FMOM=DPMOM/DZ
          IF(DZ.EQ.0) FMOM=FMOM*FLOWA/FLOWA0
          Z=Z+DZ
    X=(U-EKIN-HL)/(HG-HL)
       IF(Z.GT.ZT) GO TO 30
C     INTEGRATION COMPLETED
  90 IF(DZ1.GE.100.) GO TO 100
    PCRIT=(HSTAR**1.102*FLOWA)**1.04167*3.029E-8
    TYPE 660,PCRIT
    GO TO 110
 100 IGO=3
    GO TO 300
 110 PT=P
    RETURN
 300 IF(.NOT.LTYPE) GO TO 360
    VGOVL=VG/VL
    IF(X.GT.1.) GO TO 306
```

```
      X01MX=X/(1.-X)
      A01MA=ALPHA/(1.-ALPHA)
      IF(X.GT.0.) GO TO 302
      S=1.
      GO TO 304
  302 S=X01MX/A01MA*VGOVL
  304 QX=A01MA/(A01MA+VGOVL)
      BETA=X01MX/(X01MX+1./VGOVL)
      GO TO 308
  306 S=VGOVL
       QX=1.
      BETA=1.
  308 CONTINUE
      R1=VBAR1/VBAR2
      R3=VBAR3/VBAR2
      R4=VBAR4/VBAR2
      REFF=VEFF/VBAR2
      HBAR=X*HG+(1.-X)*HL
      FR=FLOWA2/G/D*VBAR2*VBAR2
      IF(BETA.LT.0.15) GO TO 320
      IF(BETA.LT.0.55) GO TO 310
      IF(BETA.LT.(-0.0085*FR+0.9962)) GO TO 330
      GO TO 340
  310 IF(BETA.LT.(-FR*.02+1.85)) GO TO 330
      GO TO 340
  320 FLKIND=BUBB
      GO TO 350
  330 FLKIND=SLUG
      GO TO 350
  340 FLKIND=ANNU
  350 CONTINUE
      IF(Z.GT.ZA) III=ISTAR
      T=U07ST1(2,P,1)
      DPPOT=POT*DZ
      DPFRIC=FRIC*DZ
      TYPE 650,U,EKIN,HBAR,
     1          DP,DPPOT,DPFRIC,DPMOM,
     2          X,ALPHA,QX,BETA,S,FR,
     3          T,P,Z,III,FLKIND, NINT,
     4          R4,R1,REFF,R3
      IF(Z.LE.TEMPZ) TEMPZ=TEMPZ-TYPEZ
  360 GO TO (35,45,110                                 ),IGO
C        FORMAT STATEMENTS
  610 FORMAT(' WHEN THE AQUIFER PRESSURE IS ',-6PF6.3,
     1       ' BARS, FLASHING OCCURS BELOW THE WELL BOTTOM'/
```

```
     2         ' THE PIPE IS EXTENDE  WITH A DIAMETER OF',OPF6.3,' ( ')
 620 FORMAT('      MASS FLOW=',-3PF( .2,' KGM/SEC, AQUIFER PRESSUR( ',
     1        -6PF7.2,' BARS'/
     2        '      ENTHALPY=',-7PF9.3, ' J/GM, FLASHING PRESSURE=',
     3        -6PF7.2,' BARS')
 630 FORMAT(' ')
 640 FORMAT('      D=',OPF6.3,' CM AND FLAMDA=',OPF6.3,
     1        ' FROM A DEPTH OF ',-2PF8.3,' METERS')
 650 FORMAT(' ',-7P3F8.2,-6P4F6.2,OP4F6.3,OPF7.3,OPF7.1,OPF6.1,
     1        -6PF7.2,-2PF7.1,A1,A8, I3,/,' ',8X,OPF8.2,14X,OP3F6.2)
 660 FORMAT(' JAMES'' CRITICAL PRESSURE=',-6PF7.2,' BARS')
 670 FORMAT(' ',T5,'U',T13,'EKIN',T20,'HBAR',T29,'DP',T34,'DPOT',
     1        T40,'DFRIC',T46,'DMOM',T52,'X',T57,'ALFA',T63,'Q',T69,
     2        'BETA',T77,'S',T83,'FR',T87,'TEMP',T94,'P(BAR)',
     3        T101,'D(M)',T108,'TYPE',T116,'N')
      END
       FUNCTION U07TP5(P,X,VL,VG,FLOWA,D,ALPHA)
       G=FLOWA
       HROD=1.0/VG
       HROF=1.0/VL
       FR=(FLOWA/HROF)**2/D/981
      IF(X.GT.0.02) GO TO 2001
      ALPHA=(0.833+0C.05*ALOG10(P*1.0E-6))/(1+(1-X)/X*HROD/HROF)
      GO TO 2002
2001      BETA=X*VG/(X*VG+(1-X)*VL)
      ALPHA=BETA-0.71*BETA*SQRT(1-BETA)*EXP(-0.045*ALOG(FR))
     1         *(1-P/2.212E8)
2002  CONTINUE
      U07TP5=ALPHA
      RETURN
      END
       FUNCTION U07TP4(X,ALPHA,P,VL,VG)
       F2PH=1+2547.*(X/P*1.0E6)**0.96
       U07TP4=F2PH*VL
       RETURN
       END
      FUNCTION U07VA1(ZA,PA,PSTAR,ZZ,DD,FLAM,FLOW,DZ3,VAT1,VAT2,IN,VL)
      DIMENSION ZZ(1),DD(1),FLAM(1)
      HROL=1.0/VL
      N=IN
      P=PA
      Z=ZA
      DZ=Z-AINT(Z/DZ3)*DZ3
1010  IF(N.LE.1) GO TO 1020
      IF(Z.GT.ZZ(N-1)) GO TO 1020
```

This subrotine computes void fraction α, by using ATK-model

Subrotin to compute the multiplier Φ_{lo}^2, by using Becker model.

Subrotin to compute frictional pressure gradient in the water, and to compute flash point level.

```
      GO TO 1010
1020  D=DD(N)
      FLAMDA=FLAM(N)
      VVATN=FLOW/HROL/(D*D*0.78539816)
      DPPOT=HROL*981*VAT1*DZ
      DPFRIC=VAT2*FLAMDA*HROL*VVATN**2/2/D*DZ
      DPVATN=DPPOT+DPFRIC
1030  P=P-DPVATN
      Z=Z-DZ
      IF(P.LE.PSTAR) GO TO 1050
      TYPE 1040,Z,P,VVATN,FLAMDA,D,DPVATN,DPPOT,DPFRIC
1040   FORMAT(-PF12.3,-6PF12.3,-2PF12.3,0PF12.3,0PF12.3,-6P3F12.3)
      DZ=DZ3
      GO TO 1010
1050  P=P+DPVATN
      Z=Z+DZ
      DZSTAR=(P-PSTAR)/DPVATN*DZ
      ZSTAR=Z-DZSTAR
      U07VA1=ZSTAR
      RETURN
      END
        FUNCTION U07TP4(X,ALPHA,P,VL,VG)
        X2=((1-X)/X)**2*VL/VG
        X1=SQRT(X2)
        C=1+X*VG/(X*VG+(1-X)*VL)-ALPHA
        F2=1+C/X1+1/X2
        U07TP4=F2*VL*(1-X)**2
        RETURN
        END
        FUNCTION U07TP5(P,X,VL,VG,FLOWA,D,ALPHA)
        FR=(FLOWA*VL)**2/D/981
        ALPHA1=(0.833+0.05*ALOG10(P*1.0E-6))/
     1 (1+(1-X)/X*VL/VG)
       BETA=X*VG/(X*VG+(1-X)*VL)
       ALPHA2=BETA-0.71*BETA*SQRT(1-BETA)*
     1   FR**(-0.045)*(1-P/2.212E8)
       ALPHA=AMAX1(ALPHA1,ALPHA2)
       U07TP5=ALPHA
       RETURN
       END
01   10,200,1200,0
03   142,31.6,0.15
517,22.3,0.15
1646,17.3,0.15
17.6,1.19
94.3,5
  500,   1.0,   1.0
```

Subroutine to compute the multiplier $\bar{\Phi}_L^2$ by using Chisholm model

Subroutine to compute void fraction, α, Compute maximum α for Armand model and Kowalczewski model

Data

TYPICAL OUTPUT

```
= 22.300TURB=   0.000PA=  22.300
MASS FLOW=      18.00 KGM/SEC, AQUIFER PRESSURE=  22.30 BARS
ENTHALPY=  830.309 J/GM, FLASHING PRESSURE=  14.00 BARS
D=22.300 CM AND FLAMDA= 0.005 FROM A DEPTH OF 1094.000 METERS
```

Diameter — f friction factor

U	EKIN	HBAR	DP	DPOT	DFRIC	DMOM	X	ALFA	Q	BETA	S	FR	TEMP	P(BAR)	D(M)	TYPE	N
830.31	0.00	830.31	0.00	0.00	0.00	0.00	0.000	0.000	0.000	0.000	1.000	0.1	195.1	14.00	302.8	BUBBLY	
830.28	0.00	830.28	-0.23	-0.23	-0.00	-0.00	0.002	0.042	0.000	0.181	4.988	0.2	194.3	13.77	300.0	SLUG	
829.79	0.00	829.79	-0.50	-0.48	-0.00	-0.00	0.027	0.453	0.005	0.818	5.439	3.6	182.8	10.67	250.0	SLUG	
829.30	0.00	829.30	-0.35	-0.33	-0.00	-0.00	0.046	0.622	0.008	0.906	5.831	12.4	173.8	8.66	200.0	ANNULAR	
828.81	0.00	828.81	-0.27	-0.25	-0.01	-0.00	0.062	0.718	0.010	0.940	6.205	29.6	166.0	7.18	150.0	ANNULAR	
828.32	0.01	828.31	-0.21	-0.20	-0.01	-0.00	0.075	0.780	0.012	0.959	6.578	59.7	158.9	6.01	100.0	ANNULAR	
827.83	0.02	827.81	-0.18	-0.16	-0.01	-0.00	0.088	0.825	0.014	0.970	6.960	110.0	152.3	5.06	50.0	ANNULAR	
827.34	0.03	827.31	-0.15	-0.13	-0.02	-0.00	0.100	0.859	0.015	0.978	7.365	193.7	145.9	4.25	0.0	ANNULAR	

```
4.25
--- STOP
```

- DP: Total change of pressure between sections
- DPOT: Pressure change be of potential
- DFRIC: Pressure change be of friction
- DMOM: Pressure change be of momentum
- X: Mass vapour qualit
- ALFA: Void fraction
- Q: Statical steam mass fraction
- BETA: Gas phase volumetr flow fraction
- S: Slip ratio
- FR: V^2/gd
- TEMP: Temperature (°C)
- P(BAR): Pressure (bar)
- D(M): Deapth (m)
- TYPE: Flow pattern
- N: Number of iteratio

APPENDIX VI.

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