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Connection between discharge and the time development of high/low water levels along river Jökulsá á Fjöllum

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Connection between discharge and the time development of high/low water levels along river Jökulsá á Fjöllum

Prepared for the Resources Division of Orkustofnun

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Abstract:
Based on water level measurements from Grímsstaðir, (vhm 102), Upptyppingar (vhm 162), Selfoss (vhm 453), Kreppa (vhm 233), a mathematical function is derived. The function describes the connection between discharge and the time development of high/low water levels along river Jökulsá á Fjöllum.

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Discharge, time difference, smoothing spline, depth profile, mathematical model, Jökulsá á Fjöllum, Grímsstaðir, (vhm 102), Upptyppingar (vhm 162), Selfoss (vhm 453), Kreppa (vhm 233).	**Project manager's signature:** Páll Jónsson
	Reviewed by: PJ, HA, KE

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1 INTRODUCTION

This is a follow up on an earlier report "Framgangur dægursveiflna niður Jökulsá á Fjöllum" written by Jóel Karl Friðriksson (2002) [1]. We assume the reader is familiar with [1], so in this report "Connection between discharge and the time development of high/low water levels along Jökulsá á Fjöllum" there will not be given a detailed explanation on the overall setup.

We will now in short explain the purpose of the project. Firstly, we are interested in knowing the discharge Q at a certain location, at the time where the water level is at its highest or lowest level. Secondly, we are interested in knowing the time Δt it has taken for this exact high/low water level to move down along river Jökulsá á Fjöllum, from a predetermined location to the place where the discharge is determined. Knowing Q and Δt, we then want to find the relation between these.

In 2001 the water level of Jökulsá á Fjöllum was measured every hour at Selfoss (vhm 453), Grímsstaðir (vhm 102) and Upptyppingar (vhm 162). In [1] it was concluded that from these water level measurements, it is not possible to draw any conclusions about a connection between Q and Δt. The problem was that measurements were simply not carried out frequently enough. So during the summer of 2002 measurements were taken every 5 minutes, which as we will see gives much better results. In addition, for this study we also consider the 5 minute water level measurements from Kreppa (vhm 233).

In the rest of the report, I will omit "vhm" when referring to a water level gauge.

2 DATA PROCESSING

For stations 453, 102 and 162, five minute water level measurements are used beginning on June 4th and ending on September 27th. For 233, we use hourly measurements from June 4th to July 6th and 5 minute measurements from July 6th to September 27th. At 162, the water level gauge did not work during the period between June 27th and July 7th, so these data are missing. There are some few extra gaps spread in the data set, but they are so small that the errors they give are all insignificant.

Plotting water level vs. time and joining the data points with lines gives four piecewise linear curves, one for each station. These curves represent the "exact" water level cycle; a section of one such is shown in Fig. 1 (blue curve). They are difficult to work with, because of their rapid zigzag behaviour, so we choose to smoothen them and let the smoothened curves represent the "exact" water level cycles (red curve in Fig. 1).

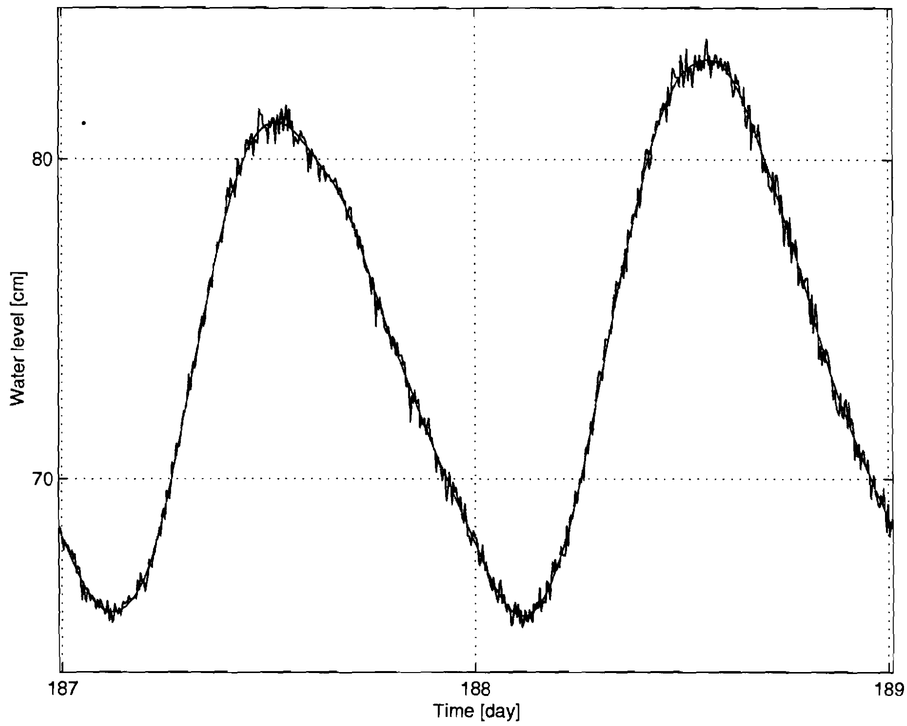

Figure 1: *Part of the water level cycle at Selfoss.*

What is done explicitly is that the Matlab function "csaps" is used to make smoothing splines from the data points. One can see e.g. in Fig 1 or Fig. 2 that the function provides remarkably good results. Function "csaps" has a variable ranging between 0 and 1; it defines how much the zigzag curve has to be smoothened. By increasing and decreasing the value of this variable (manually), we find (subjectively) that the value 0.9999 is optimal.

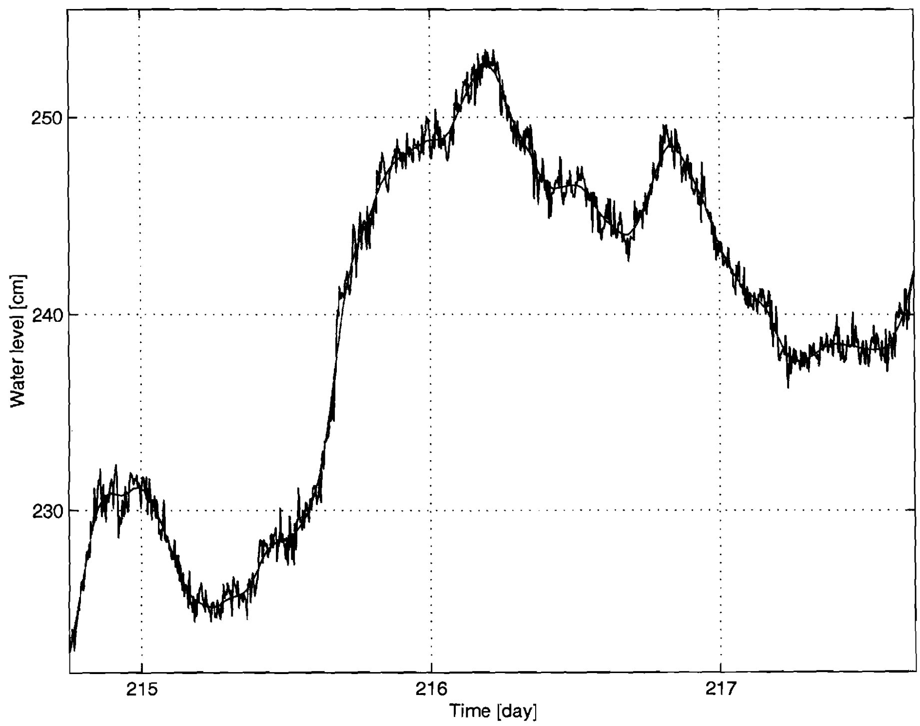

Figure 2: *Part of the water level cycle at Upptyppingar.*

Once a smoothing spline curve is found, we can determine the highest and lowest water level for each day. It is done simply by finding the largest and smallest value of the day. There are sometimes several local extrema (e.g. see Fig. 2), but then only the largest and the smallest among them are used for each day. The just mentioned procedure is performed on all four smoothing spline curves.

To understand the origin of the high and low water levels, it might be worth pointing out, that they are the result of the diurnal variation in water level due to melting of the glacier by the sun.

In order to get the most reliable data, we prefer high or low water levels related to a top or a bottom that is distinct and sharp. Meaning that some daily high or low water levels are excluded (sorted out manually). This concerns a top or a bottom that is either very "flat" or has small amplitude or is just not nice at all like the ones in Fig. 2.

Now we have to pair the tops and bottoms belonging to the same events for two curves at the time. The amount of data for stations 102 and 453 is plenty and the two water level curves are very sine like and very similar in shape, so we can be a little picky in this case. On the other hand, data points for station 162 paired with stations 453, 102 or 233 are fewer because of the gauge failure of station 162 and the curve irregularity (the curve is not so sine like). Therefore in this case we cannot allow ourselves to be very strict in omitting data. In total we pick out 196 pairs of extrema values for Grímsstaðir and Selfoss belonging to the same high/low water levels (102 high and 94 low), 162 for Upptyppingar and Selfoss (85 high and 77 low), 164 for Upptyppingar and Grímsstaðir (86 high and 78 low) and 163 for Upptyppingar and Kreppa (87 high and 76 low). See an example of the data in Fig. 3.

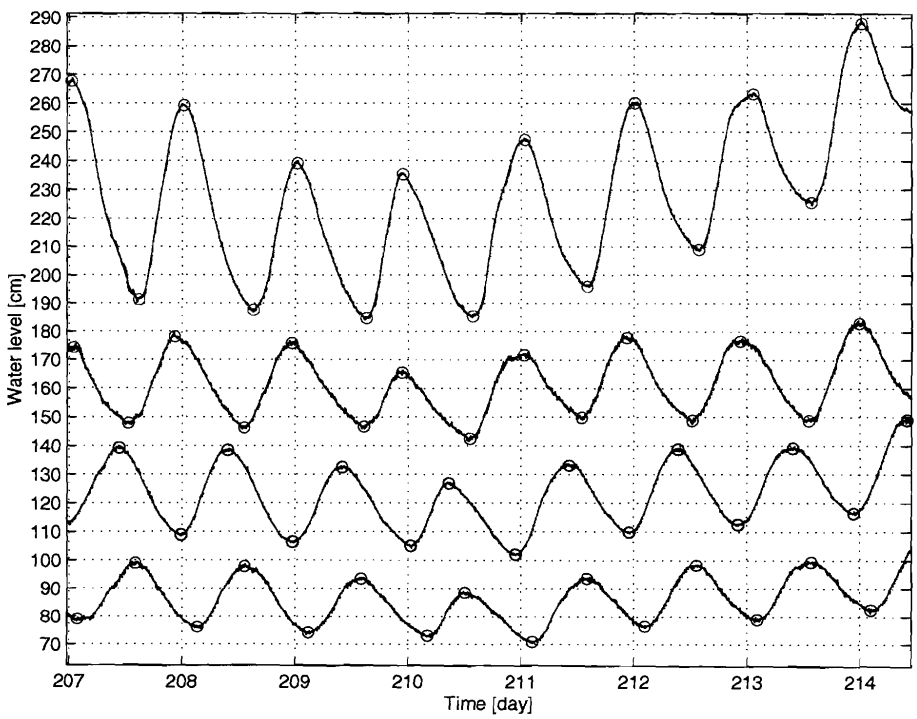

Figure 3: *Part of the water level cycles for all four stations. From the bottom and up: 453, 102, 162, 233. For better visualization, the curves are moved up/downwards mutually in order to avoid that they overlap. The colored circles are high/low water levels.*

With this selection of values, one can determine the time Δt it takes for the specific high or low water levels to move from 102 and 162 respectively to 453 and from 162 to 102, as well as the time difference between 162 and 233. Next, the discharge Q at 102 is determined from the discharge rating curve at 102. Note that the discharge rating curve at 102 is used in all the cases. In appendix A, plots of Δt vs. Q are made for Grímsstaðir-Selfoss and Upptyppingar-Selfoss with respectively all data points, highest water level and lowest water level data points, and for Upptyppingar-Grímsstaðir and Upptyppingar-Kreppa with all data points. In the plots, one can sense a little decrease of Δt with increasing Q. Note that the range of the y axis is not the same in the two different cases, meaning that comparing the data one should keep in mind that the data for e.g. Upptyppingar-Selfoss is more spread than the data for Grímsstaðir-Selfoss. The most spread data is Upptyppingar-Kreppa, assumably because of only hourly measurements until the 6th of July and also because the two gauging stations are on two separate rivers.

As a little supplement to the project we have included a flood which took place on the 8th of January 2002. It provides us with the data point (477.62, 10.077) in the Upptyppingar-Grímsstaðir case. The point is marked with a cross in the plot in Appendix A.

3 THE THEORETICAL MODEL

In order to find a good and trustworthy approximation of the data points in the $(Q, \Delta t)$ plots in appendix A, we need some understanding about the mathematical relationship between Q and Δt. In this section we will find the relationship, but note that not all calculations and considerations will be given in detail.

The discharge rating curve at station 102 is given by the equation

$$Q(h) = 729 \cdot 10^{-5}(h-37)^{2.02}$$

where h is the water level measured by the gauge, Q is measured in m^3/s and h in cm. Isolating h gives

$$h(Q) = \left(\frac{Q}{729 \cdot 10^{-5}}\right)^{\frac{1}{2.02}} + 37 \qquad (4.1)$$

which we will need later. A general expression for the discharge Q is given by

$$Q = vA$$

where v is the mean velocity at the cross section and A the area of the cross section. Assume now that v equals respectively the mean velocity between the two measuring points 102-453 and 162-453. This makes it possible for us to write

$$v = \frac{\Delta x}{\Delta t}$$

where Δx is the distance between the stations measured along the river. The equation above is then

$$Q(h)\Delta t = \Delta x A(h) \qquad (4.2)$$

We can thereby conclude that the product of Q and Δt is not constant, which means that the $(Q, \Delta t)$ data does not behave like $\frac{1}{Q}$. But then, in order to understand how the product behaves, we need to know how the area A of the cross section of the river changes as a function of h (remember Δx giving the distance between the stations is constant).

The cross section at the gauging station 102 is shown in Fig 4, and we will now make a (rough) approximation of this. First assume that the water level is never lower than around ca. 350 cm from the bottom. This means that the gap in the bottom is always filled with water, implying that the area a is independent of h, i.e. constant. The cross section above a will be approximated with a box pinched in between two identical triangles. The width of the box is set to be b and the height is set to be h', and the difference between h and h' is given by c. The slope of "alfa" is given by $\frac{1}{\alpha}$.

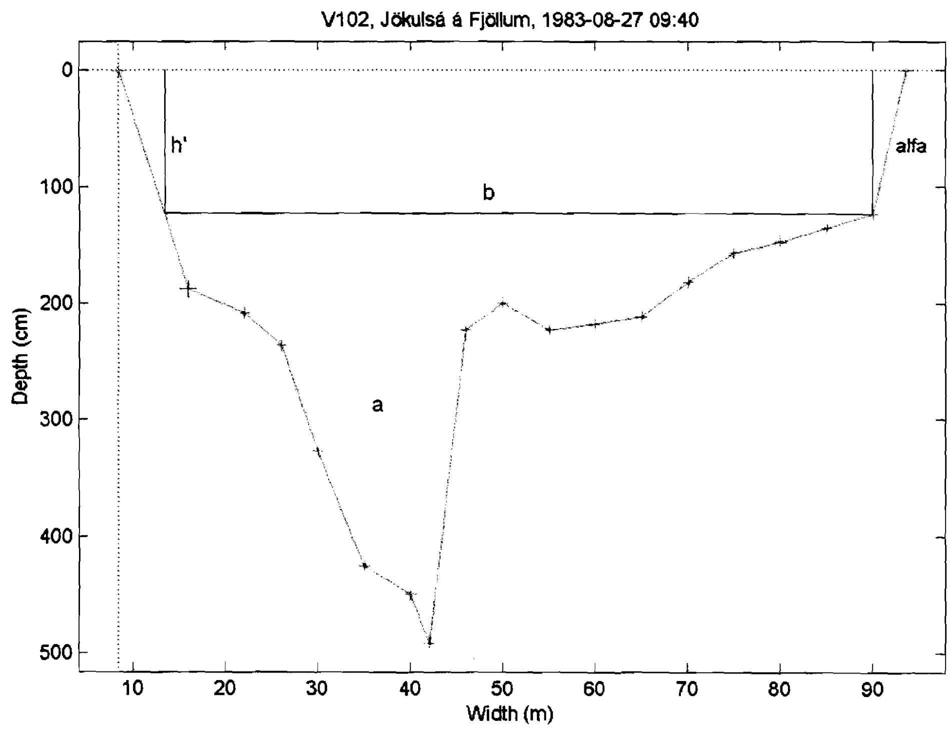

Figure 4: *Depth profile at the gauging station 102.*

We can now find a (very) simplified way to express $A(h)$, and if we combine this with equation (1), we get

$$\begin{aligned} Q(h)\Delta t &= \Delta x A(h') \\ &= \Delta x A(h-c) \\ &= \Delta x (a + b(h-c) + \alpha (h-c)^2) \\ &= \Delta x (\alpha c^2 + a - bc + (b - 2\alpha c)h + \alpha h^2) \end{aligned}$$

Insert into this the expression for $h(Q)$ in (4.1), and that gives us

$$\Delta t = \Delta x \left(\frac{\alpha c^2 + a - bc}{Q} + (b - 2\alpha c) \frac{\left(\left(\frac{Q}{729 \cdot 10^{-7}} \right)^{\frac{1}{2.02}} + 37 \right)}{Q} + \alpha \frac{\left(\left(\frac{Q}{729 \cdot 10^{-7}} \right)^{\frac{1}{2.02}} + 37 \right)^2}{Q} \right)$$

$$= \Delta x \left(\left[(\alpha c^2 + a - bc) + 37(b - 2\alpha c) + \alpha 37^2 \right] \frac{1}{Q} + \left[\frac{(b - 2\alpha c) + 2\alpha 37}{(729 \cdot 10^{-7})^{\frac{1}{2.02}}} \right] \frac{1}{Q^{\frac{1.02}{2.02}}} + \frac{\alpha}{(729 \cdot 10^{-7})^{\frac{2}{2.02}}} \frac{1}{Q^{\frac{0.02}{2.02}}} \right)$$

$$\approx c_1 \frac{1}{Q} + c_2 \frac{1}{\sqrt{Q}} + c_3$$

where c_1, c_2 and c_3 are constants. This leads us to suggest that one should try to make an approximation of the form

$$\Delta t \approx c_1 \frac{1}{Q} + c_2 \frac{1}{\sqrt{Q}} + c_3$$

But it turns out that this expression is very difficult to work with in order to make a realistic physical approximation of the dataset. So we try to change it a bit. The two first terms are joined in one, we keep the constant term (note that the constant term is very vital) and we try to make a fit with a function of the form

$$\Delta t \approx \frac{a}{Q^p} + b \qquad (4.3)$$

The constant b cannot be negative for Jökulsá á Fjöllum since this could lead to negative time differences. Furthermore, we will not expect p to be much less than 0.5 or much larger than 1.5.

4 APPLICATIONS OF THE THEORETICAL MODEL

Let us now see how the expression in (4.3) applies to the real data sets. The Matlab curve fitting toolbox is used to make a least square approximation of the datasets. The following results are all visualized in appendix B.

The curve fitting toolbox gives the function

$$f1(Q) = \frac{10{,}23}{Q^{0.4136}} + 2{,}598$$

which actually approximates the data set for Grímsstaðir-Selfoss quite well. The function has the form we would expect it to have, $p \sim 0.5$ and $b > 0$, so we conclude it must be the solution we are looking for in the Grímsstaðir-Selfoss case. Note that the confidence level is very good until discharge is a little less than 100.

In the Upptyppingar-Selfoss case, the function

$$f2a(Q) = \frac{38{,}43}{Q^{0.09277}} - 8{,}343$$

is found as an approximation to the data set, and in the Upptyppingar-Grímsstaðir case

$$f3a(Q) = -71.77\,Q^{0.02061} + 91.5$$

is found. But we notice that p in both cases is much less than expected and respectively b and a are negative which means that $f2a = \Delta t \rightarrow -8.343$ as $Q \rightarrow \infty$ and $f3a = \Delta t \rightarrow -\infty$ as $Q \rightarrow \infty$. Why does our method work so badly with the data for Upptyppingar-Selfoss and Upptyppingar-Grímsstaðir? The explanation might be that Kreppa and other streams join Jökulsá á Fjöllum between Upptyppingar-Grímsstaðir, causing disturbance to the water level cycles. This has influence on the data set in the way that it becomes spread, at least much more than in the case of Grímsstaðir-Selfoss where no streams run into the river.

If we look at the interval for which we have discharge values (ca. 150 to 500), then the more spread the data is in the vertical direction, the more "steep" a fit the least square approximation will try to make through the points. A "steep" curve is an untrustworthy approximation of the data, since it will tend to have a negative lower bound. Note that if the constant term b is omitted, it also causes the fit to be too steep, which is why this constant is so important.

A suggestion to a solution for the two problematic cases would be to keep the $p = 0.4136$ value from $f1$, and with this make a new approximation of the Upptyppingar-Selfoss and Upptyppingar-Grímsstaðir data. Doing this we get the functions

$$f2b(Q) = \frac{52.44}{Q^{0.4136}} + 9.321$$

and

$$f3b(Q) = \frac{41.42}{Q^{0.4136}} + 6.842$$

which also approximates the data quite well. By comparison it also makes good sense, that the constant term b=9.321 for $f3b$ is larger than the constant term b=6.842 for $f2b$, which again is larger than the constant term b=2.546 for $f1$. The black cross represents the data obtained from the flood in January 2002.

The Upptyppingar-Kreppa case is a little special in the sense that $\Delta t = t_{233} - t_{162}$ takes on both positive and negative values (see appendix A). This means that sometimes events in Kreppa occur before Upptyppingar. We find a first approximation to the data set given by the function

$$f4a(Q) = \frac{145.1}{Q^{0.7309}} - 0.6581$$

To be consistent we also try to make the approximation with the fixed value $p = 0.4136$. This gives

$$f4b(Q) = \frac{43.88}{Q^{0.4136}} - 2.546$$

Both $f4a$ and $f4b$ seems acceptable, so which one should we use? An idea is to try to make the same $(Q, \Delta t)$ plot just using the discharge at 233 instead of 102. We tried this, but unfortunately it gave no result. So, since both functions are reasonable solutions to the problem, we choose

$$f4b(Q) = \frac{43.88}{Q^{0.4136}} - 2.546$$

in order to be consistent in the use of a fixed $p = 0.4136$.

For Grímsstaðir-Selfoss we added a plot of the data sets for the separate months i.e. June, July, August and September, see appendix C. There is a slight decrease over the months mutually and to some extent they overlap, meaning it is not like the data from one month singles out. It does not seem like there is a deviation of the monthly data points from what we would expect.

Summing up our research, the conclusion of this report is that the Grímsstaðir-Selfoss data is by far the most reliable, and $f1$ gives quite a good description of it. In the Upptyppingar-Selfoss, Upptyppingar-Grímsstaðir and Upptyppingar-Kreppa cases the data is more spread, but by extension of $f1$ (i.e. the p value) the functions $f2b$, $f3b$ and $f4b$ are found, and they all give fairly acceptable approximations of the data.

REFERENCES

[1] Jóel Karl Friðriksson, "Framgangur dægursveiflna niður Jökulsá á Fjöllum". Greinargerð JKF-02/1, Orkustofnun, Vatnamælingar

Appendix A

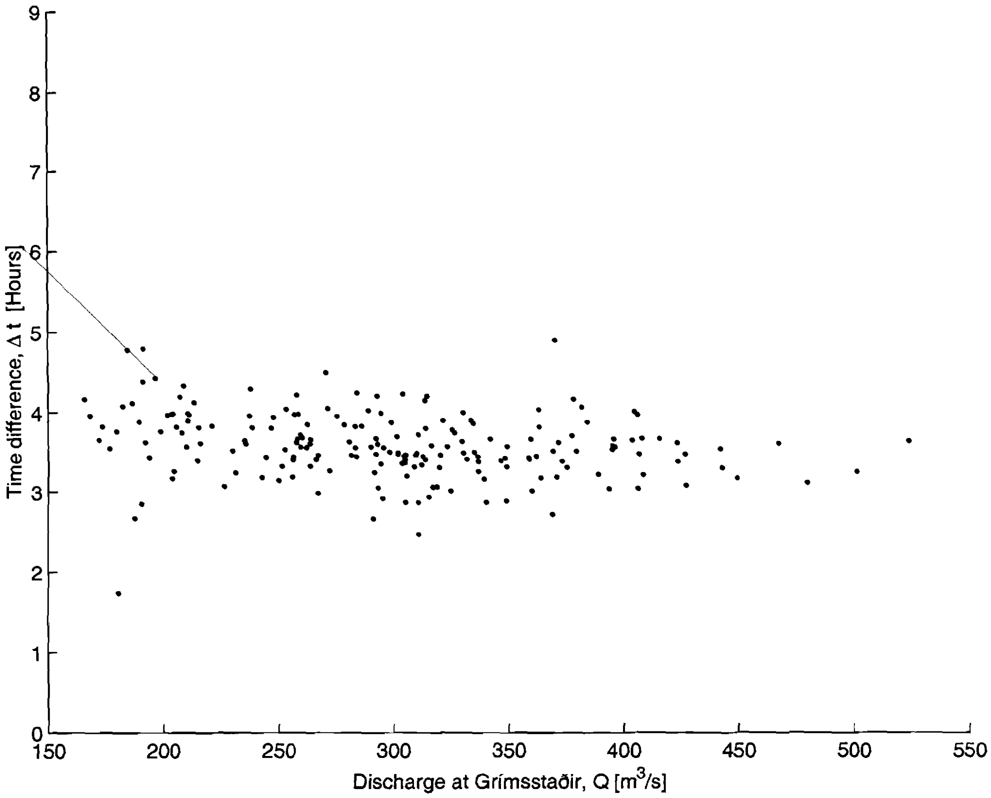

Figure 5: *Grímsstaðir-Selfoss, all data points.*

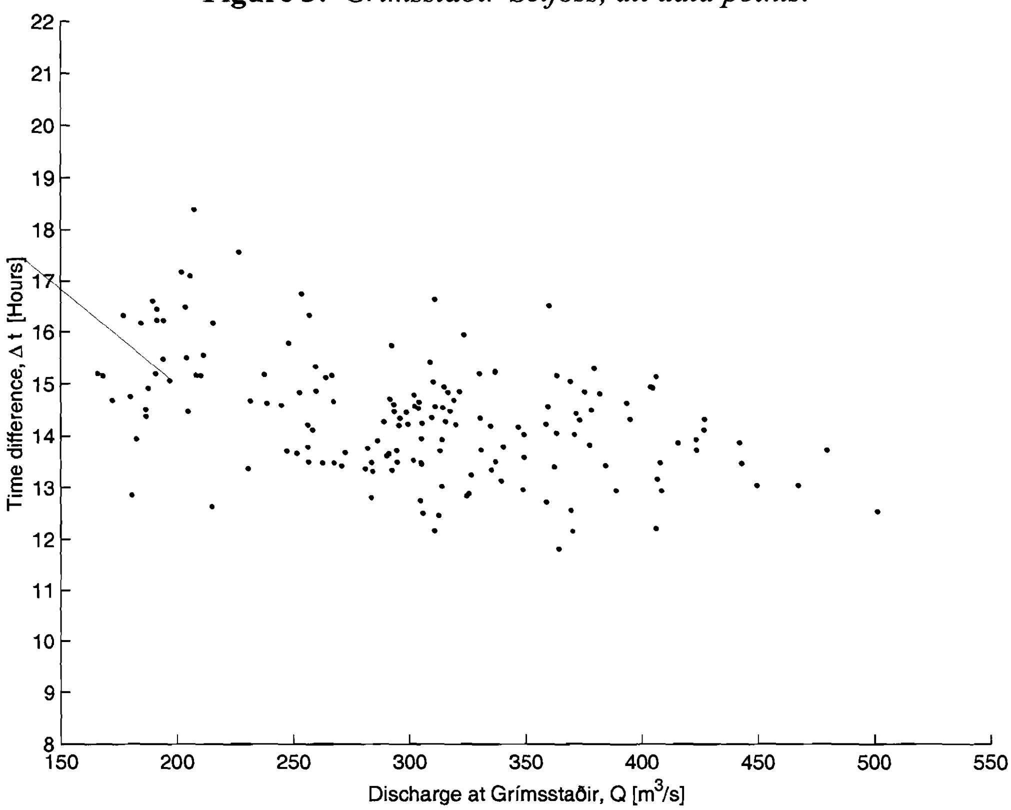

Figure 6: *Upptyppingar-Selfoss, all data points.*

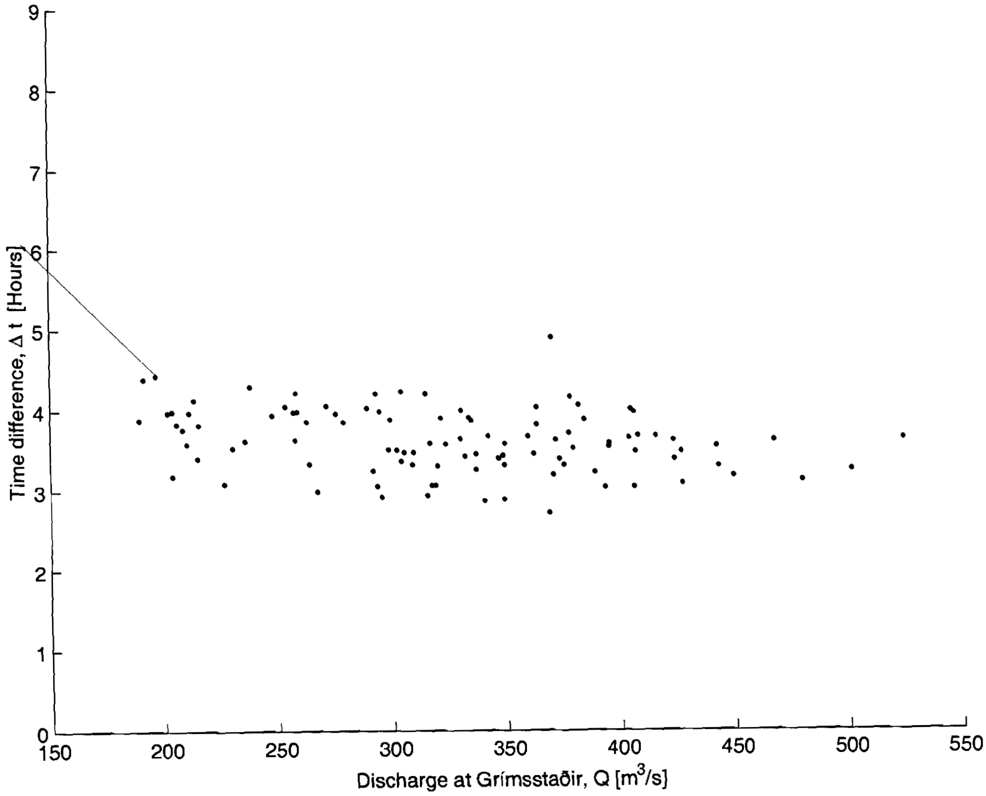

Figure 7: *Grímsstaðir-Selfoss, high water level.*

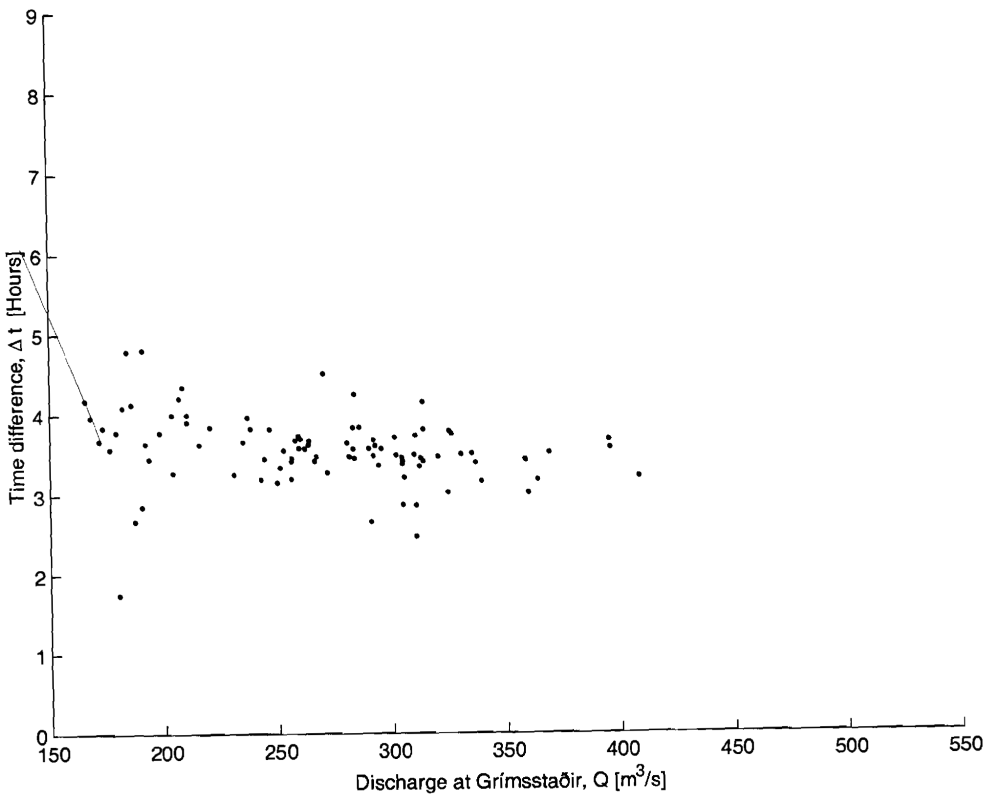

Figure 8: *Grímsstaðir-Selfoss, low water level.*

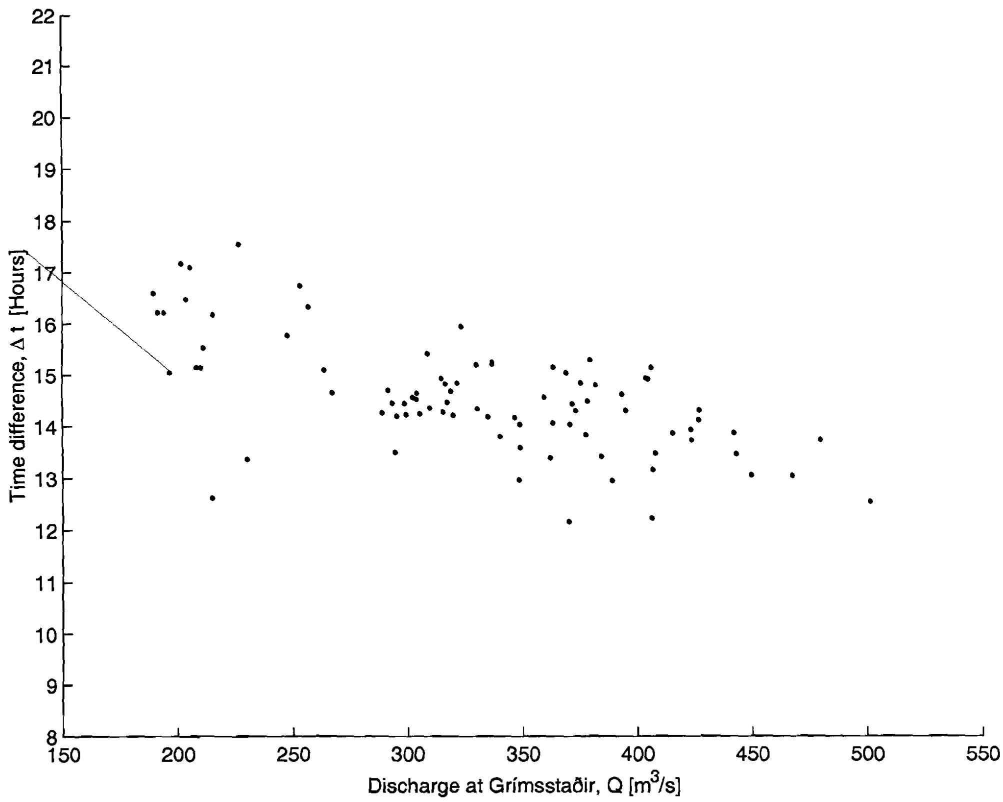

Figure 9: *Upptyppingar-Selfoss, high water level data.*

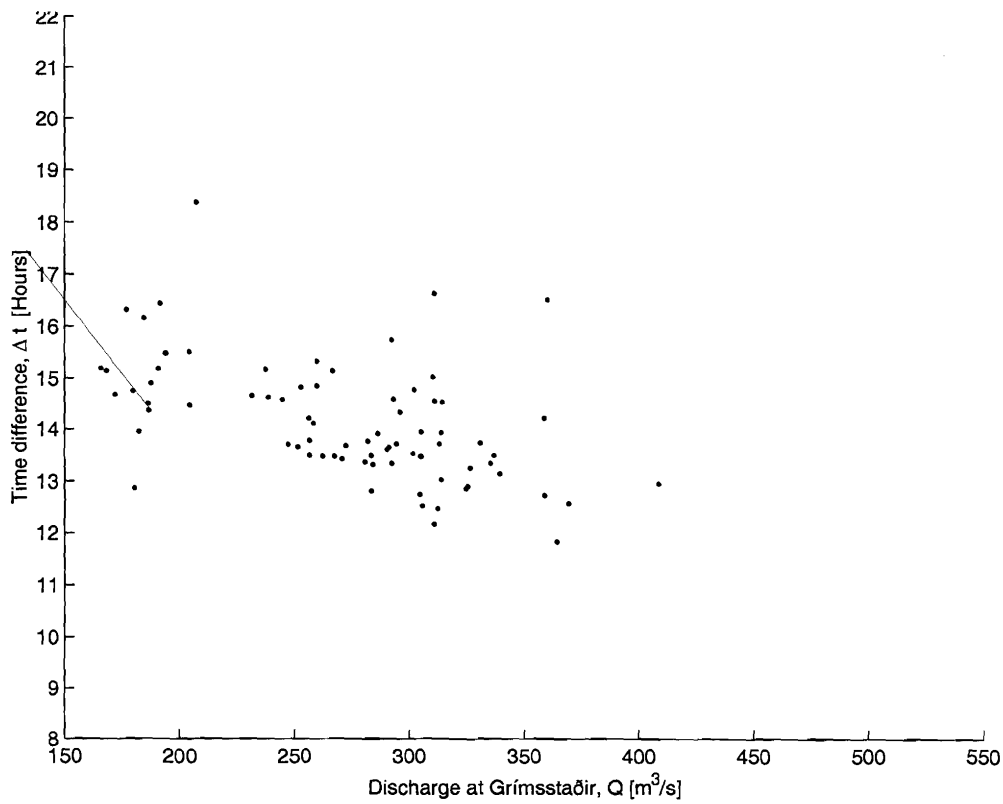

Figure 10: *Upptyppinger-Selfoss, low water level data.*

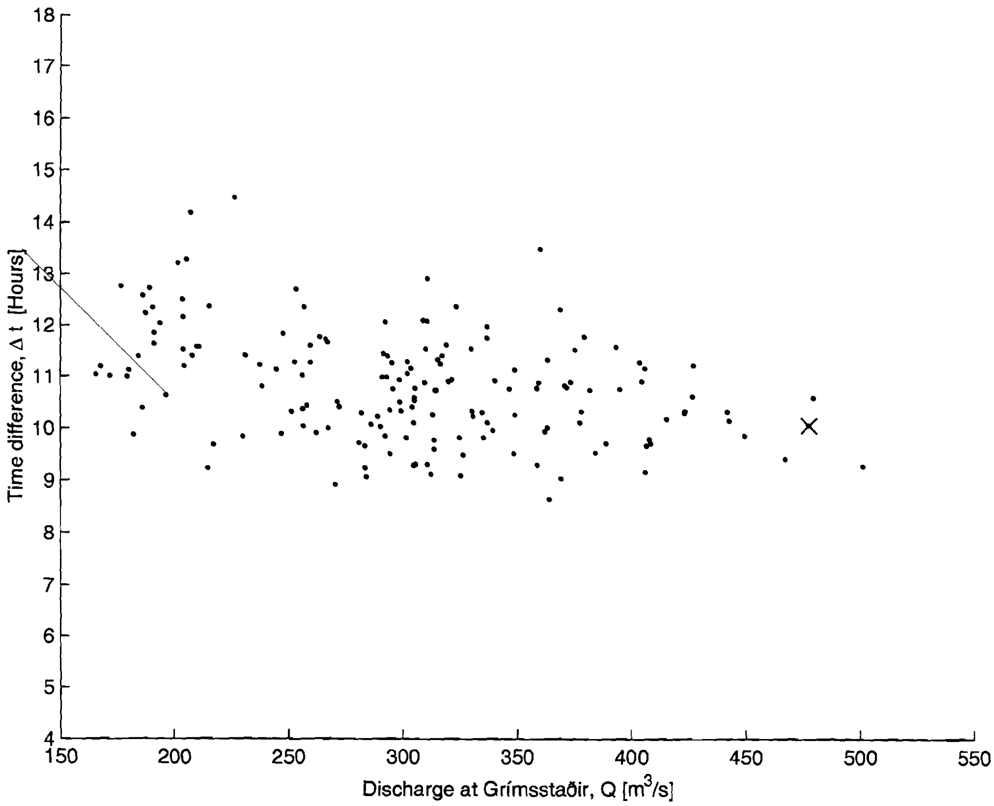

Figure 11: *Upptyppingar-Grímsstaðir, all data points.*

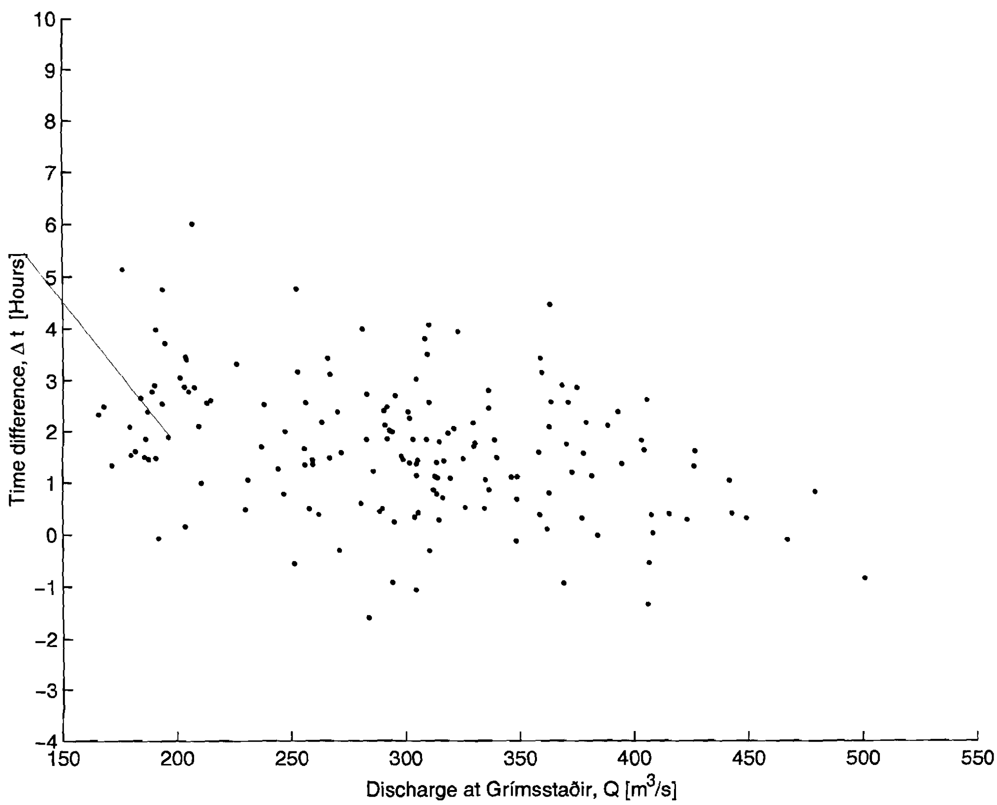

Figure 12: *Upptyppingar-Kreppa, all data points.*

Appendix B

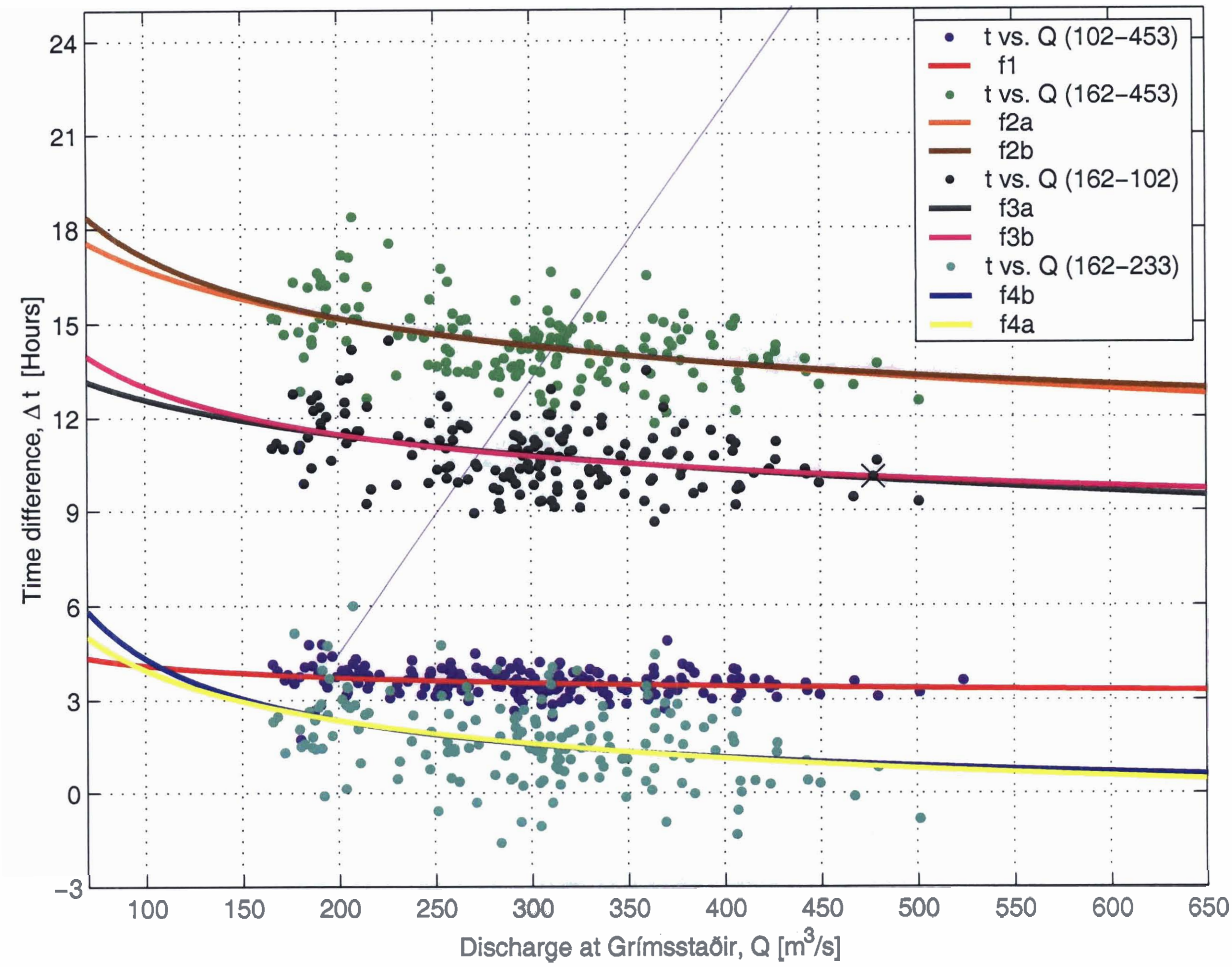

Figure 13: *Approximations of the data sets.*

Appendix C

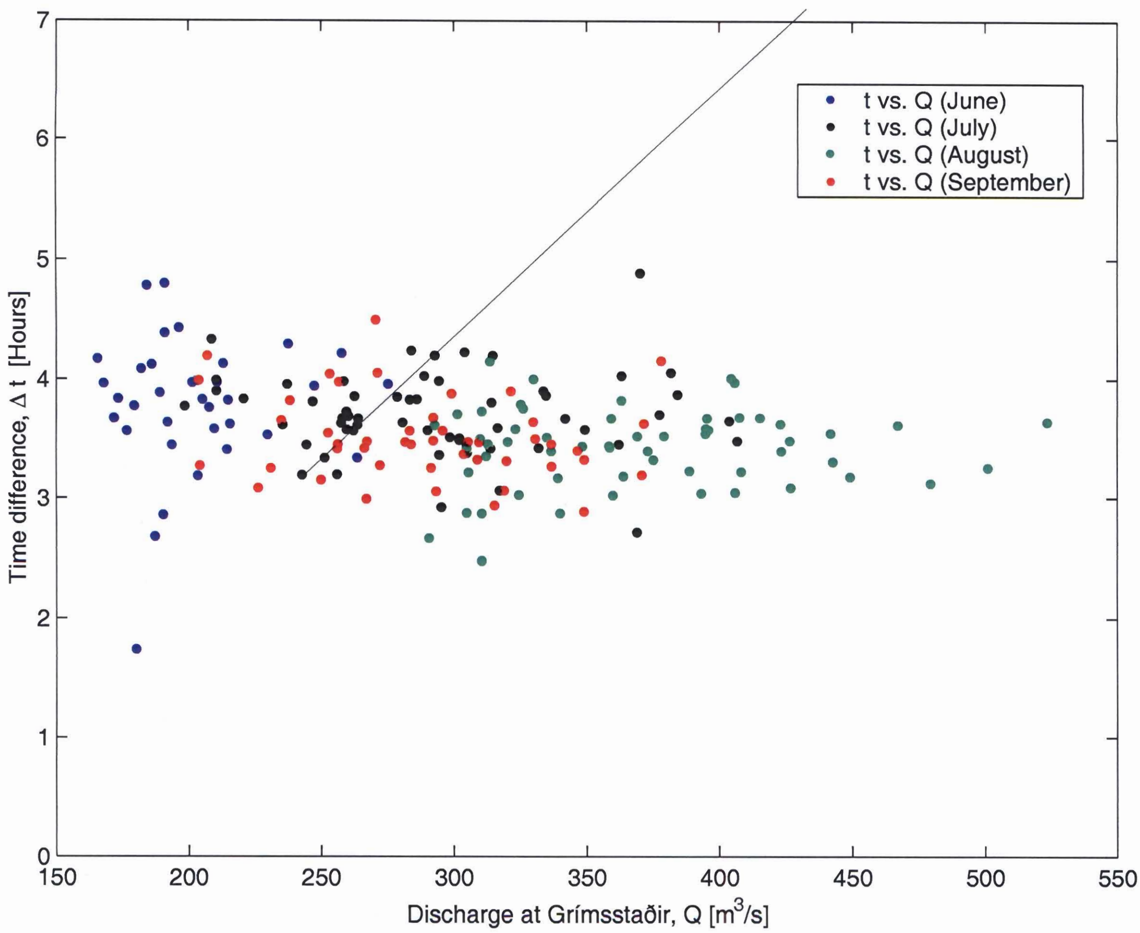

Figure 14: *Monthly data sets for Grímsstaðir-Selfoss.*