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Travel time for flow in Jökulsá á Fjöllum in relation to discharge and sediment transport

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Prepared for the Resources Division of Orkustofnun

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Abstract:

This report is a follow up on two earlier reports. The report provides improvement of earlier results. Also new results are obtained.

Based on water level measurements from Grímsstaðir (vhm 102), Upptyppingar (vhm 162), Selfoss (vhm 453) and Kreppa (vhm 233), the connection between discharge and the time development of high/low water levels along river Jökulsá á Fjöllum is investigated. Two mathematical functions are derived and applied on the data set.

Based on light absorption measurements from Grímsstaðir, (vhm 102), it is shown that high/low concentrations in suspended sediment arrives later than high/low water level events.

Keywords:	**ISBN:**
Discharge, water level, light absorption, suspended sediment, time difference, smoothing spline, depth profile, mathematical model, Jökulsá á Fjöllum, Grímsstaðir (vhm 102), Upptyppingar (vhm 162), Selfoss (vhm 453), Kreppa (vhm 233).	**Project manager's signature**
	Reviewed by: Páll Jónsson, Hákon Aðalsteinsson

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1 INTRODUCTION

This is a follow up on the reports “Framgangur dægursveiflna niður Jökulsá á Fjöllum,” (Jóel Karl Friðriksson, 2002) and “Connection between discharge and the time development of high/low water levels along Jökulsá á Fjöllum,” (Henriksen, Steen, 2003). We assume the reader is familiar with these two previous reports. This report contains new results and considerations though certain parts of it are repetitions of (Henriksen, Steen, 2003).

The aim of this project was to determine how long it takes water from the upper reaches of the river to flow down to the waterfall Dettifoss, and if how it was dependent on discharge and sediment transport.

We will now give a short explanation of the project. We are interested in knowing the discharge Q at a certain location, at the time where the water level is at its highest or lowest level. We are also interested in knowing the time Δt it has taken for this exact high/low water level event to move down the river Jökulsá á Fjöllum, from a predetermined location to the place where the discharge is determined. Once we know Q and Δt, we then want to find the relationship between them. We are also interested in the light absorption A at the location where the discharge Q is determined. A depends on the suspended sediment load and from A we want to examine if suspended sediment in the water has any effect on Δt. From A we also want to examine if a high/low water level moves faster down the river than a high/low concentration of suspended sediment or vice versa.

In the summer of 2001 the water level of Jökulsá á Fjöllum was measured every hour at three water level gauging stations; Selfoss (vhm 453), Grímsstaðir (vhm 102) and Upptyppingar (vhm 162). It was concluded that from the obtained water level measurements it was not possible to find any connection between Q and Δt. The resolution of the data was too poor. The measurements had simply not been carried out frequently enough, (Jóel Karl Friðriksson, 2002).

In the summer of 2002 water level measurements were taken every 5 minutes, and this time river Kreppa (vhm 233) was included. After the data had been plotted one could see that a resolution of 5 minutes was very satisfactory. From this data it was possible to derive some interesting results, (Henriksen, Steen, 2003).

The collecting of data was continued in 2003 and 2004. Today we have as a result much more data available. The data we are interested in is the water level over the summer of 2003 measured at Selfoss (vhm 453), Grímsstaðir (vhm 102) and Upptyppingar (vhm 162), and the similar data over the summer of 2004 at Grímsstaðir (vhm 102) and Upptyppingar (vhm 162). Light absorption data in the summer of 2003 and 2004 at Grímsstaðir (vhm 102) is also of interest.

When referring to a water level gauge in the report the name and vhm number will be used at random.

2 DATA PROCESSING

In this report we are working with the following data sets:

Water level, 2002.
Vhm 453 and vhm 102: Five minute measurements from June 6 to September 27.
Vhm 162: Five minute measurements from June 4 to June 27 and July 7 to July 27. Gauge failure from June 27 to July 7.
Vhm 233: Hourly measurements from June 4 to July 6 and five minute measurements from July 6 to September 27.

Water level, 2003.
Vhm 453 and vhm 102: Five minute measurements from June 3 to October 2.
Vhm 162: Five minute measurements June 27 to August 8.

Water level, 2004.
Vhm 102: Five minute measurements from June 15 to September 12.
Vhm 162: Measurements of mixed time intervals (five minutes and hourly) July 17 to September 9.

Light absorption, 2003.
Vhm 102: Five minute measurements from June 27 to August 8.

Light absorption, 2004.
Vhm 102: Five minute measurements from June 15 to September 12. Gauge data useless from August 11 to August 16.

There are a few small gaps spread in the data sets, but the errors they give in the final results are insignificant.

Plotting water level or light absorption data vs. time and joining the data points with lines gives piecewise linear curves. These curves represent the “exact” water level or suspended sediment cycles; a section of one such is shown in Fig. 1 (blue curve). They are difficult to work with because of their rapid zigzag behaviour. We then choose to smoothen them and let the smoothened curves represent the “exact” water level cycles (red curve in Fig. 1). Note that for a resolution of five minutes, it is possible to estimate the highest or lowest water level straight from the raw data (blue curve). The smoothing curve is just much easier to work with and it provides equally accurate results.

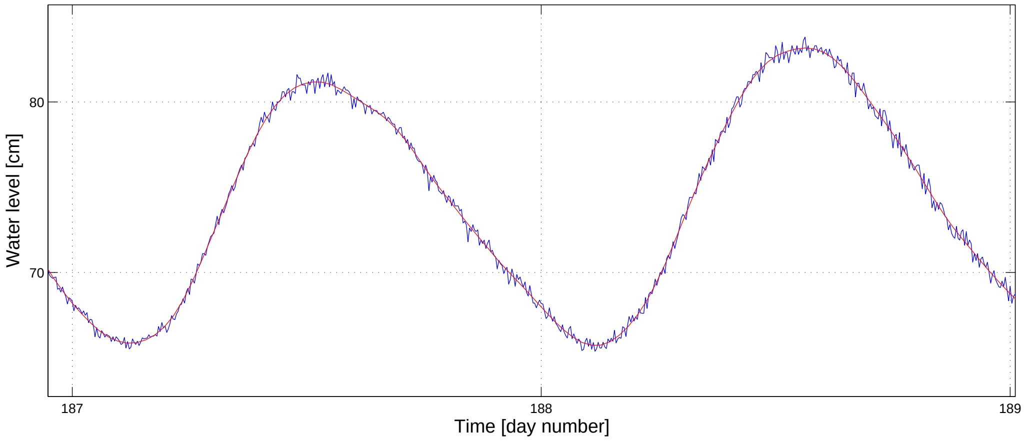

Figure 1: *Part of the water level cycle at Selfoss in 2001.*

Explicitly what is done is, that the Matlab function “csaps” is used to make smoothing splines from the data points. One can see e.g. in Fig. 1 or Fig. 2 that the function provides remarkably good results. Function “csaps” has a parameter ranging between 0 and 1, that defines how much the zigzag curve has to be smoothened. By changing the value of this variable (manually), we find (subjectively) that the value 0.9999 is optimal.

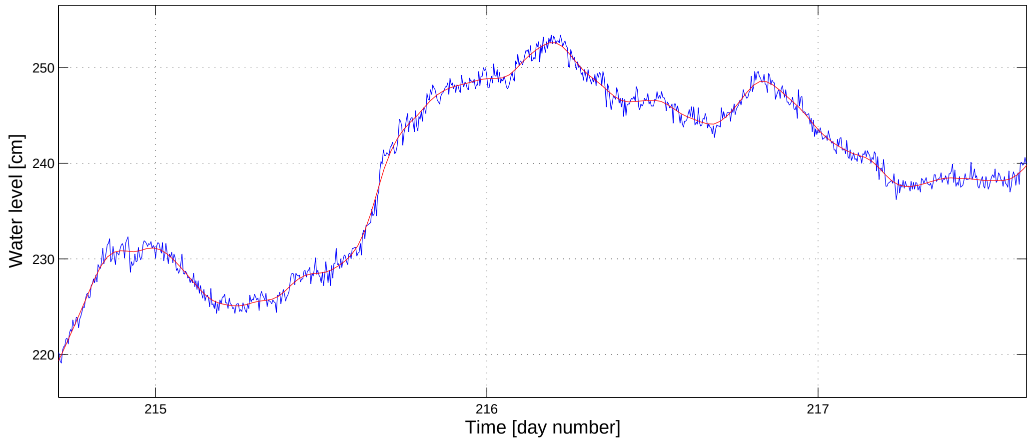

Figure 2: *Part of the water level cycle at Upptyppingar in 2001.*

Once a smoothing spline curve is found, we can easily determine the highest and lowest water level, or sediment concentration for each day. This is done by finding the largest and smallest curve value of the day. There are sometimes several local extrema (e.g. see Fig. 2), but then only the largest and the smallest among them are used for each day. The mentioned procedure is performed on all the smoothing spline curves.

In order to get the most reliable data we prefer high or low water levels related to a top or a bottom that is distinct and sharp. In other words we are concerned when a top or a bottom is either very “flat” or has small amplitude or is just not nice at all like the ones in Fig. 2. This means that some daily high or low levels are excluded (sorted out manually). The same applies for high and low values in sediment concentration.

Now we pair the tops and bottoms belonging to the same events for two water level curves at the time. We list the stations to pair and comment on the data.

- For 2002 we pick out 196 pairs of extreme values for Grímsstaðir and Selfoss belonging to the same high/low water levels (102 high and 94 low). For 2003 we pick out a total of 493 pairs, but here we do not distinguish between high and low water levels. The total amount of water level data in this case is numerous. Both years the two water level curves are sine like and very similar in shape, so the selected data is fairly reliable.

- For 2002 we pick out 162 pairs of water level for Upptyppingar and Selfoss (85 high and 77 low), 164 pairs for Upptyppingar and Grímsstaðir (86 high and 78 low) and 163 pairs for Upptyppingar and Kreppa (87 high and 76 low). The water level pairs in these cases are somewhat reduced because of the gauge failure at Upptyppingar, but also because of the curve irregularity (the curve is not so sine like). Due to the curve irregularity, the selected data might be expected to be slightly spread.

An example of the water level data is shown in Fig. 3. One can clearly see how the events occur at different times. Relevant is also to note that the diurnal variation in water level (and sediment transport) is mainly due to glacier melting.

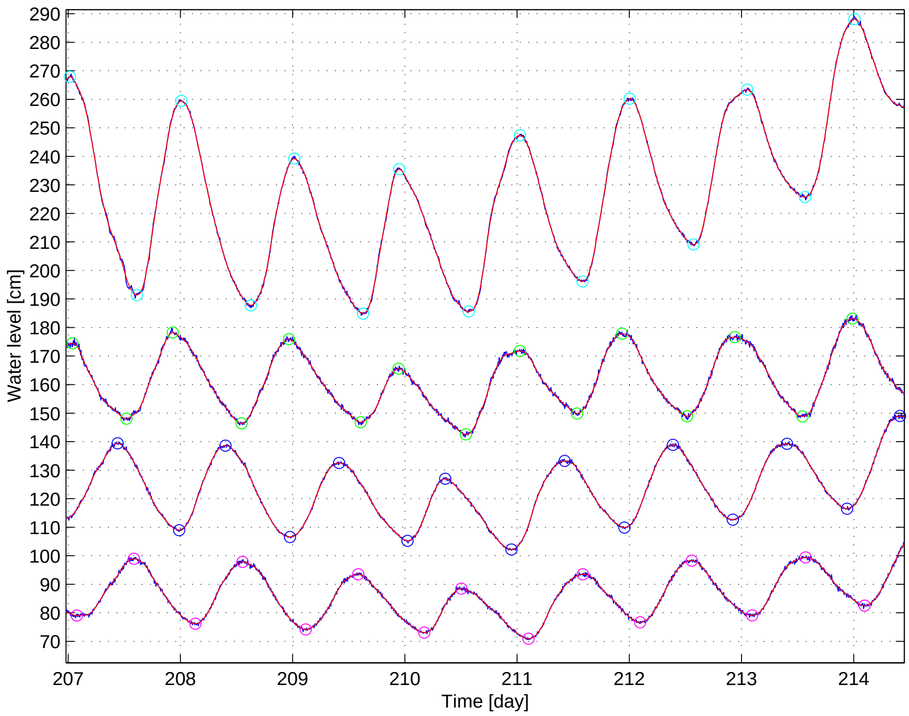

Figure 3: *Part of the water level cycles for all four stations in 2001. From the bottom and up: Vhm 453, vhm 102, vhm 162 and vhm 233. For better visualization, the curves are moved up/downwards mutually in order to avoid that they overlap. The colored circles are high/low water levels.*

We continue the pairing of data, but now include the curves of suspended sediment (light absorption). No distinguishing will be made between high and low values. When triplets are selected they all belong to the same events for three curves at the time. We list the station(s) and comment on the data.

- For 2003 and 2004 we pick out 133 pairs of water level and light absorption values at Grímsstaðir.

- For 2003 we pick out 76 triplets of water level for Grímsstaðir and Selfoss and light absorption values at Grímsstaðir.

- For 2003 and 2004 we pick out 133 triplets of water level for Grímsstaðir and Upptyppingar and light absorption values at Grímsstaðir.

- The light absorption data from Grímsstaðir are relatively sparse, considering the collecting of data has been running over two seasons. But the curves are "nice" and sine like, so this data is considered fairly reliable. The same can be said about the water levels for Grímsstaðir and Selfoss. The last case involving Upptyppingar data is on the other hand expected to be slightly more spread.

First we will concentrate on the selection of pairs, then the triplets. From the selected pairs of data, one can determine the time Δt it takes for the specific high or low water levels to move from stations 162 and 102 respectively to 453 and from 162 to 102, as well as the time difference between events at 162 and 233. One can also for 102, determine the time difference Δt between selected events in water levels and suspended sediment.

The next step in the project is to determine the discharge Q at vhm 102 for all the selected events. Important is to point out, that it is always the discharge at vhm 102 which is used. The official 2005 discharge rating curve at vhm 102 is applied. In (Henriksen, Steen, 2003) an older discharge rating curve was used, so all Q data from (Henriksen, Steen, 2003) used in this report are being recalculated by the new 2005 rating curve.

To interpret the data not involving light absorption we make plots of Δt vs. Q. These plots are shown in Appendix A. Plots of the data involving light absorption are shown in Appendix C.

We were able to supplement to the project a flood occurring on the 8th of January 2002. The flood provides us with the data point (492.6, 10.08) in the Upptyppingar-Grímsstaðir case. The point is marked with a cross in the plot in Fig. 12 in Appendix A. One might observe that it fits nicely in with the rest of the data points.

Note that the scale of the Δt axes differs in the plots in Appendix A. So if one wants to compare the different cases, then Fig. 14 in Appendix A should be used. We observe that the data from Upptyppingar-Selfoss and Upptyppingar-Grímsstaðir are more spread than Grímsstaðir-Selfoss. But the most spread data are the Upptyppingar-Kreppa case. An explanation for the data being unusually spread in the latter case might be that the two gauging stations are on two *separate* rivers. It might also have influenced that measurements at Kreppa were only made each hour until the 6th of July, 2002.

In the examination of the data, we note that all data plots in Appendix A have a significantly decreasing trend. That indicates the time difference Δt decreases as the discharge Q increases. We will look deeper into that in Section 3, but for now we will just comment on it. The trend line will not be added in the plots.

Fig. 7, 8 and Fig. 10, 11 show water levels separated in lows and highs. But it does not provide us with any interesting information unique for the highs or the lows. The plots have the same characteristics as Fig. 6 and Fig. 9, a decreasing trend, though the data points might be shifted a little towards lower discharge for the lows and higher discharge for the highs, as expected.

Splitting up the data over the months June, July, August and September is done for Grímsstaðir-Selfoss and Upptyppingar-Selfoss in Fig. 15 and 16 in Appendix B. It does not seem to provide us with any new information, apart from the June data being shifted to the left of the discharge scale, the September data being shifted to the right, and July, August centred in the middle. None of the months singles out since the data from one month always overlaps with data from other months.

The data in Fig. 17 in Appendix C has a significantly decreasing trend. Again the time difference Δt tends to decrease as the discharge Q increases. But interpreting this figure is quite interesting, since Δt in this case is the time difference between water and light absorption events. This means the water level events actually move *faster* down along the river than the suspended sediment events.

In Fig. 18 and 19 in Appendix C no significant variation in the data is present. The slopes of the trend lines are minimal. These plots were made to check whether increase in suspended sediment has any effect on flow of water level events. If for example the suspended sediment would slow down the water flow the trend line would have been significantly decreasing. But we cannot make any such conclusions from data we have available.

The approach in (Jóel Karl Friðriksson, 2002) was to use hourly measurements only. This led to the statement that data of such resolution was insufficient when determining highest or lowest water level. A highest or lowest water level might occur in between two measurements. The statement was true under the, at the time, given assumptions. But in order to avoid misunderstanding we must point out that this only applies to (Jóel Karl Friðriksson, 2002). The use of hourly measurements is today possible, because we have the smoothing spline at our disposal.

3 THE THEORETICAL MODEL

In order to find a reliable approximation of the data points in the $(Q, \Delta t)$ plots, we need some understanding of the physical relationship between Q and Δt. In this section we investigate that relationship, but note that not all calculations will be given in detail. First we show that Δt is dependent of the height of the water level wave. We then find an expression for the $(Q, \Delta t)$ relationship and then in the end we make considerations on the approximation method to use.

The physics behind the relationship is so complex, that one cannot describe it completely. But turning to fluid mechanics (Roberson, J. A. and Crowe, C. T.,1997) can give us a little insight into the problem. We will use the theory of wave celerity to describe the propagation of the high/low water level events down the river. The prerequisites for using the theory are not all fulfilled, but in the way we are going to use it, that does not matter. We seek just a rough understanding of the problem.

Wave celerity c is the velocity at which an infinitesimally small wave travels relative to the fluid in which it is travelling. The wave celerity c is given by the equation

$$c = \sqrt{gy} \tag{3.1}$$

where g is the gravitational acceleration and y is the depth of the water basin. The formula is derived under the assumption that the height of the wave Δy is small compared to the depth y of the water reservoir. In the derivation of the formula a Δy term is discarded as a consequence of that assumption. At Jökulsá á Fjöllum, Δy can be around 20% of y, so in that case Δy is no longer small and the derivation of c is more tedious. If one would keep the term linear in Δy, in the derivation, c would be at the form

$$c = \sqrt{gy} + w(y, \Delta y) \tag{3.2}$$

where $w(y, \Delta y)$ is some function of products and sums of powers of y and Δy plus a constant term. In other terms, the speed of a solitary wave is equal to the square root of the product of the depth y and g plus a $f(y, \Delta y)$ correction of some kind. The correction says that c on dependent of the height of the wave Δy. The wave celerity c is a speed relative to the fluid, but the dependency also holds for the speed v of the wave relative to the ground. Now we have found the result we were looking for. If the speed v is dependent on Δy, so is Δt.

The equation (3.2) could possibly provide us with some information about the $(Q, \Delta t)$ relationship, but that would require a lot of effort as the expression is rather implicit. Instead we take an alternative approach. An approach that relatively easy will provide us with a relationship that applies directly to our problem.

The official 2005 discharge rating curve at station 102 is explicitly given by the equations

$$\begin{aligned} Q_1(h) &= 929 \cdot 10^{-5}(h-42)^{1.9882} \quad , \quad 100 < h \leq 200 \\ Q_2(h) &= 961 \cdot 10^{-5}(h-48)^{1.9969} \quad , \quad 200 < h \leq 250 \\ Q_3(h) &= 877 \cdot 10^{-5}(h-37)^{1.9942} \quad , \quad 250 < h \leq 1000 \end{aligned} \tag{3.3}$$

where h is the water level measured by the gauge and Q is the discharge. Q is measured in m^3/s and h in cm. Since we are only concerned with the orders of magnitude, we let

$$Q(h) = k(h-h_0)^p \tag{3.4}$$

represent Q_1, Q_2 and Q_3 for all water depths h. Isolating h gives

$$Q(h) = \left(\frac{Q}{k}\right)^{\frac{1}{p}} + h_0 \tag{3.5}$$

which we will need later.

A general expression for the discharge Q is given by

$$Q = vA \tag{3.6}$$

where v now is the mean velocity at the cross section and A the area of the cross section. Assume that v equals respectively the mean velocity between the measuring points vhm 102-453 and vhm 162-453. This enables us to write

$$v = \frac{\Delta x}{\Delta t} \tag{3.7}$$

where Δx is the distance between these pairs of stations along the river. Inserting this in (3.6) above, and it follows that

$$Q(h)\Delta t = \Delta x A(h) \tag{3.8}$$

Hereby we conclude that the product of Q and Δt is not constant. That means the behaviour of the $(Q, \Delta t)$ data is in theory not hyperbolic since Δt and $\frac{1}{Q}$ are non-proportional. But then to understand how the product behaves, we need to know how the area A of the cross section of the river changes as a function of h (remember Δx giving the distance between the stations is constant). To simplify we assume that water level h equals water depth in Fig. 4.

The cross section at the gauging station 102 is shown in Fig. 4, and we will now make a (rough) approximation of it. First assume that the water level is never lower than around ca. 350 cm from the bottom. This means that the gap in the bottom is always filled with water, which implies that the area a is independent of h, i.e. constant. The cross section above a will be approximated by a box pinched in between two identical triangles. The width of the box is set to be b and the height is set to be h', and the difference between h and h' is given by c. The slope of "alfa" is given by $\frac{1}{\alpha}$.

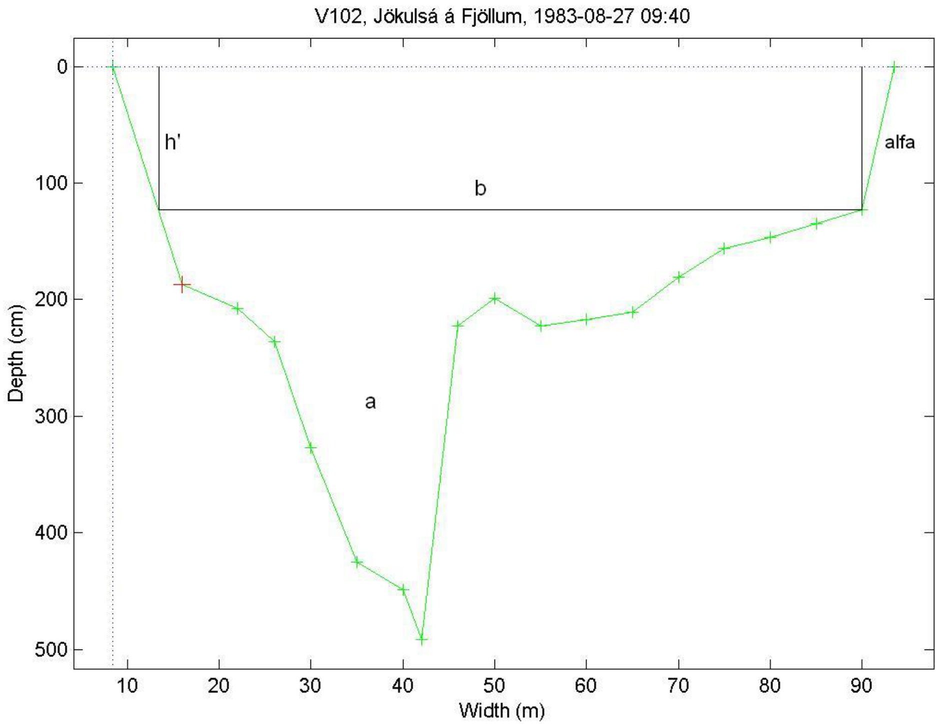

Figure 4: *Depth profile at the gauging station 102.*

We have now found a (very) simplified way to express $A(h)$, and if we combine this with equation (3.8), we get

$$\begin{aligned} Q(h)\Delta t &= \Delta x A(h') \\ &= \Delta x A(h-c) \\ &= \Delta x(a + b(h-c) + \alpha(h-c)^2) \\ &= \Delta x(\alpha c^2 + a - bc + (b-2\alpha c)h + \alpha h^2) \end{aligned} \tag{3.9}$$

Insert into this the expression $h(Q)$ from (3.5)

$$\begin{aligned} \Delta t &= \Delta x\left(\frac{\alpha c^2 + a - bc}{Q} + (b-2\alpha c)\frac{\left(\left(\frac{Q}{k}\right)^{\frac{1}{p}} + h_0\right)}{Q} + \alpha\frac{\left(\left(\frac{Q}{k}\right)^{\frac{1}{p}} + h_0\right)^2}{Q} \right) \\ &= \Delta x\left(\left[(\alpha c^2 + a - bc) + (b-2\alpha c)h_0 + \alpha h_0^{\,2}\right]\frac{1}{Q} + \left[\frac{(b-2\alpha c) + \alpha 2h_0}{k^{\frac{1}{p}}}\right]\frac{Q^{\frac{1}{p}}}{Q} + \frac{\alpha}{k^{\frac{2}{p}}}\frac{Q^{\frac{2}{p}}}{Q} \right) \\ &\approx c_1\frac{1}{Q} + c_2\frac{1}{\sqrt{Q}} + c_3 \qquad \text{(equation (3.3) use } p \approx 2) \end{aligned}$$

where c_1, c_2 and c_3 are constants. This leads us to suggest that one should try to make an approximation of the form

$$\Delta t \approx c_1 \frac{1}{Q} + c_2 \frac{1}{\sqrt{Q}} + c_3 \quad (3.10)$$

Before we start approximating the data we make some preparatory considerations. The discharge Q is given by the discharge rating curve which is a function of the water level. The rating curve itself gives the relationship between water level and discharge based on measurements and this means that there is a measurement error in the discharge and the water level. Two different top/bottoms of the same water level w might not have the same wave heights (amplitude). The equation (3.2) shows that for two different top/bottoms of the same water level w, the time it takes to move down river may be different. That means there is also an error in the time difference Δt. In the $(Q, \Delta t)$ plots one should therefore bear in mind that there are errors in both variables. This is relevant when approximating the data. Using least squares methods will not be correct since it only accounts for error in one variable. The best choice would be to use orthogonal least squares regression, since we have errors in both variables, but that is not an option in the tool we are using (Matlab R14). So due to the current lack of software, we are limited to use traditional least squares methods. Note the reasoning in trend lines (normal least squares) from Section 2 holds, because the trends are so significant in all cases that they dominate the errors in the two variables.

4 APPLICATIONS OF THE THEORETICAL MODEL

To approximate the data we will try to use power functions and sums of such functions. Both formulas (3.2) and (3.10) suggest that. The fitting results in this report are quite different from what was obtained in (Henriksen, Steen, 2003).

The curve fitting toolbox in Matlab R14 is used to make least squares approximations of the data sets. We focus on the three following functions

$$f_1(Q) = k_1 Q^p \quad (3.11)$$

$$f_2(Q) = k_1 Q^p + k_2 \quad (3.12)$$

$$f_3(Q) = k_1 Q^p + k_2 Q^q + k_3 \quad (3.13)$$

For f_1 the least square method is linear, for f_2 and f_3 the method is non linear. The fitting process using f_1 is straight forward and the function applies well to the data. We will not bother to use f_2 because when the condition $p < 0$ is applied to f_2 the fit converges towards the power function f_1. The function f_3 is more complex. A lot of effort in adjusting initial values and different fitting constraints has to be set in order to get a good fit.

Though the errors for a f_3 fit are a little smaller than for a f_1 fit, we choose to use the f_1 fit. That is because the f_1 fit is so much quicker and easy to obtain, and for our purpose the data is approximated satisfactorily by the function. If in the future a more detailed research should be required, one might suggest trying f_3. The disadvantage of f_3 is that the function does not apply for very low discharge, but in our case that is actually not a problem since the discharge for Jökulsá á Fjöllum hardly ever goes below approximately 90 m^3/s.

A f_1 and f_3 fit are shown in Fig. 5. The dotted lines show the 95% confidence levels. Note that the confidence level for f_3 is very wide until discharge reached around 80 m^3/s.

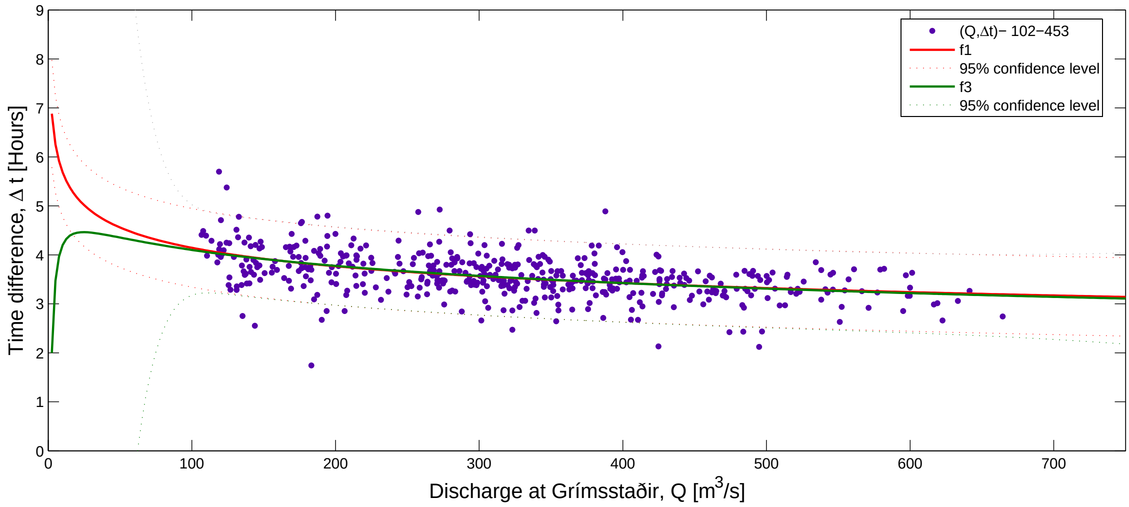

Figure 5: *Fits with prediction bounds. Grímsstaðir-Selfoss, all data points.*

To check different possibilities we have also tried fitting with exponential functions, sums of such functions and even n'th-degree polynomials. However, they did not approximate the data as good as the power functions f_1 and f_3.

During the fitting process the curve fitting toolbox options for f_1 were set as follows. Robust was off, the algorithm was Trust-Region and the remaining settings were default. We will now give the final results of the fittings. They are visualized in Fig. 20 in Appendix D.

In the following formulas Q is measured in m^3/s and $f_1(Q)$ in hours. For the Grímsstaðir-Selfoss case, the function

$$f_1^{GS}(Q) = 7.81Q^{-0.1376} \tag{3.14}$$

is found as an approximation to the data set. In the Upptyppingar-Selfoss case, the function

$$f_1^{US}(Q) = 32.15Q^{-0.1415} \tag{3.15}$$

is found, and in the Upptyppingar-Grímsstaðir case

$$f_1^{UG}(Q) = 25.15Q^{-0.1481} \tag{3.16}$$

There is a dependency in wave height, so $p \neq -0.5$ as it would be expected for the power p. Rather, the power p seems to be close to $p = -1/7$.

We observe that Kreppa and other streams join Jökulsá á Fjöllum between Upptyppingar and Grímsstaðir/Selfoss, causing disturbances to the water level cycles. This has the influence on the data set in the way that it becomes spread, at least much more than in the case of Grímsstaðir-Selfoss where no significant streams run into the river. The more spread the data is in the vertical direction, the "steeper" a fit the least square approximation will try to make through the data points.

We observe that $f_1(Q) \to 0$ as $Q \to \infty$. This is reasonable since we would not expect the time difference to be negative, at least not in the three cases we just examined.

The Upptyppingar-Kreppa case is a special case in the sense that $\Delta t = t_{233} - t_{162}$ takes on both positive and negative values (see Fig. 13 in Appendix A). This means that sometimes events in Kreppa occur before Upptyppingar. The explanation is most likely that they are two separate rivers. We find an approximation to the data set given by the function

$$f_1^{UK}(Q) = 332.1Q^{-0.9336} \tag{3.17}$$

According to f_1^{UK} there is trend towards Δt being positive. This can of course be disputed since some values of Δt are negative. The black f_1^{UK} curve is the "steepest" of the four curves, but the data is also very spread in the vertical direction. This case is very difficult to approximate.

5 CONCLUSION

In conclusion, the consensus of this report arrives to the point that the Grímsstaðir-Selfoss data is by far the most reliable. The function f_1 gives quite a good description of this data. In the Upptyppingar-Selfoss, Upptyppingar-Grímsstaðir and Upptyppingar-Kreppa cases the amount of data are fewer and more spread. Though the data is difficult to approximate, f_1 still provides a fairly acceptable approximation. An alternative function f_3 exists.

Furthermore we have concluded that the water level events actually move *faster* down the river than the corresponding suspended sediment events. A suspended sediment event is never overtaken by a water level event.

Increase in suspended sediment does not seem to have any effect on the flow velocity of water level events, to examine this issue further we can only ask for more data.

To improve the results of this report one could try using orthogonal least squares regression instead of standard regression. One could also investigate the function f_3 closer.

It is important to be aware of the fact that this analysis only applies to Jökulsá á Fjöllum. Other rivers might have a very different behaviour.

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Appendix A, $(Q, \Delta t)$-plots

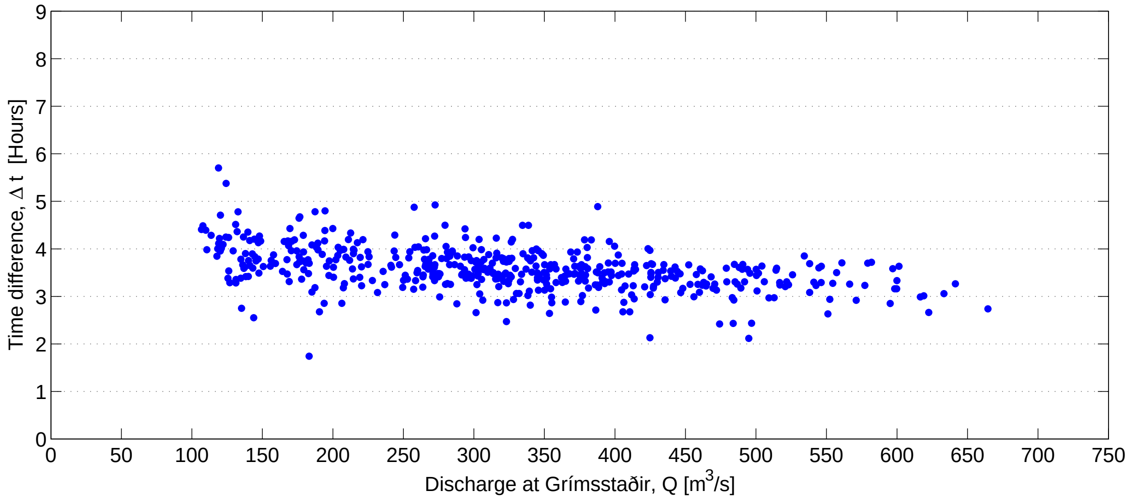

Figure 6: *Grímsstaðir-Selfoss, all data points.*

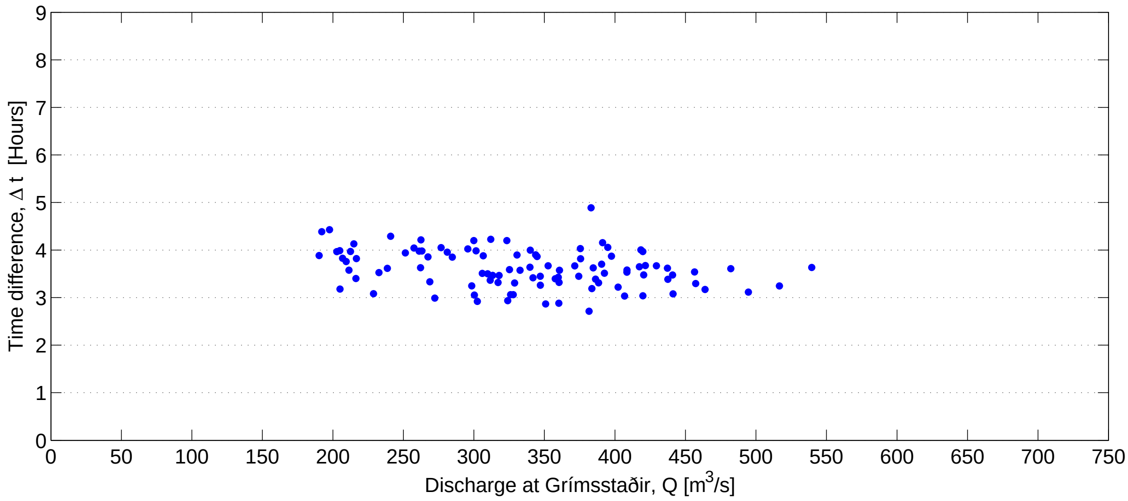

Figure 7: *Grímsstaðir-Selfoss, high water level.*

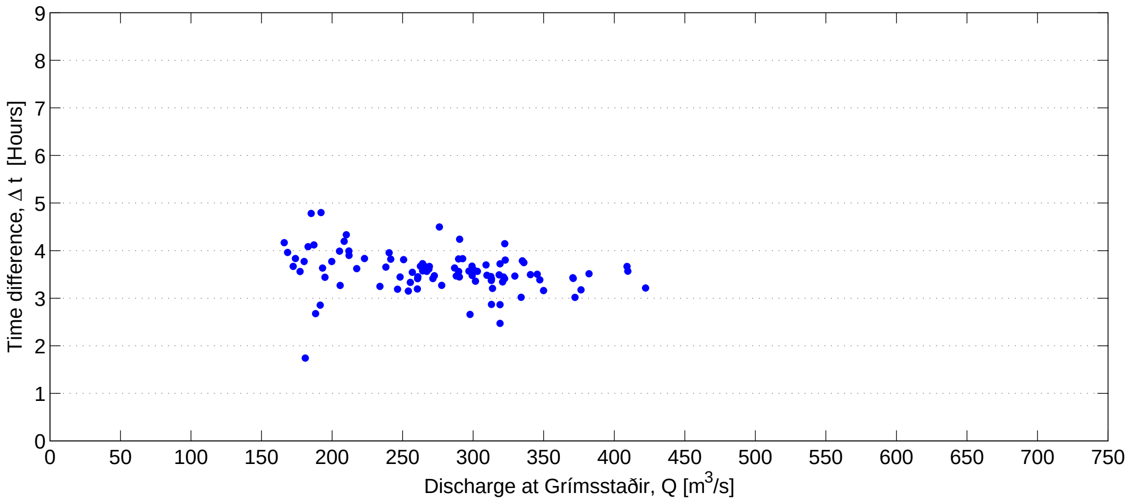

Figure 8: *Grímsstaðir-Selfoss, low water level.*

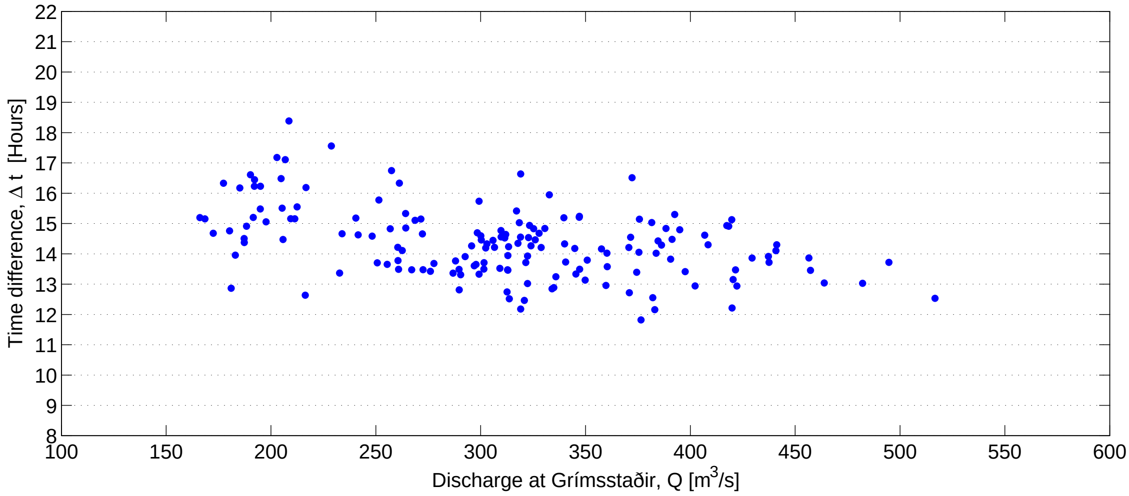

Figure 9: *Upptyppingar-Selfoss, all data points.*

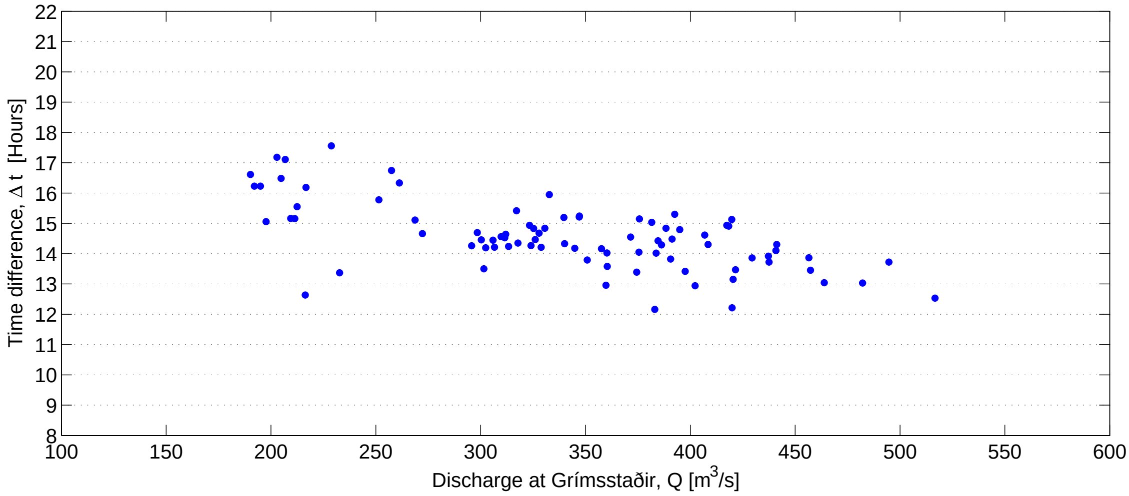

Figure 10: *Upptyppingar-Selfoss, high water level data.*

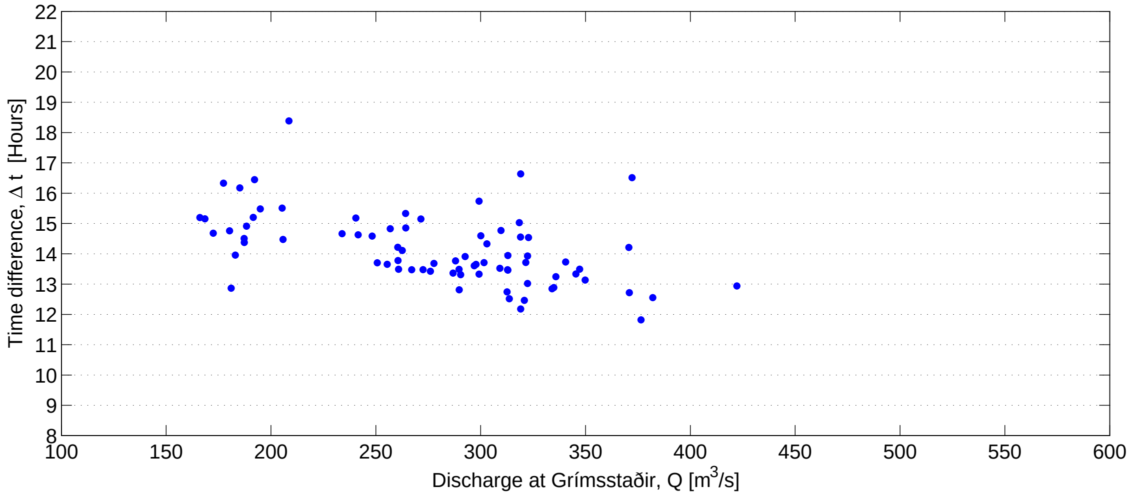

Figure 11: *Upptyppinger-Selfoss, low water level data.*

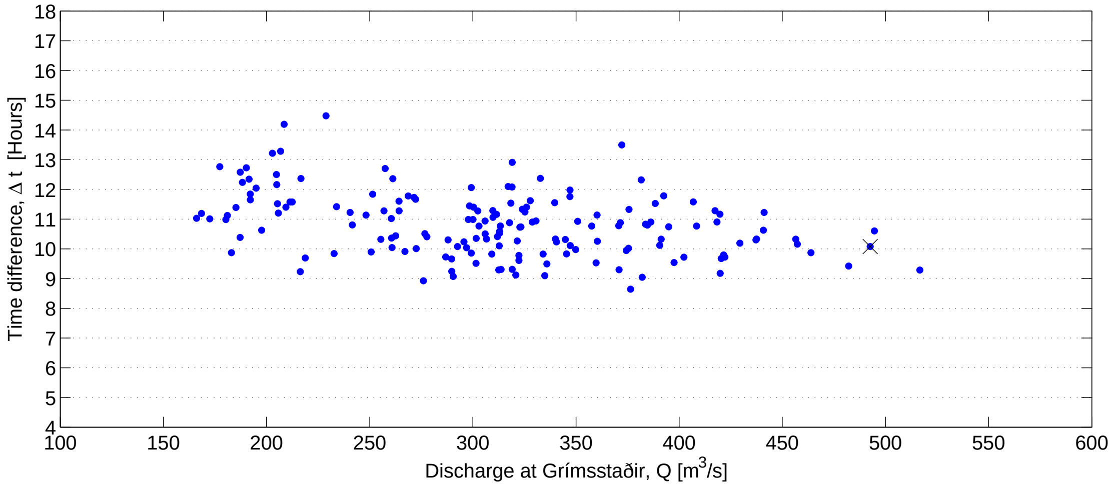

Figure 12: *Upptyppingar-Grímsstaðir, all data points.*

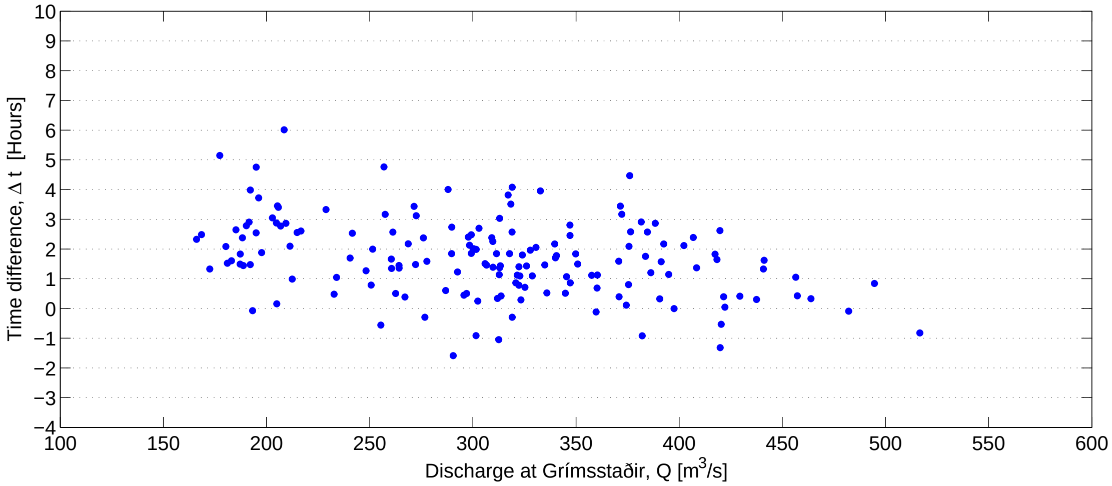

Figure 13: *Upptyppingar-Kreppa, all data points.*

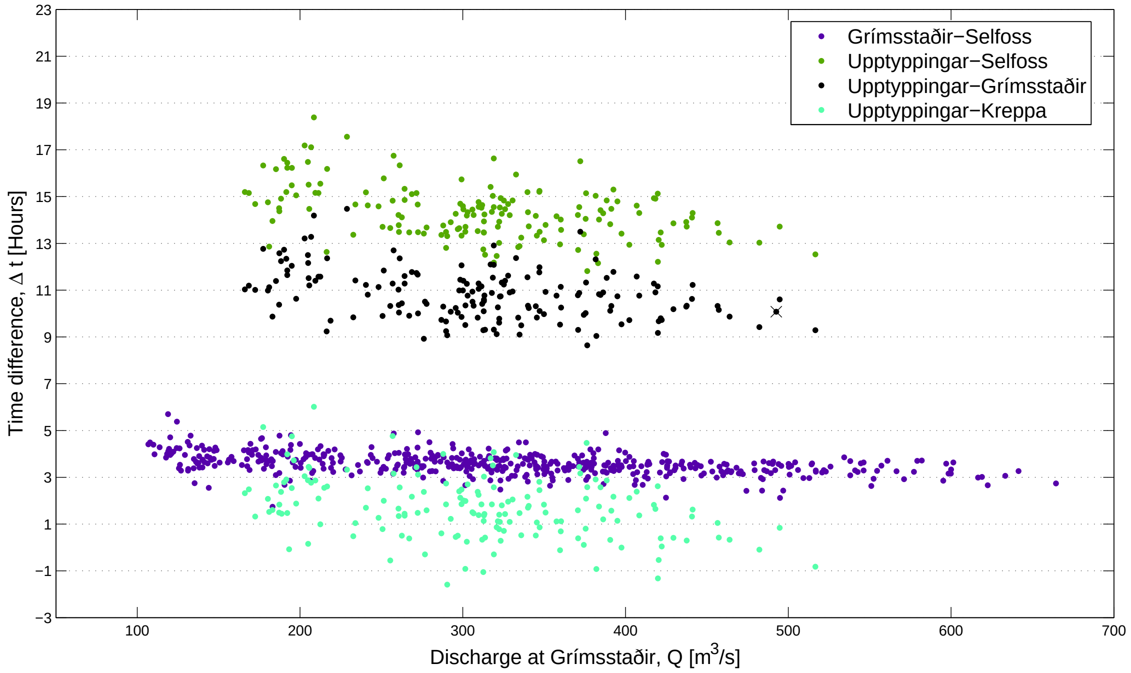

Figure 14: *All data sets.*

Appendix B, Monthly $(Q, \Delta t)$-plots

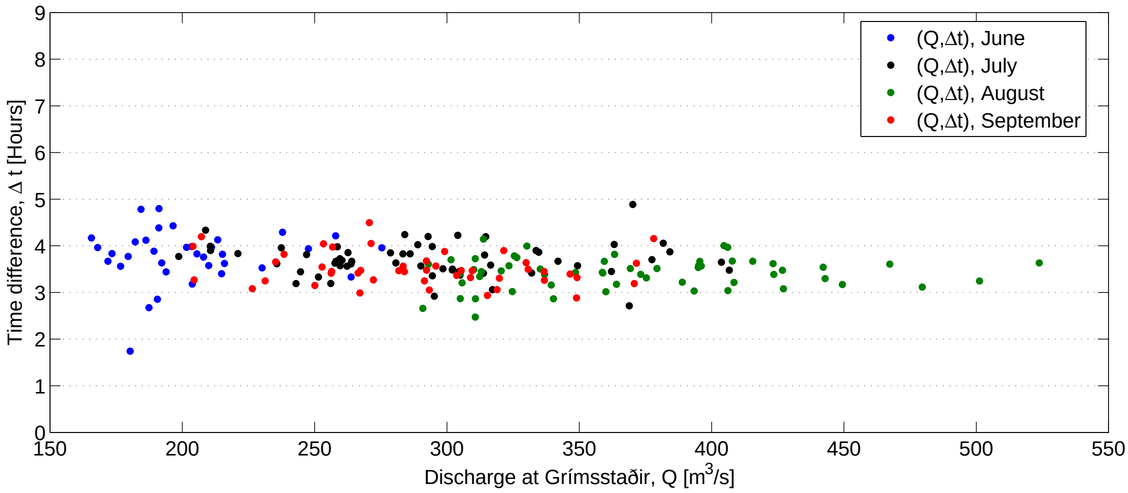

Figure 15: *Monthly data sets for Grímsstaðir-Selfoss.*

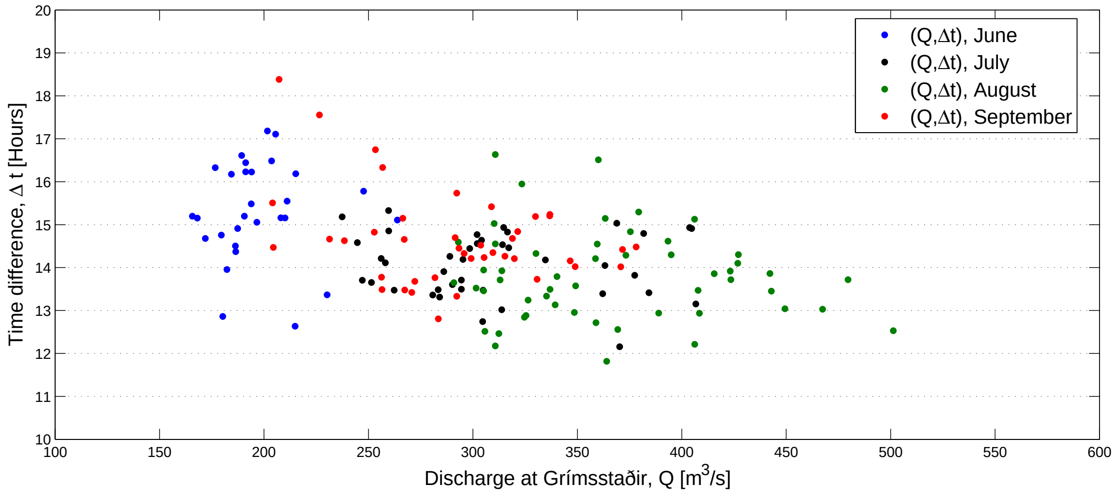

Figure 16: *Monthly data sets for Upptyppingar-Selfoss*

Appendix C, Light absorption $(Q, \Delta t)$-plots

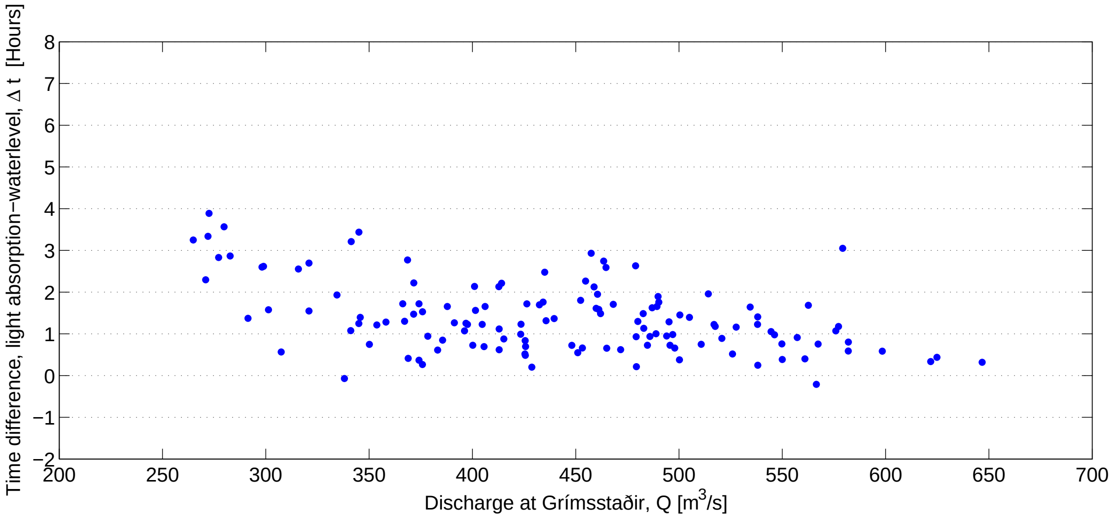

Figure 17: *Time difference in light absorption and water level vs. discharge.*

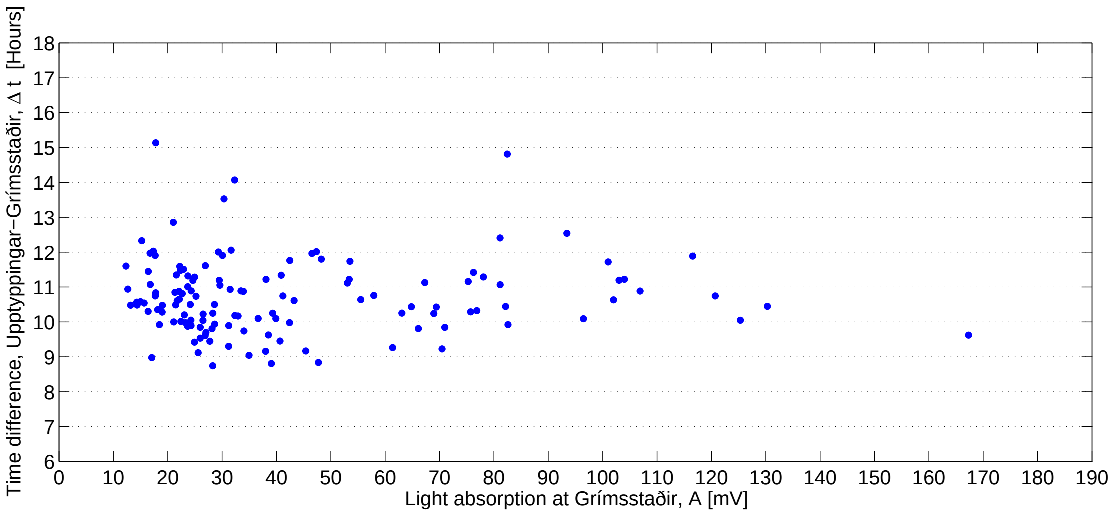

Figure 18: *Time difference in water level events vs. light absorption.*

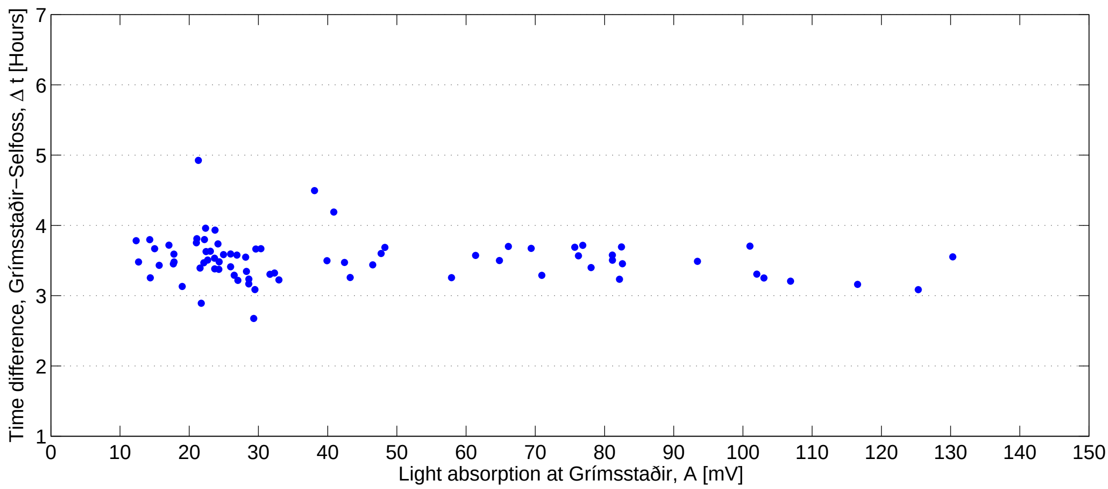

Figure 19: *Time difference in water level events vs. light absorption.*

Appendix D, Approximations

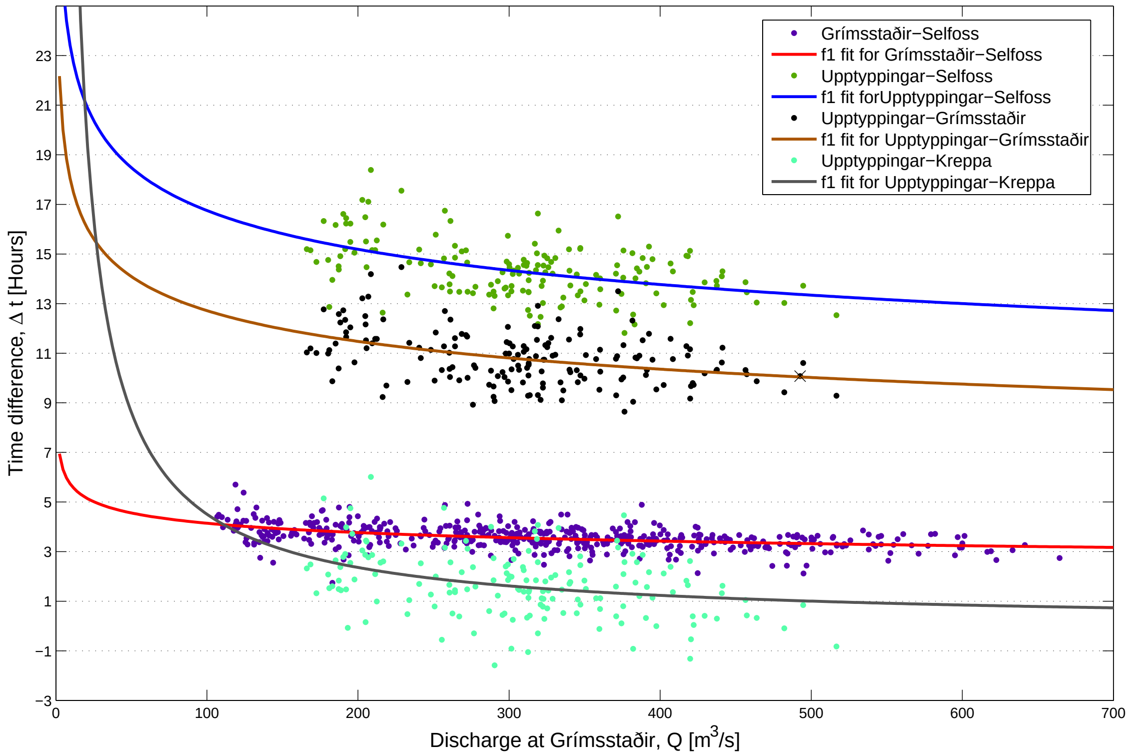

Figure 20: *Approximations of the data sets.*