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Prices vs. Quantities: The Irrelevance of Irreversibility*

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Abstract

We explore the efficacy of price and quantity controls in a dynamic setup in which the decisions of some agents are irreversible. The assumption of irreversibility is shown to improve the performance of a tax relative to that of a system of tradable quotas and significantly alter the equilibrium behavior of agents. We nevertheless conclude that taking into account the fact that agents' decisions may be irreversible does not lead to policy implications significantly different from those reached in a simpler model in which irreversibility is ignored.

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JEL classification: *D*81; *D*9; *H*23; *L*51; *Q*28; *Q*38

I. Introduction

Recent research has shown that irreversibility matters; it matters for behavior at the micro level and it is a key concept in explaining many important, aggregate economic phenomena; see e.g. Dixit and Pindyck (1994). It therefore seems likely that irreversibility also matters for the choice of policy instruments. We examine this conjecture by re-examining the old question of "prices versus quantities?", as posed by Weitzman (1974) in his seminal study of optimal regulation. He studied the costs and benefits of two market-based instruments—taxes and quotas—in a static setup in which some amount of a certain good can be produced at a given cost, yielding well-defined benefits. In this setting, he demonstrated that uncertainty with respect to the costs of producing the good affects the choice between a

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price and a quantity control: controlling the quantity makes the marginal cost uncertain, whereas a price control leaves the quantity produced uncertain. Weitzman showed that if the goal is to maximize net benefits—i.e., the benefit of availability of the good less the cost of its production—then (under certain assumptions) a quantity control performs better than a price control if and only if the marginal benefit curve is steeper than the marginal cost curve.[1] In this paper we extend the framework of Weitzman and others to allow for the possibility that some agents under regulation face irreversible decisions.

We start from the observation that the two types of policy instruments differ, not only because a quantity control regulates the availability of the good while a price control regulates its costs, but also in the way they affect the dynamic characteristics of agents' opportunity cost.[2] Consider, for instance, Weitzman's original motivating example concerning the question of whether it would be better to control certain forms of pollution by a system of tradable emission quotas or by charging pollution taxes.[3] Suppose a producer can, at each point in time, choose between emitting a pollutant by holding emission quotas or by investing in abatement equipment. With reference to the theory of irreversible investment, under uncertainty the degree of irreversibility of the investment decision may then be expected to have an important effect: the stronger the irreversibility, the higher the return on investment at the moment of decision. If abatement equipment is long lived and cannot be used for anything else, then—for investment to be rational—a substantial premium has to be observed relative to the case where equipment is short-lived, or can be sold off at a price close to the value at installment. Consequently, if the market price of emission quotas is highly variable, producers will avoid abatement technologies that involve a large amount of sunk costs.[4] If emissions are instead controlled by taxes,

[1] See Stavins (1996) and Hoel (1998) for discussions. According to Stavins, parts of Weitzman's insight can be traced back to Lerner (1971) and Upton (1971), and were formalized independently by a number of other authors, including Adar and Griffin (1976), Fishelson (1976) and Roberts and Spence (1976).

[2] Opportunity cost uncertainty also matters if agents are risk averse; see Baldursson and von der Fehr (2004).

[3] Much of the literature on the Weitzman-type problem does in fact originate from the study of environmental management; see Fisher, Barret, Bohm, Kuroda, Mubazi, Shah and Stavins (1996) for references. Stavins (1999) contains a review and evaluation of the experience with market-based instruments. One strand of this literature has studied how the choice of policy instruments affects incentives to adopt new technologies; see e.g. Magat (1978), Milliman and Prince (1989) and Jung, Krutilla and Boyd (1996). Irreversibility would seem to be particularly relevant in such decisions.

[4] Chao and Wilson (1993) provide an analysis of a firm's choice of abatement technology when facing a stochastic permit price. They also discuss the extent to which abatement technologies differ with regard to sunk costs. Saphores and Carr (2000) and Xepapadeas (1999) also use the irreversible investment approach to study related issues.

such uncertainty would not arise as the opportunity cost of pollution (or investment foregone) will then equal the tax rate.[5]

We have chosen a modeling framework that highlights the difference between high and low irreversibility as clearly as possible. Specifically, there are two technologies for abatement. The first technology does not involve sunk costs and consequently the amount of abatement undertaken may be varied over time at no additional cost. The other technology is completely irreversible, so that once investment has taken place, costs are entirely sunk.[6] Variation in quota demand may originate from the cost of abatement, the number of polluters, or other product and factor market sources.

In Section II we illustrate the main insights of our analysis in a simple two-period model: the presence of irreversibility does not affect the optimal tax or quota; it nevertheless improves the performance of a tax relative to that of a quota, although the effect is quantitatively modest. In Sections III–VI we investigate the robustness of these results in a fully dynamic model. In Section IV we find that when the damage of emissions is in direct proportion to their flow an emission tax minimizes the total cost of emissions and their abatement. At the other extreme, with a critical level of emissions such that if exceeded damages increase without bounds, a system of tradable quotas is optimal, as we show in Section V. Next, in Section VI, we consider intermediate cases where marginal damages are bounded but increase with the level of emissions. Irreversibility then has profound effects on the dynamic evolution of the economy; in particular, investment occurs later and the market price of quotas is more volatile than with perfectly reversible technologies. However, the issue of irreversibility is of limited importance for the choice of policy. We illustrate these points in Section VII by considering a parameterized and numerical version of the dynamic model. While our analysis is strictly speaking only illustrative, the underlying intuition would seem to point to a more general result.

II. A Two-period Model

We start by considering a simple model, designed so as to bring out the fundamental results in a transparent setting.

[5] A similar observation may be made in the case of non-tradable emission quotas. In such a regime, the opportunity cost of abatement equals zero when emissions are below allowable levels and is (infinitely) large beyond those levels; consequently, agents will choose the abatement technology that minimizes the costs of reducing emissions to allowable levels.

[6] For example, in electricity generation, some firms reduce SO_2 emissions by making costly investment in scrubbers, others by using low-sulfur coal; see Schmalensee, Joskow, Ellerman, Montero and Bailey (1998) and Chao and Wilson (1993).

We consider a model with two types of firms with access to different abatement technologies.[7] Firms of type I may reduce emissions by irreversible investment, where the cost of reducing emissions by k is given, in annuity terms, by $q(k)=0.5k^2$ (this implies that investment cost is given by $q[1+r]/r$). Firms of type R, on the other hand, do not face irreversible abatement decisions but may undertake emission reduction on a fully reversible, no-sunk cost basis, with the (flow) cost of achieving abatement a expressed as $c(a)=0.5[a+\beta]^2$, where β is a random variable. For each type, there is a continuum of identical firms of total mass 1. All firms are risk-neutral profit maximizers and price takers in all markets. Prior to any abatement activities, each firm produces 1 unit of the pollutant.

The market is open for two periods. To facilitate comparision with the fully dynamic model of the next section, we let the length of period 0 be 1, while the remainder of the time horizon is summed up in period 1, which consequently is infinitely long. Uncertainty enters the model as follows: in period 0, $\beta\equiv 0$; in period 1, with equal probability, $\beta=-\Delta$ and $\beta=\Delta$. The value of β is revealed at the start of each period before firms make their abatement decisions.

Assuming symmetric behavior among firms of similar type, we can write expected discounted total cost of abatement over the two periods as follows:

$$F=\tfrac{1}{2}k_0^2+\tfrac{1}{2}a_0^2+\frac{1}{1+r}\left\{\frac{1}{2}\frac{1+r}{r}\left[\tfrac{1}{2}\left[k_1^-\right]^2+\tfrac{1}{2}\left[a_1^- -\Delta\right]^2\right]\right.$$
$$\left.+\frac{1}{2}\frac{1+r}{r}\left[\tfrac{1}{2}\left[k_1^+\right]^2+\tfrac{1}{2}\left[a_1^+ +\Delta\right]^2\right]\right\}, \tag{1}$$

where subscripts denote time periods and superscripts $-$ and $+$ denote events associated with negative and positive values of β, respectively. The first two terms represent abatement cost in period 0, while the third term represents discounted expected abatement costs in period 1.

For simplicity we assume that damages in each period are independent of cumulative emissions and depend solely on the flow of emissions. Denoting total (flow) emissions by $Y=1-k+1-a$, the (flow) social damage these emissions cause is measured by the function $d(Y)=0.5\delta Y^2$, while the expected present discounted value of damage is

$$D=\frac{\delta}{2}Y_0^2+\frac{1}{1+r}\left\{\frac{1}{2}\frac{1+r}{r}\frac{1}{2}[Y_1^-]^2+\frac{1}{2}\frac{1+r}{r}\frac{1}{2}[Y_1^+]^2\right\}. \tag{2}$$

The optimal policy aims at minimizing the total cost to society of emissions and abatement

[7] An equivalent formulation would involve one type of firm with access to two different technologies.

$$S = D + F. \tag{3}$$

In the spirit of Weitzman (1974), we restrict our analysis to two commonly considered economic instruments for the reduction of emissions: a tax τ per unit of emitted pollutant and an upper bound $\bar{Y}$ on total emissions (individual firm quotas are initially distributed in some way such that their total sum is indeed equal to $\bar{Y}$). Both parameters are assumed to be credibly fixed forever, once they are set. In general, of course, neither of these policies will be optimal when compared to a wider class of taxes and quotas which might, for example, depend on the state of affairs at each point in time (in our model, the number of type R firms and the number of type I firms that have not yet invested). There will also be sources of uncertainty other than those we have considered in our model. In particular, there is likely to be regulatory uncertainty, e.g. regarding the willingness of future governments to pursue any policy that is in effect today. We do not deal with these issues here (implicitly assuming that such uncertainty affects the tax and the quota regimes symmetrically). Furthermore, for simplification we assume that the initial capital stock in type I firms is zero, i.e., that no investment has taken place at the outset.

We first consider the quota regime. Denoting the market price of quotas by p, type R firms will abate up to the level where $c'(a) = a + \beta = p$. Given capital installed in type I firms, the necessary abatement in type R firms such that aggregate emissions are equal to $\bar{Y}$ is $a = 2 - k - \bar{Y}$, so the equilibrium quota price is $p = c'(2 - k - \bar{Y}) = 2 - k - \bar{Y} + \beta$. The desired capital stock in type I firms in period 1 is given by equating marginal cost of investment to price of quotas: $q'(k) = k = p_1$. However, due to the irreversibility constraint, this is only possible if the desired stock is at least as high as the capital stock carried over from period 0. This constraint is only binding when $\beta = -\Delta$ and the quota price is low. Therefore, $k_1^- = k_0$ while $q'(k_1^+) = k_1^+ = p_1^+ = 2 - k_1^+ - \bar{Y} + \Delta$ or $k_1^+ = 0.5[2 - \bar{Y} + \Delta]$. In period 0, type I firms will, at the margin, equate investment cost and the present value of savings on quota purchases and future investment. Hence,

$$q'(k_0) = k_0 = p_0 + \frac{1}{1+r}\left[\frac{1}{2}\frac{r+1}{r}p_1^- + \frac{1}{2}\frac{r+1}{r}k_0\right]. \tag{4}$$

Given that firms invest up to the level k_0, the term $p_1^-[1+r]/r$ represents the (marginal) savings on quota purchases if prices go down and no further investment takes place, whereas the term $k_0[1+r]/r\,(= q'(k_0)[1+r]/r)$ is the reduction in costs due to previous investment if prices go up and further investment is desirable. Solving for k_0 we get $k_0 = p_0 2r/[1+2r] + p_1^-/[1+2r]$. Since $p_1^- = 2 - k_0 - \bar{Y} - \Delta < 2 - k_0 - \bar{Y} = p_0$ we conclude that $p_1^- < k_0 < p_0$. Thus marginal investment cost is lower than the quota price in period 0, but higher than the quota price in period 1 when $\beta = -\Delta$. Substituting for p_1^- and solving for k_0 we get

$$k_0 = \frac{1}{2}[2 - \bar{Y}] - \frac{\Delta}{2[1 + 2r]}. \tag{5}$$

Given these results on the behavior of firms, minimization of total cost S leads to the optimal quota $\bar{Y}^Q = 1/[1 + 2\delta]$, while minimum total cost is

$$S^Q = \frac{1 + r}{r}\frac{\delta + 2[1 + \delta]^2}{[1 + 2\delta]^2} + \frac{\Delta^2}{r} + \frac{\Delta^2}{2[1 + 2r]}. \tag{6}$$

We now compare these results with a setting in which investments of type I firms are perfectly reversible. In this case, each period may be considered in isolation, and hence marginal costs are equalized to quota prices in each period, i.e., $c'(a) = q'(k) = p$. We find $a_0 = k_0 = 0.5[2 - \bar{Y}]$ and $a_1^{\mp} = k_1^{\pm} = 0.5[2 - \bar{Y}] \pm 0.5\Delta$. Minimizing total cost in this case, the optimal quota turns out to be the same as before, while total cost becomes

$$S^R = \frac{1 + r}{r}\frac{\delta + 2[1 + \delta]^2}{[1 + 2\delta]^2} + \frac{\Delta^2}{r}. \tag{7}$$

It is evident that irreversibility has a non-negligible effect on the dynamic behavior of firms; in particular, initial investment is lowered in order to reduce excess capacity later on. Nevertheless, the optimal quota is the same in both cases. The irreversibility constraint—which implies that marginal abatement costs are generally not equalized across technologies—leads to higher total costs. This inefficiency can only be reduced by a reallocation of abatement across technologies so as to bring marginal costs closer together. It turns out that, in the reduced form, the quota enters marginal costs linearly and with the same coefficient. Therefore, a change in the quota shifts both costs equally and hence does not affect the difference between them. More generally, the scope for controlling marginal costs will be determined by their curvature, i.e., irreversibility may affect the optimal choice of policy. We explore this issue further in later sections.

In the tax regime, the opportunity cost of investment is fixed, so there is no uncertainty at the firm level associated with investment decisions. Therefore the standard neoclassical investment model applies and type I firms will invest in period 0 so that, at the margin, the discounted savings on tax payments over the planning horizon equal the cost of abatement equipment, or, in flow terms, $q'(k) = k = \tau$. Since the tax rate is permanently fixed there will be no further investment in period 1. Similarly, abatement in each type R firm, a, is given by $c'(a) = a + \beta = \tau$. Since the value of β varies randomly in period 1, abatement, and hence emissions, in type R firms are stochastic in this period. Total emissions become $Y = 1 - k + 1 - \alpha = 2[1 - \tau] + \beta$. Expected discounted total costs of emission and abatement may be written $D = 0.5\delta\{4[1 - \tau]^2\ [1 + r]/r + \Delta^2/r\}$ and $F = \tau^2[1 + r]/r$, respectively. The optimal tax rate is $\tau^* = \delta/[1 + 2\delta]$ and the corresponding minimum total costs are

$$S^T = \frac{1 + r}{r}\frac{\delta + 2[1 + \delta]^2}{[1 + 2\delta]^2} + \frac{\delta\Delta^2}{r}. \tag{8}$$

We first note that $S^T < S^R$ if and only if $\delta < 1$. This is the Weitzman result: tax performs better than quota if and only if the slope of marginal damage is less than the slope of marginal cost. Furthermore, $S^T < S^Q$ if and only if $\delta < 1 + 0.5r/[1 + 2r]$. In other words, the range over which tax performs better than quota is larger when irreversibility is taken into account. However, this effect is modest; that is, with typical values of discount rates $1 + 0.5r/[1 + 2r] \approx 1$. Note also that, in this particular case, the degree of uncertainty does not affect the choice of policy instrument.

III. A Dynamic Model

The two-period setup analyzed in the preceding section does not allow for realistic modeling of the dynamic aspects of irreversible investment in the presence of uncertainty. We now address this issue. It is well known that dynamic models involving stochastic processes quickly become analytically intractable. Therefore, we assume away all aspects not absolutely crucial for our analysis.[8] In particular, this also leads to a slightly different structure of the model from that considered above.[9] It does not seem likely that the simplifying assumptions significantly affect the nature of our results. Indeed, the fact that we obtain essentially the same results in the two models is in itself an indication of their robustness.

We again consider a model with two types of firms, type I and type R, all of which are infinitesimally small, risk-neutral profit maximizers and price takers in all markets. Apart from certain technical conditions, the main difference relative to the setup considered above is that uncertainty now originates from the number of type R firms. Below we set out the analytical details of the dynamic model.

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[8] A list of such issues would include stock effects, derivative effects, lending and borrowing, futures markets and lead times for the implementation of investment decisions.

[9] See Baldursson and von der Fehr (2003) for a discussion of modeling choices.

For a firm of type I, the total cost of investing an amount k in abatement equipment is, in annuity terms, $q(k) > 0$. We assume that $q(k)$ is increasing, convex and twice continuously differentiable. We also assume that there is a bound to feasible abatement, i.e., $q'(k) \uparrow \infty$ as $k \uparrow 1$. Let K_t denote investment at time t by type I firms. It is clear that capital stock must be increasing in t, but not necessarily continuous, since firms may invest a positive amount at the same time.

For a firm of type R, the flow cost of achieving abatement a is $c(a) \geq 0$. The function c is increasing, convex and twice continuously differentiable, with marginal cost starting at zero (i.e., $c'(0) = 0$) and going to infinity as abatement possibilities are exhausted (i.e., $c'(a) \uparrow \infty$ as $a \uparrow 1$). The mass of firms of type R at time t, denoted by N_t, varies exogenously and is given by a geometric Brownian motion process with drift parameter μ and diffusion coefficient σ:

$$dN_t = \mu N_t dt + \sigma N_t dW_t, \tag{9}$$

where $W = \{W_t, t \geq 0\}$ is a standard Brownian motion process. Note that if all type R firms choose the same level of abatement a and their number is n, then total emissions of type R firms are given by $n[1-a]$.

We assume that if the future path of total emissions is given by the stochastic process $Y = \{Y_t, t \geq 0\}$, the social damage these emissions cause may be measured by a functional D, which depends on Y. To facilitate comparison with standard static models, we assume that damages in each period are independent of cumulative emissions and depend on the flow of emissions only. As will be seen below, in this setting taxes and quotas can both be obtained as optimal policies, depending on the shape of the damage function.[10] The damage functional can then be written

$$D = E_{k,n}\left\{\int_0^\infty e^{-rt} d(Y_t)dt\right\}. \tag{10}$$

The subscripts k and n on the expectations operator indicate conditioning on the event $\{K_0 = k, N_0 = n\}$. The function d is increasing, convex and twice continously differentiable.

Suppose a particular emissions process Y is the result of an investment process in abatement equipment $K = \{K_t, t \geq 0\}$ for type I firms and an abatement process $A = \{A_t, t \geq 0\}$ for type R firms, i.e., $Y_t = [1-K_t] + [1-A_t]N_t$. Then the total cost of abatement is given by

[10] In a more general formulation, where damages in any given period are allowed to depend on the stock of emissions as well as on the flow, simple policy rules, such as a constant tax or a quota, are never optimal; see for instance Farzin (1996).

$$F = E_{k,n}\left[\int_0^\infty e^{-rt}\frac{q'(K_t)}{r}dK_t + \int_0^\infty e^{-rt}c(A_t)N_t dt\right], \qquad (11)$$

and consequently the total cost to society of emissions and abatement is given by

$$S = E_{k,n}\int_0^\infty e^{-rt}[d(Y_t) + q(K_t) + c(A_t)N_t]dt - \frac{1}{r}q(k). \qquad (12)$$

IV. The Tax Regime

The analysis of the tax regime proceeds essentially as in the two-period model, *viz*. marginal abatement costs are equated to the tax rate at all times, $q'(k_\tau) = c'(a_\tau) = \tau$. Since the tax rate is permanently fixed, the extent of abatement will not change from these initial decisions and there will be no further investment in abatement in type I firms nor changes in abatement in type R firms. The flow of total emissions is stochastic and given by $Y_t^\tau = [1 - k_\tau] + [1 - a_\tau]N_t$. As noted earlier, a single tax rate will in general not constitute an optimal instrument for regulating emissions. However, in the particular case in which the damage of emissions is measured simply by their expected value, the tax regime is in fact optimal. We state this as:

Proposition 1. *Suppose the social damage of emissions is in direct proportion to the present value of their volume, i.e.,* $d(Y) = \delta Y$; *then the tax rate* $\tau = \delta$ *and the corresponding, fixed levels of investment and abatement,* $K_t \equiv k_\tau$ *and* $A_t \equiv a_\tau$, *minimize S, the total cost of emissions and their abatement.*

The case considered in the proposition is the dynamic equivalent of the constant marginal benefit case of the static Weitzman-type model. In the static model, a tax is the optimal policy instrument as long as the marginal benefit of reduced emissions is independent of total emissions. In a dynamic model, more is required to obtain a similar result. First, in any given period, marginal damages must be independent of total emissions within that period. Second, marginal damages must be independent of the time profile of emissions. Under these assumptions the marginal cost of increasing emissions is the same at any given point in time, and hence subjecting firms to a tax rate equal to this marginal cost is a first-best policy.

V. The Quota Regime

We next turn to the case in which emissions are controlled by a system of tradable quotas. We begin by discussing the decentralized competitive equilibrium and indicating some of its properties. We then consider cases in which a system of tradable quotas constitutes an optimal policy.

Assume that an upper limit $0 < \bar{Y} < \infty$ has been set for the aggregate flow of emissions from all firms at each point in time and that quotas have been distributed in some way initially. We assume there are no transaction costs or other market imperfections associated with trade in quotas, so the initial distribution will not affect the resulting equilibrium.[11] Recall that abatement costs in type R firms are purely variable with no investment or adjustment costs involved. Hence, if the restriction on emissions is binding, each type R firm will simply determine its abatement at every point in time, A_t, such that variable cost of abatement equals the current market price of quotas, P_t. If the restriction is not binding, there will be no abatement in type R firms and the market price of emission quotas will be zero. Since $Y_t = [1 - K_t] + [1 - A_t]N_t$, we can write the price process as

$$P_t = c'(A_t^+), \tag{13}$$

where

$$A_t^+ = \max\{0, A_t\} = \max\left\{0, \frac{N_t + [1 - K_t] - \bar{Y}}{N_t}\right\}. \tag{14}$$

This implies that, if $N_t \leq \bar{Y} - 1 + K_t$, the restriction on total emissions is not binding and $A_t^+ = 0$, so no abatement will take place in type R firms.

Suppose that at time 0 type I firms have invested an amount k in abatement equipment. Firms now face the problem of choosing an investment strategy that will minimize the expected present value of the cost of emissions at the margin. These are given by the flow of opportunity cost of holding a quota unit, P_t, until the time of investment when a one-time outlay of $q'(k)/r$ is required. In technical terms the investment strategy is given by a stopping time θ, and the present value to be minimized is[12]

$$g(n, k|K, \theta) = E_{k,n}\left[\int_0^{\theta} P_t e^{-rt} dt + \frac{1}{r} q'(k) e^{-r\theta}\right]. \tag{15}$$

The above problem is an irreversible investment problem of the "exit" type discussed in Dixit and Pindyck (1994), but since the form of the cost flow $P_t = c'(A_t^+)$ as a function of the driving stochastic process N is unusual,

[11] Atkinson and Tietenberg (1991) provide a general discussion of market failures in emission trading. See Stavins (1995) for a formal analysis of tradable quotas in a setting with market transaction costs.

[12] A stopping time is a rule that indicates at each point in time whether to stop or not, based only on observations of random events made up to that point. In other words, it is not prescient.

we cannot appeal directly to any previously solved problem of this type and have to work out a solution from first principles. This yields the behavior of an individual firm given the aggregate investment process K. The resulting competitive equilibrium may be derived by similar methods, as in Leahy (1993) and Baldursson and Karatzas (1997). The technical details are given in Baldursson and von der Fehr (2003).

The following proposition summarizes the connections between the competitive equilibrium, the individual investor's problem and the social optimum. In particular, the social optimality of the cumulative investment process generated by a competitive quota regime is established.

Proposition 2. (*i*) *The competitive equilibrium cumulative investment process generated by a decentralized quota regime minimizes the aggregate economic costs of abatement.*

(*ii*) *The optimal investment time for an investor in the decentralized quota regime coincides with the first time investment is made in the social optimum.*

Corollary 1. *If the social damage of emissions increases without bounds when emissions exceed a certain level* $\hat{Y}$, *and is sufficiently low below that level, then a decentralized quota regime with the aggregate quota* $\bar{Y} = \hat{Y}$ *minimizes the total cost of emissions and their abatement.*

VI. Taxes vs. Quotas

The analysis of the two previous sections reveals that both policy instruments may achieve the first-best solution to the problem of minimizing the total costs of emissions and their abatement. In particular, a tax is optimal in the extreme case in which marginal damage costs are independent of the total amount of emissions, while a quota is optimal at the opposite extreme where damage costs are infinitely high once a certain damage level is reached, but sufficiently low below that level. We now turn to the question of how the two policy instruments compare for intermediate cases in which marginal damage costs are bounded but depend positively on total emissions.

For concreteness, we consider a parameterization of the flow damage function, *viz.* $d(Y) = Y^{\frac{1}{\alpha}}$. Then $\alpha = 1$ corresponds to the case in which damage is proportional to the expected, present value of future emissions, but as α grows smaller, increasing weight is assigned to values of Y_t above 1. In the limit, as $\alpha \downarrow 0$, there is no damage associated with paths lying strictly below 1, while if the collection of paths straying above 1 has positive probability, the damage will be infinite.

Corresponding to the analysis of the two-period model, we define $S^T(\alpha)$ to be the minimal total cost of emissions and their abatement for a given α (and for given n and k) when emissions are controlled by an (optimal) tax. Similarly, define $S^Q(\alpha)$ to be the corresponding minimal total cost when emissions are controlled by an (optimal) quota. Note that by Corollary 1 we have $S^Q(0) < S^T(0)$, and from Proposition 1 it follows that $S^T(1) < S^Q(1)$. These functions are continuous in α and consequently there exists at least one value $\alpha^*_{TQ} \in (0, 1)$ at which they are equal.

Next, define $S^R(\alpha)$ to be the cost corresponding to $S^Q(\alpha)$ in an otherwise identical model except that the investment decisions of type I firms are completely reversible, i.e., there are no sunk costs. Note that since irreversibility represents a restriction on the investment process, $S^R(\alpha) \leq S^Q(\alpha)$. This implies that the range of parameter values for which an (optimal) tax system performs better than an (optimal) tradable quota system is greater when type I firms' abatement costs are sunk than when they are not. We state this result as:

Proposition 3. *Let* $\Omega^{TQ} = \{\alpha | S^T(\alpha) \leq S^Q(\alpha)\}$ *and* $\Omega^{TR} = \{\alpha | S^T(\alpha) \leq S^R(\alpha)\}$. *Then* $\Omega^{TQ} \supseteq \Omega^{TR}$.

Conjecturing the existence of a unique $\alpha^*_{TQ} \in (0, 1)$ such that $S^Q(\alpha^*_{TQ}) = S^T(\alpha^*_{TQ})$, and that $S^T(\alpha)$ is downward sloping, we would have $\alpha^*_{TQ} \leq \alpha^*_{TR}$. From an intuitive point of view such a result seems reasonable. The smaller the slope of the marginal damage function, the less costly it is to allow large volumes of emissions. Consequently, the tax instrument performs relatively better compared to the quota instrument the larger α is. As shown below, this intuition is confirmed in some numerical examples. However, we have not been able to demonstrate that this is true in general and, more specifically, we cannot prove that there is a unique $\alpha^*_{TQ} \in (0, 1)$ or that $S^T(\alpha)$ is downward sloping.

VII. A Numerical Example

To illustrate the properties of the dynamic model, we analyze a particular parameterization by numerical methods. First, we show the difference in the market outcome over time with and without the assumption of irreversibility imposed on type I firms. Subsequently, we demonstrate that, in spite of this, the expected present value of social cost, and hence policy choices, are hardly affected. Details are given in Baldursson and von der Fehr (2003).

In our baseline case, we use the following functional forms and parameters: $N_0 = 1$, $q'(k) = k/[1-k]$, $c'(a) = a/[1-a]$, $d(Y) = Y^{\frac{1}{\alpha}}$, $r = 0.05$, $\mu = 0$ and $\sigma = 0.2$. The choice of parameter values is, of course, to some extent arbitrary. However, a sensitivity analysis shows that the results are robust with respect to substantial variations in these values.

As we have seen, irreversibility has no bearing in the case of tax regulation. Therefore, we concentrate on the study of the quota regime. The quota is set at $\bar{Y} = 1$ and the initial abatement capital stock is $K_0 = 0$. Note that initially the two groups of firms are symmetric, with regard to both the mass of firms ($N_0 = 1$) as well as the level and shape of the abatement (flow) cost functions.

Figure 1 shows the investment path for a particular realization of N. Some investment is triggered immediately, and the capital stock in abatement equipment continues to grow as the number of type R firms, and hence the price of quotas, increases. As time goes by, and the capital stock has reached a certain level, there may well be long periods during which no investment takes place.

For comparison, the figure also includes the investment path in a corresponding model in which type I firms' abatement investments are completely reversible. In such a model the total amount of abatement equipment installed will, of course, vary over time in response to fluctuations in the price of quotas. As a consequence there will typically be periods in which the stock of abatement capital is smaller than in the irreversibility case. In particular, when abatement costs are sunk the average capital stock will tend to be lower early on, and correspondingly larger in later periods, relative to the outcome in the case in which investments are completely reversible.

The sluggishness of adjustment in type I firms when investment is irreversible is translated into a more volatile price—both locally and globally—for quotas in this case; in particular, there may be periods in which the aggregate constraint on emissions is non-binding and the quota price falls to zero. The evolution of capital stocks is consequently mirrored in the development of

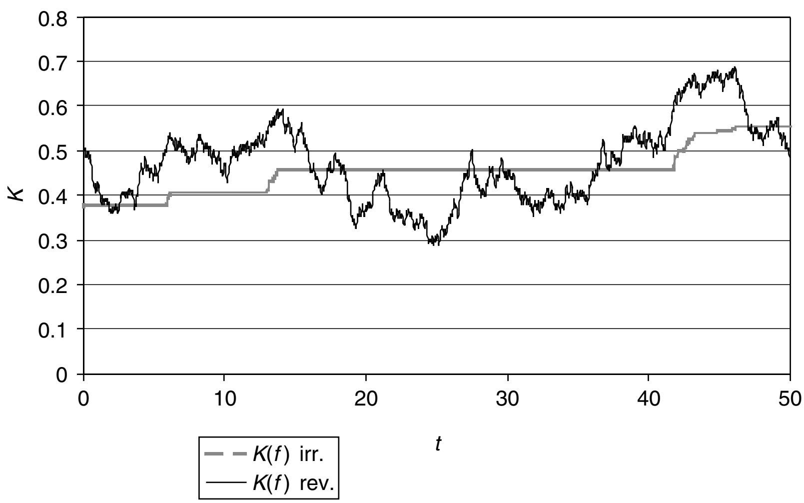

Fig. 1. Investment paths in type I firms

price levels. Compared to the case when abatement decisions are reversible the quota price is typically higher early on, and correspondingly lower later, when costs are sunk. Nevertheless, over time average prices are quite similar.

The quota price at which investment is triggered, $p(\bar{n}(k))$, is well above the flow cost of abatement investments, $q'(k)$, reflecting the expectation that the quota price will go down in the future. The relative difference may be quite considerable and is greater for low-cost firms; for example, the critical value of $[p - q']/q'$ equals 270% when $k = 0.2$, 177% when $k = 0.4$, 134% when $k = 0.6$ and 106% when $k = 0.8$.

In Figure 2, we show the numerical solution of the three minimal cost functions $S^T(\alpha)$, $S^Q(\alpha)$ and $S^R(\alpha)$ as defined in Section VI for the same functional forms and parameter values as above. $S^T(\alpha)$ is decreasing, while $S^Q(\alpha)$ and $S^R(\alpha)$ are both increasing. Consequently, there is a unique intersection of the tax and the quota curves, respectively, with the two intersection points satisfying $\alpha^*_{TQ} < \alpha^*_{TR}$.

In view of the considerable differences between the investment trigger price and the flow cost of investment, one might perhaps have expected a larger difference between $S^Q(\alpha)$ and $S^R(\alpha)$. The cost difference is quite modest, however, and so is the difference between α^*_{TQ} and α^*_{TR}. Moreover, the optimal quotas turn out to be very similar in the cases with and without irreversibility. The optimal quota is generally set larger when type I firms'

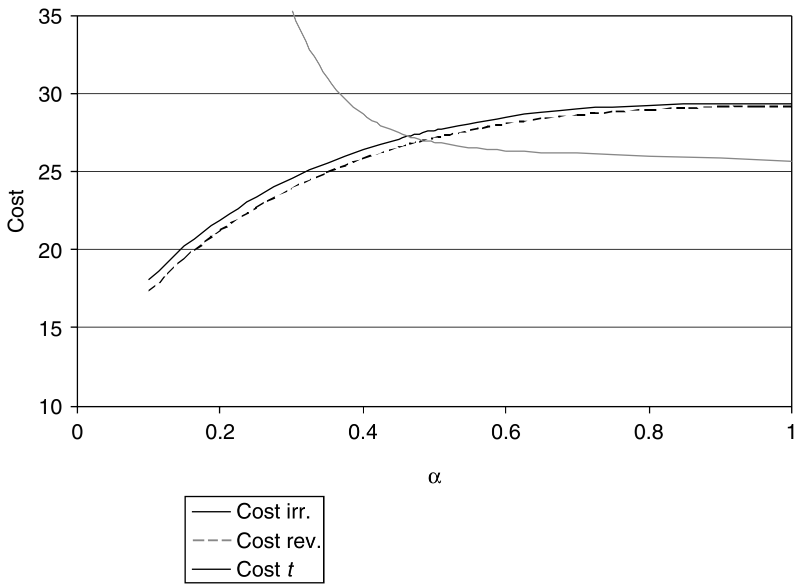

Fig. 2. Expected total costs of emissions

abatement costs are sunk, and hence their willingness to invest is lower, to avoid the quota price becoming too high and a consequently high level of costly abatement in type R firms. It turns out, however, that only a fairly modest increase in the quota is necessary to achieve this.

It would appear that the result that irreversibility is of limited importance as far as choice of policy instrument is concerned does not depend significantly on the particular parameter values and functional forms considered. In fact, the model features a number of characteristics which would seem to enhance, rather than diminish, the importance of irreversibility. First, we have assumed that the investment costs of type I firms are entirely sunk. More generally, we would expect such costs to be partly recoverable and hence the option value of having access to the quota market to be correspondingly smaller. Second, in the numerical analysis, the standard deviation of the number of type R firms equals 20%. For many industries this would represent an unusually high variability in the net entry rate and if this parameter is lowered to 10%, the assumption of irreversibility becomes practically irrelevant. Third, the extent to which variability in the number of type R firms affects quota price uncertainty depends on the flexibility of the reversible technology. We have assumed that returns to scale decrease quite rapidly at the micro level and, with less pronounced diseconomies of scale, the quota price uncertainty would have been correspondingly smaller. Lastly, we have assumed an exogenous exit and entry process for type R firms. Intuitively, endogenizing exit and entry decisions would tend to reduce the variability in the prices of quotas and this would be more pronounced in the case when investment in type I firms is irreversible. We investigated the importance of these features, as well as other modeling assumptions, by conducting a sensitivity analysis and the conclusion is that the main results are robust.

We saw in the two-period model that irreversibility was of limited importance as far as the choice of policy instrument is concerned. In particular, when marginal costs are linear and symmetric across technologies, the optimal quota is exactly the same with and without irreversibility. This is not the case in the infinite horizon model, but the difference is small. To understand why irreversibility does not matter much for choice of policy instrument here, it is important to keep in mind the different patterns of the investment and quota price paths under the two assumptions about technology in type I firms. Although patterns may differ quite considerably for a given outcome, there is a tendency for differences to cancel out on average and over time. In particular, relatively low levels of investment early on in the irreversibility case are compensated for by larger capital stock later on.[13] Issuing more emission

[13] Abel and Eberly (1999) show that the effect of irreversibility on capital accumulation under uncertainty is ambiguous; see also Bertola (1998).

quotas mitigates the added costs caused by irreversibility, and hence increases initial investments, but leads to an even higher level of abatement capital in later periods when an abundance of such capital may turn out not to be warranted. Therefore, only modest changes in policy are necessary to account for irreversibility.

VIII. Conclusion

We have extended the static framework of Weitzman (1974) and others to consider the effects on the choice between quantity controls and price controls when allowing for the fact that some agents may face irreversible decisions.

In general, when quantities are regulated by quotas, the behavior of agents at the micro level, and hence market outcomes, will be very different with and without irreversibility. In particular, investment in the irreversible technology will occur later and price paths will be more volatile than indicated by an analysis without irreversibility. However, this does not necessarily mean that on average, over time, there are large differences in investment or, indeed, costs. An immediate implication is that, from a policy point of view, the irreversibility aspect of agents' decisions may be neglected when calculating the optimal policy and deciding whether to use a price-based or a quantity-based regulatory instrument.

Our analysis has been framed within the context of environmental management and is based on a particular modeling setup. Obviously any claims to generality are limited by the simplicity and specificity of our framework. Nevertheless, the underlying intuition suggests that our irrelevance-of-irreversibility result may have captured a general feature of many policy problems.

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