Models for simulating the temporal development of Siberian larch (*Larix sibirica*) plantations in Hallormsstaður Iceland

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ABSTRACT
Siberian larch (*Larix sibirica*) is one of the main tree species used in afforestation in Hallormsstaður (65.5ºN and 14.45ºW) and the adjacent area of north-east Iceland. Models for predicting the future growth and yield of managed and unmanaged stands are necessary components of modern forest management planning systems. Pioneering work was done by Pesonen et al. (2009) in modelling the growth of Siberian larch in Hallormsstaður. However, it was recently discovered that these models may overestimate the growth of dense stands and underestimate the growth variation between trees within a stand. This article reports updated models for Siberian larch based on a larger number of observations, most of which come from permanent sample plots. The new set of models consists of an algebraic difference model for dominant height, individual-tree models for diameter increment and tree height, and a logistic model for tree survival. The new model set was found to have little bias and behave logically in long-term simulations.

Keywords: Siberian larch, growth model, individual-tree model, simulation model

YFIRLIT
Jöfnur sem lýsa vexti lerkis (Larix siberica) á mismunandi aldursskeiðum á Hallormsstað.
Síberíulerki (*Larix sibirica*) hefur verið aðaltrjátegundin í skógrækt á Hallormsstað og innanverðu Austurlandi og Norðausturlandi. Vaxtarföll sem áætla vöxt trjáa eru óaðskiljanlegur hluti af nútíma áætlanakerfi fyrir skógrækt. Án þeirra er ekki hægt að áætla lotulengd skógarins eða hvaða umhirðuaðgerðir skila mestum arði. Frumathuganir á vexti lerkis á Hallormsstað voru gerðar árið 2006 af Pesonen et al. (2009). Samanburður á þeim jöfnum og niðurstöður mælinga á föstum mæliflötum hafa hins vegar leitt í ljós að þær jöfnur ofmeta vöxt í þéttum skógi og vanmeta vaxtarmun á milli trjáa innan skógar. Í þessari grein eru birtar nýjar jöfnur fyrir síberíulerki og eru þær byggðar á stærra gagnasafni frá föstum mæliflötum sem hafa verið mældir frá 10 og upp í 50 ár. Nýju vaxtarjöfnurnar nota algebruföll við útreikninga á yfirhæð, þvermáls- og hæðarvexti einstakra trjáa og logistic aðhvarfsgreiningu til að lýsa sjálfgrisjun skógar. Til að athuga áreiðanleika nýju fallanna var hermilíkan látið framreikna vöxt skógar í 150 ár. Niðurstöðurnar gefa vísbendingar um að föllin séu vel aðlöguð, hafi litla skekkju og hegði sér rökrétt þegar vöxtur er framreiknaður langt fram í tímann.

INTRODUCTION

Siberian larch (*Larix sibirica* Ledeb.) is one of the main tree species used in afforestation in the northern and north-eastern parts of Iceland. It grows well on infertile and dry sites and has therefore been the most planted tree species in Iceland during 1945–2000 (Pétursson 1999). The first stand was planted in Hallormsstaður in 1937 (Snorrason 1986). In Iceland, Hallormsstaður and the surrounding area are the main production sites of larch, which will be a species of increasing commercial value in the coming years.

Predicting the future growth and yield of managed and unmanaged stands is necessary in modern forest management planning. Without yield models there are no means to evaluate which rotation length or thinning schedule would give the most favourable yield of different products (Pukkala & Pohjonen 1993). In recent years, forestry in Iceland has developed rapidly and the need to introduce and adapt growth models for commercial species has become more evident. Pioneering work was done by Pesonen et al. (2009) in modelling the growth of Siberian larch in Hallormsstaður and by Juntunen (2010) in modelling the growth of lodgepole pine (*Pinus contorta* Dougl.) for the whole country.

When comparing the models made by Pesonen et al. (2009) with data from permanent sample plots it was found that the models, if they were not calibrated, tended to overestimate the growth of dense stands and underestimate the growth variation between trees within a stand. New updated models were therefore required for more accurate growth calculations. The aim of this study was to develop a model set for predicting stand development. The developed set of models consisted of a dominant height model, and individual-tree models for diameter increment, tree height and tree survival. The taper model developed earlier by Heiðarsson & Pukkala (2011) complements the set of models. These models will be used in forest planning to evaluate the yield of a stand in alternative management schedules, so as to help decision making and provide better projections of Icelandic forest resources.

MATERIALS

The data for this study were collected in Hallormsstaður (65.5ºN and 14.45ºW), eastern Iceland. The mean annual temperature (1961–1990) at a synoptic station at Hallormsstaður was 3.4°C and the mean annual precipitation was 738 mm (Sigurdsson et al. 2005). For the same period, the mean maximum daytime temperatures were 12.4, 14.1 and 13.4°C in June, July and August, respectively (Sigurdsson et al. 2005). Hallormsstaður is situated 70 km from the Atlantic Ocean and thus the climate in the area is less oceanic than closer to the coast (Bløndal 2001).

Two different data sets were used for growth modelling: the data set of Pesonen et al. (henceforth referred to as the Pesonen data) collected in temporary plots (Pesonen et al. 2009) and another data set collected in permanent sample plots (PSP data). The Pesonen data were collected in 2006 and consist of 149 diameter growth observations from temporary sample plots in even-aged plantations of Siberian larch (Table 1). The past 5 year diameter increment was measured from drilled increment cores. Using these increments, the stand status of each plot 5 years earlier was reconstructed, allowing the researchers to develop a model for future growth.

The PSP data consist of measurements in 6 permanent sample plots established in even-aged plantations of Siberian larch (Table 1). Two of the plots have been remeasured at a 3-year interval. The increments were converted into 5-year increments by multiplying 6-year increments by 5/6 (Pukkala et al. 2009). The 5-year periods used were: 1952–1957, 1960–1965, 1965–1970, 1971–1976, 1977–1982, 1993–1998 and 2000–2005. The other set of PSPs was remeasured at 5-year intervals and the plots had been thinned at different densities. In these plots the 5-year periods used were: 1998–2003 and 2003–2008.

Only the PSP data were used for dominant height and survival modelling since the Pesonen data did not include information on dominant height increment and tree survival. The data set for dominant height modelling consisted of 28 pairs of stand age and dominant height measurements, with age ranging from 15 to 70 years and dominant height ranging from 5.9 to 19.3 m. The number of observations used for individual-tree height modelling was 2165. Trees that were not removed in thinning were used for survival modelling, and survivors that were not removed in thinning (1681 trees) were used in diameter increment modelling. There was mortality only among the small trees of the unthinned sample plots.

Table 1. Characteristics of the two data sets used for diameter growth modelling, Pesonen and PSP data. S.D. = standard deviation, site index is the dominant height in stand at age 80 and age is from planting.

	Pesonen (n=149)			PSP (n=1681)		
Variable	**Mean**	**S.D.**	**Range**	**Mean**	**S.D.**	**Range**
Diameter, cm	13.1	6.4	2.7–38.3	14.6	6.4	2.8–36.3
Dom. height, m	8.9	3.0	3.6–15.7	10.7	3.3	5.5–18.3
Age, years	31.7	11.7	14.0–64.0	31.0	12.4	14.0–61.0
Basal area, m^2ha^{-1}	15.8	8.4	1.0–37.6	26.5	9.4	12.1–45.0
No. of stems per ha^{-1}	1538	813	400–3900	1700	1007	540–4980
Site index, m	15.6	1.7	10.0–21.5	19.6	1.27	16.8–22.0
Increment, cm/5 yrs	2.89	0.99	0.98–6.24	1.88	0.98	0– 5.43

Pesonen data represented lower stand basal areas than the data collected from the permanent sample plots (Figure 1). In both data sets, diameter increment decreased similarly with increasing stand basal area, basal area in larger trees (BAL) and diameter at breast height (dbh) of the tree. The scatter plot (Figure 1) indicates more diameter growth in the Pesonen data than in the PSP data.

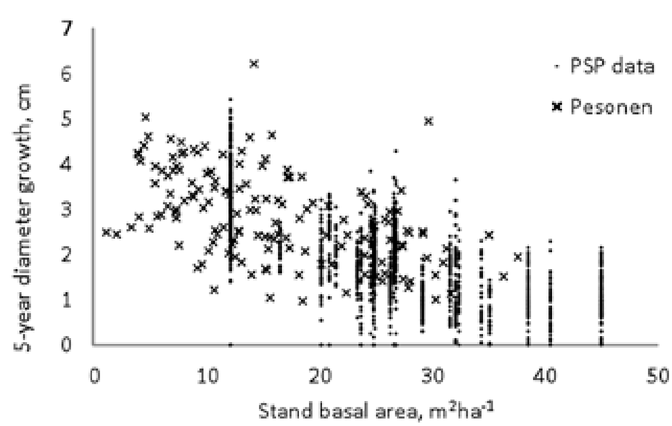

Figure 1. Relationship between diameter increment and stand basal area in the data set used for diameter increment modelling.

METHODS

Site index and dominant height modelling

Several different algebraic models developed from the equations of Chapman-Richards (Richards 1959), Schumacher (1939), McDill and Amateis (1992), and Lundqvist and Korf (Korf 1939) were tested for prediction of dominant height with age. All models predict the dominant height H_2, at a certain time point T_2, using current dominant height H_1 and current age T_1 as predictors:

$$H_{2k} = f(T_{1k}, H_{1k}, T_{2k}) + \boldsymbol{\varepsilon}_k \quad (1)$$

where subscript k refers to plot. When T_1 is replaced by index age and H_1 is replaced by site index (dominant height at index age) the model gives the dominant height at age T_2 for site index H_1. If H_1 is the measured dominant height at age T_1 and H_2 is index age, the model gives the site index.

Different models were evaluated and compared on the basis of the degree of explained variance (R^2), mean of squared errors (MSE), residuals, and biological consistency of predictions. It turned out that the model fit was very good for most of the equations but the model behaviour outside the range of the empirical data was logical only for a few equations. Therefore, the main criterion when choosing among the models was biological consistency.

Single tree height modelling

The selected tree height model was a modification of the model of Hossfeld (Peschel 1938) fitted as a non-linear mixed-effect model:

$$h_{ik} = 1.3 + \frac{a_0 + \beta_{1k} + a_1 H_k}{1 + b_1 / d_{ik} + b_2 / d_{ik}^2} + \beta_{2k} + \varepsilon_{ik} \qquad (2)$$

where h_{ik} is the height of tree *i* in plot *k* (m), d_{ik} is its diameter at breast height (cm), H_k is dominant height (m), a_0, a_1, b_1 and b_2 are constants, β_{1k} and β_{2k} are random plot factors and $\boldsymbol{\varepsilon}_{ik}$ is a residual. β_{1k}, β_{2k} and $\boldsymbol{\varepsilon}_{ik}$ are assumed to be independent and identically distributed with mean equal to zero and constant variances equal to σ^2_{plot1}, σ^2_{plot2}, σ^2_{tree}, respectively. Dominant height in the numerator allows the height curve to rise when the stand develops and its dominant height increases.

Diameter increment modelling

The intention was to develop a model for the future 5-year diameter increment, which incorporates the influence of tree size, competition and site productivity. Tree size was described by dbh, competition by stand basal area and the basal area in larger trees (m^2ha^{-1}), and site productivity by site index. Stand age and dominant height were also used, which together describe site productivity. They also describe the mean tree size and the stage of stand development. Several transformations and combinations of predictors were also tested. Non-linear regression analysis was used to fit alternative models of the following form:

$$id_{ijk} = \exp(a_0 + \beta_k + \beta_{jk} + a_1 G_{jk} \ldots) + \boldsymbol{\varepsilon}_{ijk} \qquad (3)$$

where id_{ijk} is the diameter increment of tree *i* in measurement *j* of plot *k*, β_k is random plot factor and β_{jk} is random measurement occasion factor, a_0 and a_1 are fixed parameters and $\boldsymbol{\varepsilon}_{ijk}$ is residual. G_{jk} is the stand basal area of plot *k* in measurement *j* (one potential predictor). Alternative models were tested using MSE and the statistical significance of the model and its coefficients, as well as by checking that the signs of all regression coefficients were logical and corresponded to the assumed influence of different factors. For example, increasing competition should decrease growth, whereas improving the site index should increase it. The diameter increment should decrease with older ages and larger tree diameters. Distributions of residuals were also examined for any biases.

Survival modelling

Binary logistic regression analysis was used to fit a model for the probability of a tree surviving for the next five years. The same variables that were used in diameter increment modelling were also used as potential predictors in the survival model. The resulting model was a logistic model which constrains the survival probability to vary between 0 and 1.

RESULTS

Site index and dominant height

The following model developed from Schumacher's equation had a good fit ($R^2 = 0.980$, MSE = 0.173) and logical behaviour, and was therefore selected:

$$\hat{H}_{2k} = \exp\left[\ln(H_{1k}) + 9.865\left(\frac{1}{T_{1k}^{0.604}} - \frac{1}{T_{2k}^{0.604}}\right)\right] \qquad (4)$$

Some other models, such as different modifications of the Chapman-Richards equation and the Lundqvist-Korf equation, had a slightly better fit but they gave unrealistically fast dominant height increments at young ages for sites better than those included in the modelling data. The index age was set to 80 years, which means that the site index of Siberian larch plantations in Hallormsstadur is equal to stand dominant height at 80 years. Figure 2

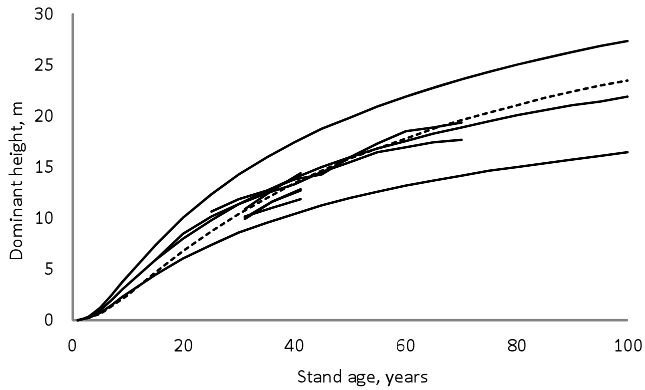

Figure 2. Dominant height development by age in site indices H80 = 15 m, 20 m and 25 m (index age is 80 years) according to the fitted dominant height model (solid curves). The model of Pesonen et al. (2009) is indicated by dashed line. The measured dominant height development in the plots used as modelling data is also shown (shorter solid lines).

shows that the model is well in line with the measured dominant height development of the permanent plots that were used as modelling data. The model is also fairly similar to the model developed earlier by Pesonen at al. (2009).

Single tree height model

The tree height model gives the heights of individual trees (m) as a function of diameter (d, cm) and stand dominant height (H, m):

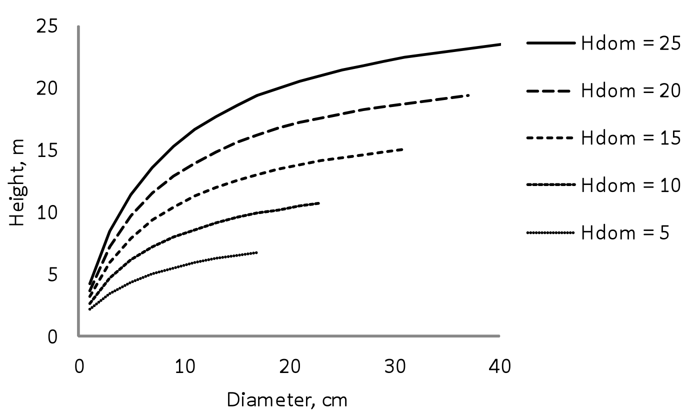

Figure 3. Modelled relationship between single tree dbh and height in an even-aged Siberian larch plantation at five different dominant heights.

$$\hat{h}_{ij} = 1.3 + \frac{3.428 + 0.931 H_j}{1 + 8.126 / d_{ij}} \quad (5)$$

The fit of the model was good (R^2 = 0.963, MSE = 0.433). Inverted squared diameter (d^{-2}) was excluded from the model as it was not significant (see Equation 2). Random plot factors were not included in the model since they were not significant (t value was 1.64 for β_{1k} and 1.84 for β_{2k}, see Equation 2). They were also not significant when included one at a time. The model and Figure 3 show that the dbh-height curve rises when dominant height increases, which means that a tree with a certain diameter is taller when it grows in an older stand.

Diameter increment model

The diameter increment model is as follows:

$$\hat{id}_{ijk} = \exp(2.525 - 0.0203 G_{jk} - 0.0705 BAL_{ijk} / \ln(d_{ijk} + 1) - 0.344 \ln(T_{jk})) \quad (6)$$

where id_{ijk} is future 5-year diameter increment (cm) of tree i in measurement *j* of plot *k*, *G* is stand basal area (m^2 ha^{-1}), *BAL* is basal area in larger trees (m^2 ha^{-1}) and *T* is stand age (years). The R^2 of the model is 0.659 and its MSE is 0.352. The random plot effect was not significant (*t* value 0.37) and the measurement effect was hardly significant (*t* value 2.47) and weak, explaining only a small fraction of the residual variance. Random effects were therefore not included in the model. Site index was not a significant predictor, due to the fact that there was very little site variation in the modelling data. As can be seen

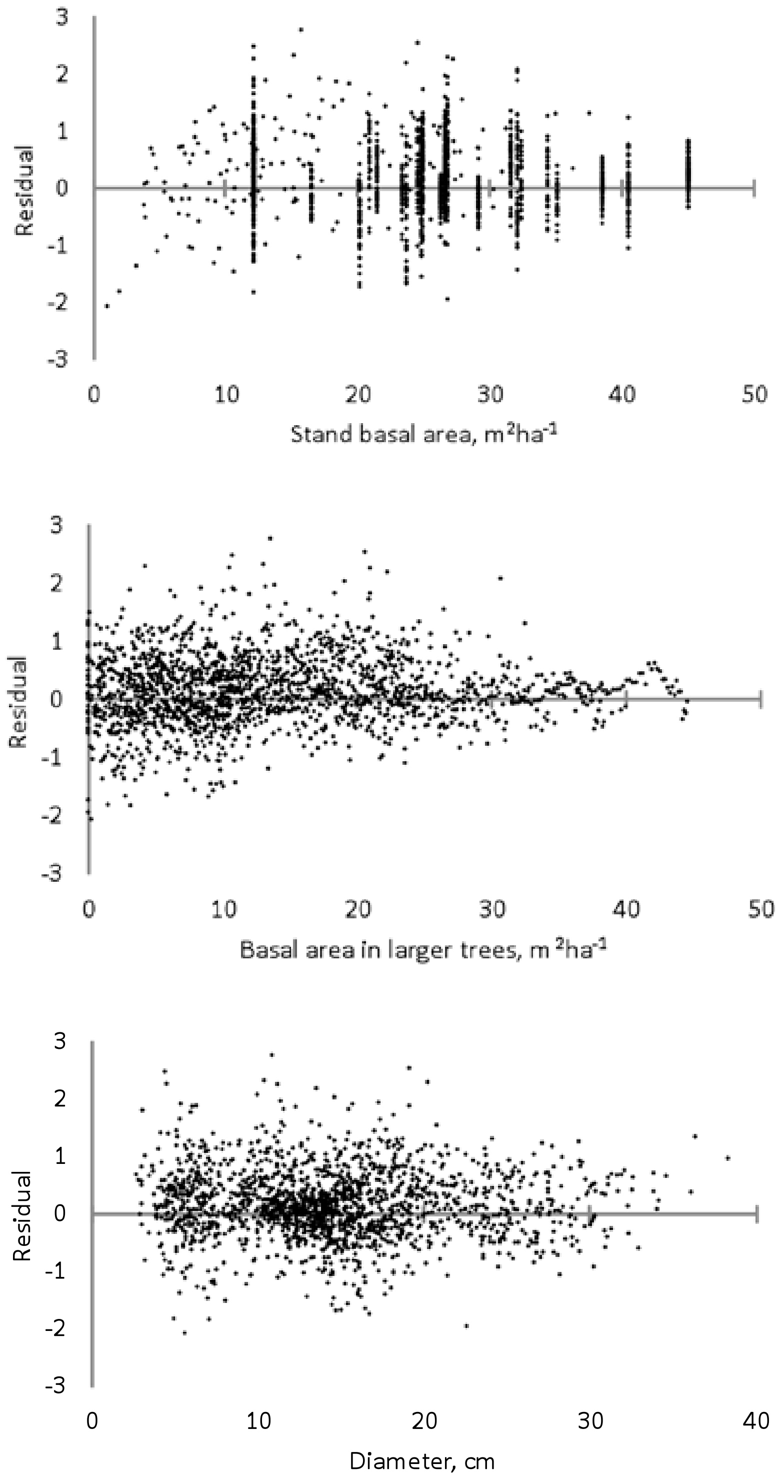

Figure 4. Residuals of the diameter increment model plotted against stand basal area (4a), basal area in larger trees (4b), and dbh(4c).

from Figure 2, the plots on which most of the PSP data were collected, had a site index close to 20 m. There was more site variation in the Pesonen data but since this data set included only 149 diameter growth observations, which were much less than the 1681 observations in the PSP data-set, its influence on model parameters and their significance was very small.

According to the model, diameter increment decreases with increasing basal area (G) and basal area in larger trees (BAL). However, the influence of BAL becomes smaller when tree diameter increases [i.e. $BAL/\ln(d+1)$ decreases with increasing diameter]. Within a certain stand, BAL decreases from G to zero when diameter increases from the minimum diameter to the maximum of the stand. As a consequence, competition affects most the growth of small and suppressed trees. Diameter increment also decreases with increasing stand age. The residuals of the model show a constant variance when plotted against G, BAL or dbh (Figure 4). Analysis of the residuals show-ed that prediction errors did not correlate with G, BAL and dbh.

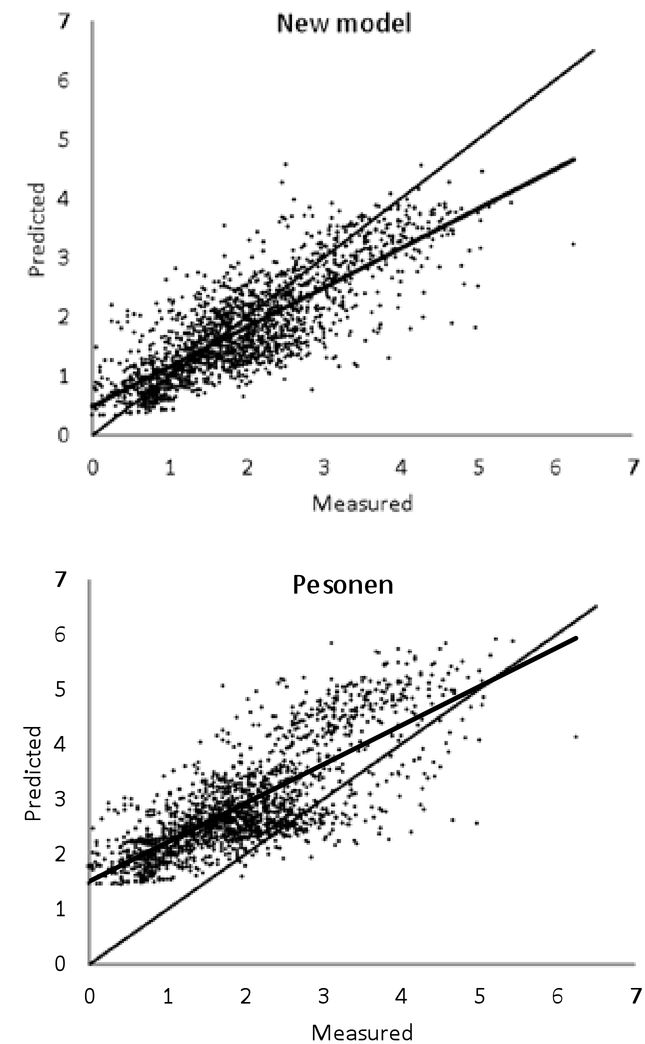

Figure 5. Relationship between measured and the predicted 5-year diameter increment (cm) by the diameter increment model of this study (5a) (New model) and of Pesonen et al. (2009) (5b). The thin line is 1/1 line (100% fit) and the thick line is a trend line (linear regression between measured and predicted value).

Plotting model predictions against measured diameter increments revealed that predictions varied less than measured growth (Figure 5). The same was even more evident with the model of Pesonen et al. (2009). The average overestimate of Pesonen et al. (2009) was 0.93 cm in 5 years whereas the new model of this study slightly underestimated growth, 0.16 cm per 5 years, on average.

Survival model

The survival model indicates that tree survival decreases with increasing basal area in larger trees (BAL):

$$\hat{s}_{ik} = \frac{1}{1 + \exp\left[-\left(17.871 - 0.381 BAL_{ik}\right)\right]} \tag{7}$$

where s_{ik} is the probability of tree i of plot k to survive for the coming 5 years. Figure 6

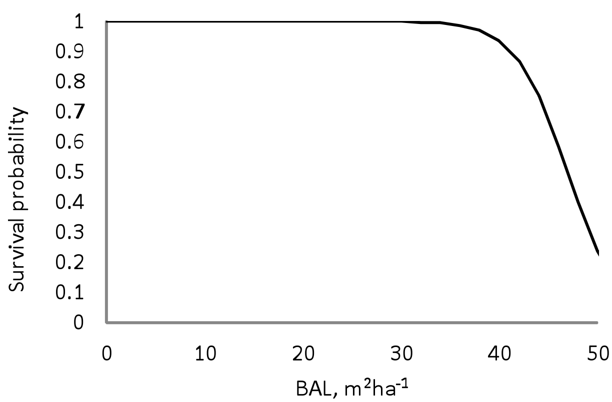

Figure 6. Dependence of 5-year survival rate on the basal area in larger trees (BAL).

graphically depicts the model. In general, mortality was very low except for the most suppressed trees of very dense stands.

Simulation examples

The models presented in this study can be used to simulate the development of Siberian larch plantations in the Hallormsstaður area. The input data consist of stand age and dominant height, as well as the tree diameters within a plot, or number of trees within different diameter classes. The width of the diameter class can be chosen freely. Stem volumes and assortment volumes can be calculated with the taper model of Heiðarsson and Pukkala (2011).

Stand age and dominant height are used to calculate site index using Equation 4. Then, the simulation proceeds as follows:

1. Calculate tree height for different trees that represent the stand in simulation (e.g., midpoint trees of diameter classes) using Equation 5. Calculate stem and assortment volumes of the trees with the taper model, using predefined minimum top diameters and piece lengths.
2. Calculate the total stand volume, total assortment volumes and any other stand characteristics of interest (e.g., mean and median diameter), using tree volumes, tree diameters, tree heights, and numbers of trees in different diameter classes.
3. Calculate dominant height after 5 years using Equation 4.
4. Calculate 5-year diameter increments of trees (Equation 6) and add them to the current diameters.
5. Calculate the new frequencies of diameter classes by multiplying the current frequency with the 5-year survival probability of a tree in the diamet--er class (Equation 7).
6. Return to Step 1.

If the stand is represented by a plot instead of midpoint trees of diameter classes, mortality can be simulated by the Monte Carlo method, by keeping the tree alive with probability equal to the predicted survival rate. Another possibility is to kill all trees having a survival probability less than a certain threshold, e.g. 0.5. The latter approach may be too systematic compared to how trees die in reality, but it has the advantage of being deterministic.

Figure 7 shows the development of key characteristics at medium (site index 20.3 m) and good sites (27.8 m) simulated for 140 years, starting from a 10-year-old plantation with 2000 trees per hectare and ending in a 150 year old stand. The simulations were based on rectangular plots, and tree mortality was simulated with the Monte Carlo method.

The simulations showed that stand volume and mean tree diameter grew faster on the better site although site index was not a predictor in the diameter increment model. This was because diameters were initially larger on the better site, which reduced the effect of BAL in Equation 6. Height development, which is driven by the dominant height model (Equation 4), is also faster on the better site. Tree mortality began earlier on the better site. Stand volume reached 750 m^3 ha^{-1} on the better site and 550 m^3 ha on the medium site. Volume increment was fastest at 20–35 years.

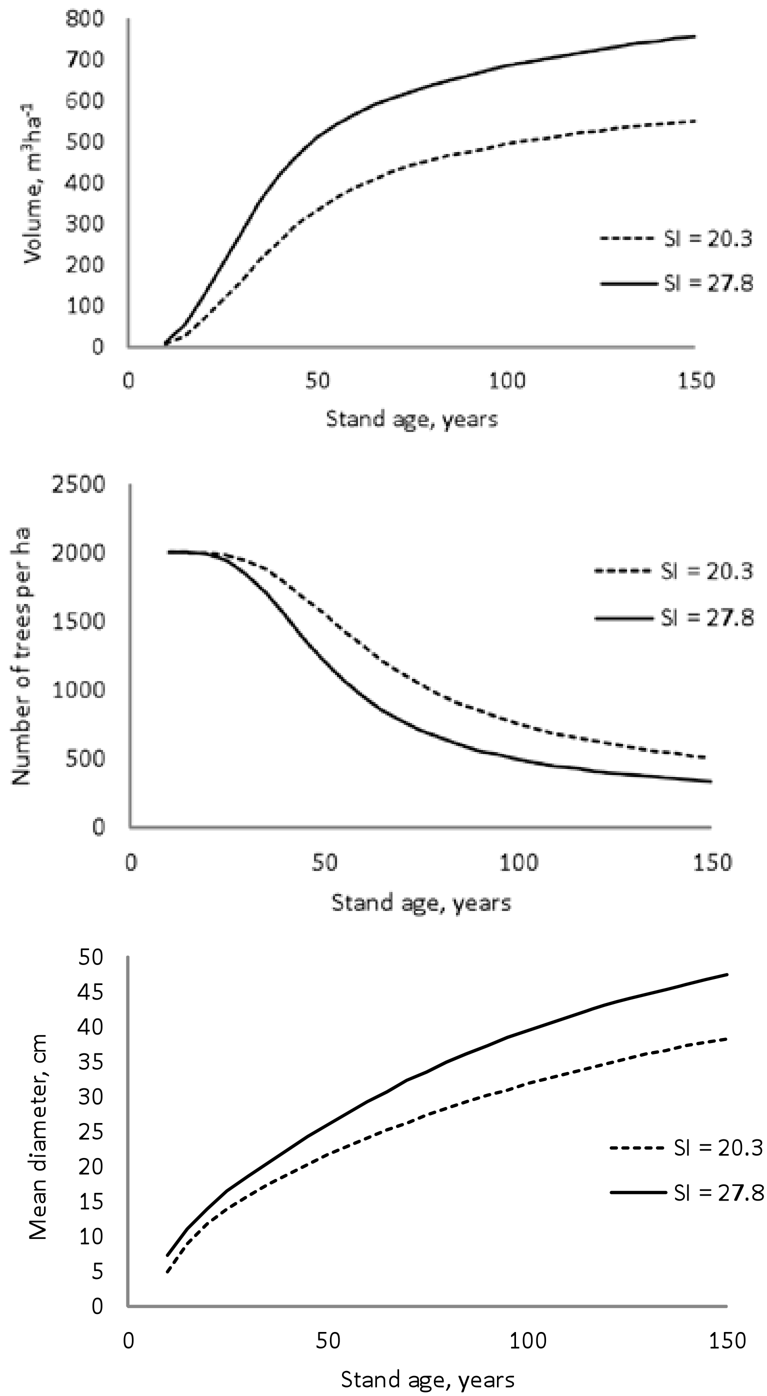

Figure 7. Simulated stand development of Siberian larch plantation on a medium and good site according to the models presented in this study. Tree volume is calculated with the taper equation of Heiðarsson and Pukkala (2011). Mean diameter has been calculated by using tree basal area as a weight variable.

DISCUSSION

This study presents a set of models which enables simulation of the development of pure even-aged larch stands in Hallormsstaður, Iceland. The set consists of a dominant height model, and individual-tree models for diameter increment, tree height and tree survival. In model selection, special emphasis was placed on the logical behaviour of the models. A simulation for 140 years, starting from a 10-year-old plantation with 2000 trees per hectare, was carried out. The results showed that the model set behaves logically in long-term simulations.

The PSP data had less variation in site index than the Pesonen data, but since the Pesonen data included only 149 diameter growth observations, their influence on model parameters was small. The outcome that site index was not included in the diameter increment model as a predictor means that the model is most suitable for medium sites. However, if simulation is started from an initial stand, larger tree diameters at better sites lead to higher growth predictions, implying that the influence of site productivity is also incorporated in the diameter increment model (see Figure 7, bottom).

If the simulation of stand development is started from an open area or a very young stand, it is advisable to simulate only dominant height development at first, and generate individual-tree diameters and heights at a certain dominant height. If the site index is not known from a previous rotation or adjacent stands, it should be set to about 20 m if the site is a typical planting site of Siberian larch in the Hallormsstaður area. The individual-tree height model (Equation 5, Figure 3) pinpoints the relationships between dominant height, tree height, and tree diameters. For example, when dominant height is 5 m, most tree heights will be within the 4–5 m range, which means that tree diameters should also range from 4 to 5 cm.

The models developed in this study are simple but behave logically in simulations. Nonlinear mixed-effect models (random parameter models) were fitted as the first alternative for height and diameter increment. The advantage of mixed models is that they can be calibrated for a particular stand. It turned out that plot and measurement effects were not strong and they were therefore not included in the final models. When the mixed-effect and fixed-parameter versions of the diameter increment model were compared, it turned out that the mean squared error (MSE) was 4.5 % larger for the fixed part of the mixed-effect model.

Siberian larch is an important species in North and East Iceland and further development of the growth models is needed, especially for the northern part of the country. Additional data should be collected from different site indices for a better modelling of site influences. With the new models presented in this article it is possible to make more accurate growth and yield estimates for stands. It is also possible to optimize schedules and rotation lengths. Recently, Icelandic Forest Service introduced a new forest management planning system. The new models are an important component of that system because the stand structure and amount of timber in the forests can be evaluated and the forest resources can now be used more effectively.

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