Taper functions for lodgepole pine (*Pinus contorta*) and Siberian larch (*Larix sibirica*) in Iceland

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ABSTRACT

The aim of this study was to test and adjust taper functions for plantation-grown lodgepole pine (*Pinus contorta*) and Siberian larch (*Larix sibirica*) in Iceland. Taper functions are necessary components in modern forest inventory or management planning systems, giving information on diameter at any point along the tree stem. Stem volume and assortment structure can be calculated from that information. The data for lodgepole pine were collected from stands in various parts of Iceland and the data for Siberian larch were collected in and around the Hallormsstadur forest, eastern Iceland. The performance of three functions in predicting diameter outside bark were tested for both tree species. The results of this study suggest that the model Kozak 02 is the best option for both Siberian larch and lodgepole pine in Iceland. The coefficient of determination (R^2) for the fitted Kozak 02 functions are of the same magnitude as reported in earlier studies. The Kozak II functions gave similar results for the two species analysed in this study, which means that either of them could be used.

Keywords: Lodgepole pine, management planning system, Siberian larch, stem form, taper model

YFIRLIT

Mjókkunarföll fyrir stafafuru (Pinus contorta) og síberíulerki (Larix sibirica) á Íslandi.

Meginmarkmið rannsóknarinnar var að prófa og aðlaga þekkt mjókkunarföll þannig að þau yrðu nothæf fyrir stafafuru og lerki á Íslandi. Mjókkunarföll eru nauðsynleg í nútíma umhirðu- og áætlanakerfum í skógrækt, en þau gefa upplýsingar um breytingar á þvermáli trjábols með hæð. Út frá þessum upplýsingum er hægt að áætla rúmmál trjábola á mismunandi hæðarbilum út frá mælingum á einungis hæð og þvermáli í brjósthæð. Slíkar upplýsingar eru nauðsynlegar til að meta og mæla þær viðarafurðir sem skógurinn gefur af sér. Gögnin fyrir stafafuru komu af öllu landinu en gögnin fyrir síberíulerki voru frá Hallormsstað og aðliggjandi skógum. Þrjú föll sem áætla þvermál með berki voru prófuð, aðlöguð og síðan borin saman fyrir báðar trjátegundirnar. Föllin nota þvermál í brjósthæð og heildar hæð trés sem óháðar breytur. Niðurstöðurnar benda til þess að fallið Kozak 02 henti best til að meta mjókkun bols, bæði stafafuru og síberíulerki. Fylgnistuðullinn R^2 sýnir að fallið Kozak 02, aðlagað af íslenskum mælingum, gefur svipaðar niðurstöður hér á land og erlendis. Fallið Kozak II gefur mjög svipaðar niðurstöður og Kozak 02 fyrir báðar tegundirnar og er munurinn það lítill að notast má við bæði föllin.

INTRODUCTION

In recent years, forestry in Iceland has developed rapidly and the need to introduce and adapt models for estimating tree taper and bole volume at different tree heights and for different tree species in Iceland has become more evident. Taper functions are the basis of computer algorithms for calculating stem volumes at any height and assortment structure, a prerequisite for successful forest planning and management (Kublin et al. 2008). The importance of taper functions is demonstrated by the high number of models published and used, varying in complexity. As yet, no single theory or model exists that adequately explains the variation in stem form for all species (Newnham 1988).

Many studies in this field have involved polynomials of order two or greater (Bruce et al. 1968, Kozak et al. 1969, Goulding & Murray 1976, Laasasenaho 1982). According to Sterba (1980) the weakness of this model type has been the inability to characterize the lower portion of a tree with significant basal swelling. Another type is the segmented polynomial model, which uses joining points to link stem sections along the bole at the joining points (e.g. Max & Burkhart 1976, Demaerschalk & Kozak 1977). Later, variable-exponent taper functions where introduced, which use changing exponents to describe the shape of a bole from the ground to the top (e.g. Kozak 1988, 2004, Newnham 1988, 1992). They assume that within a tree the form changes continuously along the stem. These functions enable the exponent to change with relative tree height, which allows a single function to describe the stem profile. Other types of taper functions can also be found in the literature, such as trigonometric (Thomas & Parresol 1991) and nonparametric (Lappi 2006) models.

According to Kozak (2004), taper functions provide forest managers with estimates of (1) diameter at any point along the stem, (2) total stem volume, (3) merchantable volume and merchantable height of the stem to any top diameter and from any stump height, and (4) volume of stem sections of any length and at any height from the ground. They can also provide an estimate of the height of a tree at which a particular diameter occurs (Fonweban et al. 2011). The flexibility of the taper function is an essential factor and must be taken into account in the model construction because stem taper is predicted to various sizes of trees (Eerikäinen 2001).

Iceland has a short forest history, with organised forestry having started at Thingvellir in 1899 with planting of a pine stand. Legislation was approved in 1907 to protect the remaining woodlands and to create new forests (Aradottir & Einarsson 2005). During the past 60 years, emphasis has been on afforestation through planting of trees (Einarsson 2009). The main task has been to find appropriate species and provenances that are adapted to Icelandic climate and growing conditions. Functions to estimate the total stem volume of the most common species used in forestry in Iceland have been published (Norrby 1990, Snorrason & Einarsson 2006) but no models has been available for predicting tree taper in Iceland. The aim of this study was to develop flexible taper functions for plantation grown lodgepole pine (*Pinus contorta* Dougl.) and Siberian larch (*Larix sibirica* Ledeb.) that can be used to predict stem diameter at any given height along the tree bole.

MATERIALS AND METHODS

The data used for modelling were obtained from different research projects. The data for lodgepole pine were collected in 2008 and were originally gathered for the construction of growth models made by Juntunen (2010) (Table 1). The data consist of 87 felled sample trees from even-aged plantations, sampled in various parts of Iceland. The measured tree characteristics were: total height, past five years height growth, stump height and stump diameter. Diameters over bark were measured at the following relative heights, which are given as percentages of the total tree height: 1, 2.5, 5, 7.5, 10, 15, 20, 30, 40, 50, 60, 70, 80 and 90%. The Siberian larch data (Table 1) were collected between 1995 and 2008 from even-aged plantations in Hallormsstadur (65.5ºN and 14.45ºW), eastern Iceland. The sample sites were permanent sample plots and plots in a thinning experiment. The trees measured were felled during thinning operations. The tree characteristics measured were total height, dbh and stump diameter. Diameters over bark were measured at the following relative heights: 1, 2.5, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 95 %.

Table 1. Summary statistics for tree attributes for lodgepole pine and Siberian larch.

	Lodgepole pine (n = 87)			**Siberian larch (n = 89)**		
Variable	**Mean**	**S.D.**	**Range**	**Mean**	**S.D.**	**Range**
D (cm)	13.4	3.9	4.4 – 33.3	20.6	6.6	7.9 – 34.5
H (m)	8.1	2.4	3.1 – 18.4	13.5	3.7	5.7 – 19.0
Age (years)	37.7	9.0	18 – 68	50.9	16	29 – 71

D is diameter at 1.3 m above ground level; H is total height; S.D. is standard deviation

Data analysis

The following functions were fitted for both tree species: Kozak II (Kozak 1997), Kozak 02 (Kozak 2004) and the Biging function (Biging 1984). Kozak II was found to be the best function for Calabrian pine (*Pinus brutia*) in Syria among the 32 functions tested by de Miguel et al. (2011). On the other hand, Kozak (2004) found another function, namely Kozak 02 to give the best results for several species. The Biging function has the advantage of being simple, its most parsimonious version having only two parameters.

Equations for the three tested functions are as follows:

Kozak II

$$\hat{d} = a_0 D^{a_1} H^{a_2} \left[\frac{1-\sqrt{q}}{1-\sqrt{t}}\right]^{b_1\left(\frac{1-\sqrt{q}}{1-\sqrt{t}}\right)^{0.1} + b_2 q^4 + b_3 \arcsin\left(1-\sqrt{q}\right) + b_4\left(\frac{1}{e^{\frac{D}{H}}}\right) + b_5 D^{\left(\frac{1-\sqrt{q}}{1-\sqrt{t}}\right)}} \quad (1)$$

Kozak 02

$$\hat{d} = a_0 D^{a_1} H^{a_2} X^{b_1 q^4 + b_2\left(1/\exp(D/H)\right) + b_3 X^{0.1} + b_4\left(1/D\right) + b_5 H^{Q} + b_6 X} \quad (2)$$

Biging

$$\hat{d} = D^{a_1}\left[b_1 + b_1 \ln\left\{1-\left(1-\exp\left(\frac{-b_1}{b_2}\right)\right)\left(\frac{h}{H}\right)^{b_3}\right\}\right] + \left(\frac{H}{D}\right)^{b_4} \quad (3)$$

where d is diameter (cm) at height h (m), D is dbh (cm), H is total tree height (m), q is h/H, t is 1.3/H, X is $(1-q^{1/3})/(1-(1.3/H)^{1/3})$, and Q is $(1-q^{1/3})$. The parameters a_0, a_1, a_2, and b_1, b_2, b_3, b_4, b_5 are to be estimated for each function by fitting the functions to the data. The parsimonious version of the Biging function is (Biging 1984):

$$\hat{d} = D\left[b_1 + b_1 \ln\left\{1-\left(1-\exp\left(\frac{-b_1}{b_2}\right)\right)\left(\frac{h}{H}\right)^{1/3}\right\}\right] \quad (4)$$

In this study, the more complex version of the Biging function with five parameters (Equation 3) was tested.

All the functions were fitted to both P. *contorta* and *L. sibirica* data. If any of the parameters was non-significant according to the t-test, the corresponding predictor was dropped from the function, resulting in a function with fewer parameters. R^2 and RMSE (root of the mean of squared errors) for the stem diameter were calculated for every function. In addition, residuals were plotted against classes of h/H. The class width was 0.1, i.e., the h/H classes were 0–0.0999, 0.1–0.1999, 0.2–0.2999, etc.

The functions were fitted for untransformed diameter. Fitting the functions for squared diameter was also tested but this would have resulted in very similar models. The comparisons of functions were based on the fitting statistics (R^2 and RMSE) and the residual

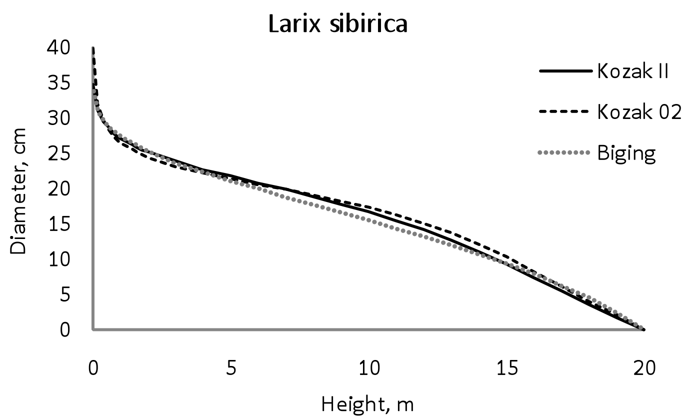

Figure 1. Stem profiles according to the three taper functions for a Siberian larch with dbh 25 cm and height 20 m.

plots. The RMSE and the coefficient of determination (R^2) were computed from untransformed values of measured and predicted diameters. The functions were fitted as fixed parameter models using nonlinear regression analysis in SPSS. This is because the fixed part of a random parameter model (only the fixed part is used when mixed models are applied in forestry practice) does not usually perform better than a fixed parameter model (de Miguel et al. 2011).

RESULTS

Siberian larch

Parameter b_4 of Kozak II was not significant for Siberian larch. The equation without b_4 is as follows:

$$\hat{d} = 0.983D^{0.839}H^{0.197}\left[\frac{1-\sqrt{q}}{1-\sqrt{t}}\right]$$

$$0.570\left(\frac{1-\sqrt{q}}{1-\sqrt{t}}\right)^{0.1} + 0.478q^4 - 0.353\arcsin\left(1-\sqrt{q}\right) + 0.014D^{\left(\frac{1-\sqrt{q}}{1-\sqrt{t}}\right)}$$

In Kozak 02, parameter a0 was not significantly different from 1. The equation for Siberian larch is therefore:

$$\hat{d} = D^{0.928}H^{0.087}X$$

$$0.564q^4 - 0.849\left(1/\exp(D/H)\right) + 0.473X^{0.1} + 2.494\left(1/D\right)$$

$$+0.069H^{Q} - 0.207X$$

In the Biging function, the last parameter b_4 was not significant. The final equation is therefore:

$$\hat{d} = D^{0.957}\left[2.289 + 0.568\ln\right.$$

$$\left.\left\{1-\left(1-\exp\left(\frac{-2.289}{0.568}\right)\right)\left(\frac{h}{H}\right)^{0.097}\right\}\right]$$

Figure 1 shows the stem taper for a Siberian larch tree with dbh of 25 cm and height of 20 m, calculated with the above equations. It can be seen that, in the middle of the stem, the Biging function predicts smaller diameters than the two Kozak functions. Kozak 02 predicts more cylindrical stem forms than Kozak II.

Lodgepole pine

When the Kozak II function was fitted to the lodgepole pine data, it was found that parameter a_0 (in Equation 1) was not significantly different from one. The equation for the function which includes only the significant parameters is as follows:

$$\hat{d} = D^{0.932}H^{0.085}\left[\frac{1-\sqrt{q}}{1-\sqrt{t}}\right]$$

$$0.619\left(\frac{1-\sqrt{q}}{1-\sqrt{t}}\right)^{0.1} + 0.570q^4 - 0.155\arcsin\left(1-\sqrt{q}\right) - 0.566\left(\frac{1}{e^{\frac{D}{H}}}\right) + 0.0042D^{\left(\frac{1-\sqrt{q}}{1-\sqrt{t}}\right)}$$

Also the Kozak 02 function was simplified since a_0 did not differ significantly from one and a_2 did not differ from zero. The final equation for the function is:

$$\hat{d} = D^{0.997}X$$

$$0.634q^4 - 0.948\left(1/\exp(D/H)\right) + 0.507X^{0.1}$$

$$+0.903\left(1/D\right) + 0.032H^{Q} - 0.069X$$

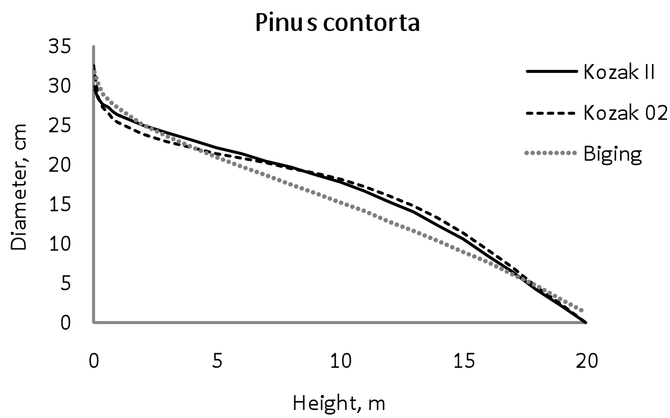

Figure 2. Stems profiles according to the three taper functions for lodgepole pine with dbh 25 cm and height 20 m.

The equation for the Biging function for lodgepole pine is as follows:

$$\hat{d} = D^{0.957}\left[1.482 + 0.640\ln\left\{1-\left(1-\exp\left(\frac{-1.482}{0.640}\right)\right)\left(\frac{h}{H}\right)^{0.304}\right\}\right] + \left(\frac{H}{D}\right)^{-0.595}$$

Similarly to Siberian larch, the Biging function predicts smaller diameters than the Kozak functions in the middle of the stem (Figure 2). Kozak 02 and Kozak II are close to each other but Kozak II predicts slightly more conical stem forms than Kozak 02.

Evaluation of the functions

Table 2 shows the fitting statistics for all the tested functions. It can be seen that the Biging function is clearly the weakest for both species. In terms of fitting statistics, the best function for both species is Kozak 02. However, differences between the two Kozak models are small.

Table 2. Fitting statistics for the taper functions. High R^2 and low RMSE imply good fit.

Model	Siberian larch		Lodgepole pine	
	R^2	RMSE	R^2	RMSE
Kozak II	0.985	1.041	0.979	0.889
Kozak 02	0.986	1.024	0.979	0.888
Biging	0.980	1.205	0.970	1.054

Another way to evaluate and compare the functions is to look at the graphics of the residuals. The residuals are the differences between measured and predicted diameters. Positive residuals mean underestimation and negative mean overestimation. The residuals suggest that the two Kozak functions behave similarly and fit better than the Biging function (Figure 3). The residuals are smaller for lodgepole pine than for Siberian larch, which is in accordance with the RMSE results. The Biging function has more positive residuals than negative ones in the middle of the stem (underestimation) whereas the Kozak functions often underestimate diameters near the tree top. However, this underestimation has a minor effect on volume estimates because both tree diameter and volume are much smaller near the top.

The same conclusions can be drawn from Figure 4, which shows the bias formulated as the mean of the re-siduals at different relative heights. The Biging function clearly underestimates diameter at relative heights of 0.3–0.6. The Kozak functions also have some biases but they are smaller than for the Biging function. The two Kozak functions behave very similarly.

DISCUSSION

The results of this study suggest that the function described by Kozak (2004) and tested and adjusted in this paper, referred to as Kozak 02, is the best option for both Siberian larch and lodgepole pine in Iceland. These results differ from those of de Miquel et al. (2011) who found another function by Kozak, the Kozak II function, to be the best choice for *Pinus brutia* in Syria among 32 taper equations tested. However, we found that Kozak II and Kozak 02 gave similar results for the two species analysed in this study, which means that either of

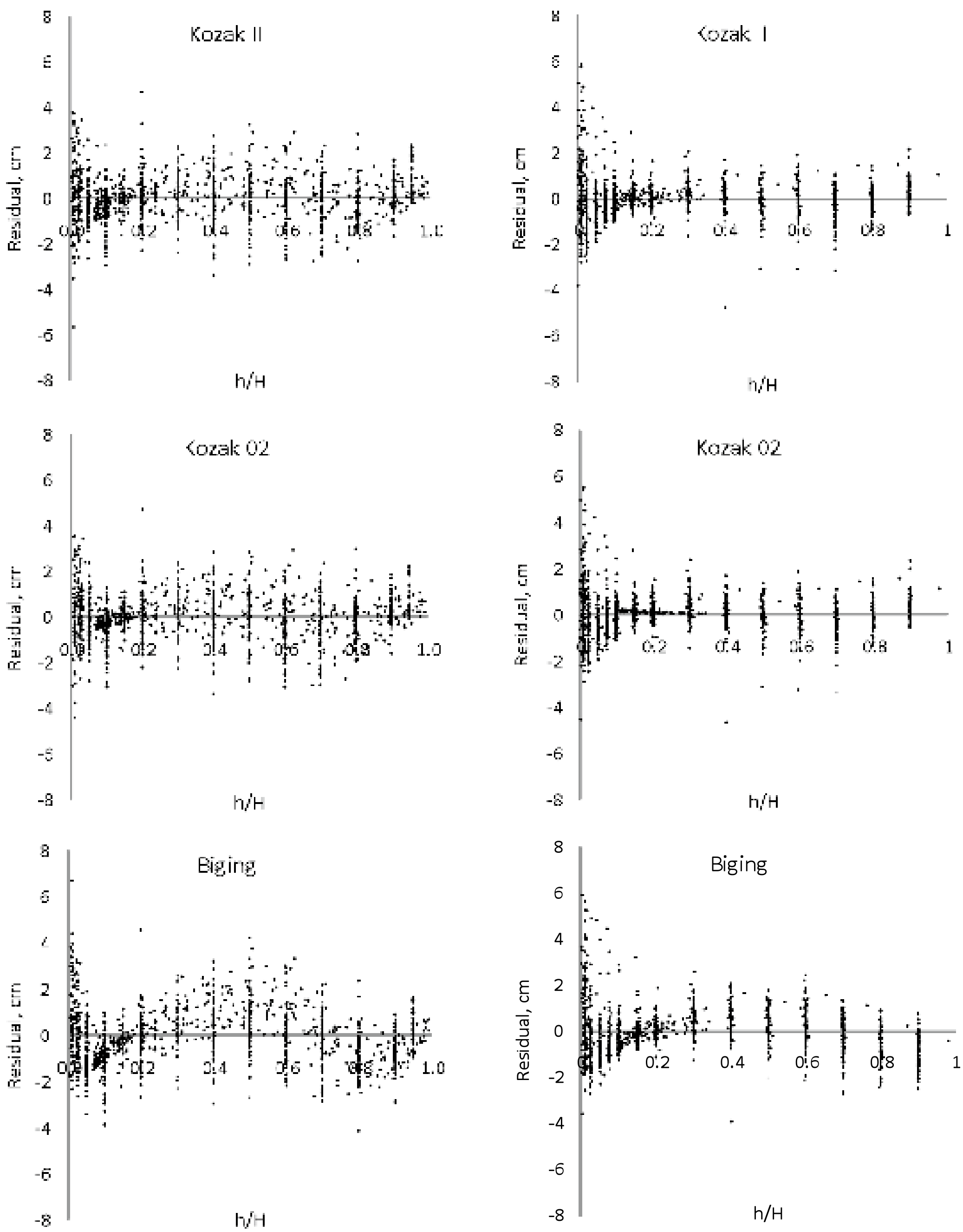

Figure 3. Residuals (measured minus predicted diameter) of the three taper functions for Siberian larch (left) and lodgepole pine (right) plotted against relative height (h is the height of the estimated/measured diameter and H is the total tree height).

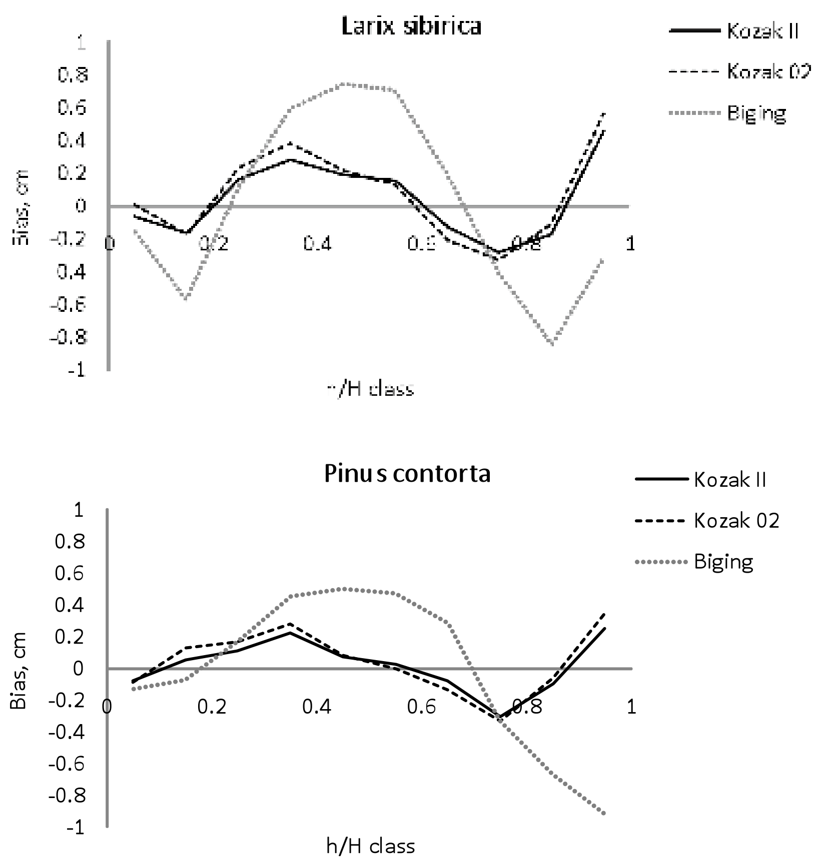

Figure 4. Mean residual (bias) in different h/H classes (measurement height/total tree height).

them could be used. The Kozak 02 function was found to be the preferred one for maritime pine (*Pinus pinaster*) in North-western Spain when 31 different taper models were tested and compared (Rojo et al. 2005).

The coefficient of determination (R^2) of Kozak 02 for the Icelandic data was of the same magnitude as reported in earlier studies (e.g. Bi 2000, Rojo et al. 2005, de Miguel et al. 2011). The RMSEs were about the same or slightly smaller than in the other studies (e.g. Rojo et al. 2005, Brooks et al. 2008). Comparisons of fitting statistics are not always straightforward since the predicted variable may be in a different form in different studies. The diameter (d) may have been transformed to d^2, d/D (D is dbh) or $\ln(d)$, for example, before fitting the function. Since the fitting statistics were computed from the modelling data, they may have slightly overestimated the performance of the functions in independent data or when the taper functions are used in forestry practice. Another shortcoming is that the taper measurements were taken from trees that were removed in thinning; the measured trees may have represented a biased sample since the removed trees may have been different than the trees on average.

Of the two species, lodgepole pine has a more cylindrical stem form than Siberian larch according to the fitted Kozak 02 functions (Figure 5). The comparison in Figure 5 reveals the importance of developing separate taper functions for different species. Therefore, future studies should develop taper functions for all the main species used in Icelandic forestry.

Taper models are one of several necessary components in a modern forest management planning system. In forestry planning, prediction of the future stand development on different sites and silvicultural systems is essential. This prediction requires models for tree growth and survival. If natural regeneration is used, regeneration models are also required. Pioneer planning models have already been developed for Siberian larch (Pesonen et al. 2009) and lodgepole pine in Iceland (Juntunen 2010), but there is still much work to be done in this field. Together with taper and volume functions, growth and yield models would make management planning and the economic measures of wood utilisation and investments in new plantations more reliable than today.

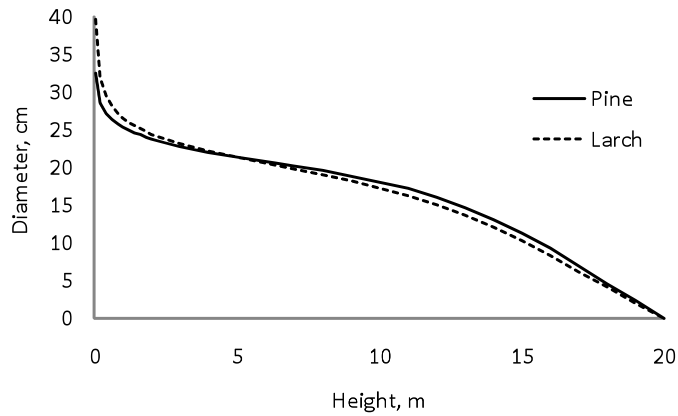

Figure 5. Comparison of the predicted stem taper in lodgepole pine and Siberian larch according to the fitted Kozak 02 functions for a tree with dbh 25 cm and total height 20 m.

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