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The Representative Household's Demand for Money in a Cointegrated VAR Model

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Abstract

This paper develops a constant, data congruent, error correction model of broad money demand in Iceland. A representative household model with liquidity services directly in the utility function is developed. Solving for the steady state gives a linear, long-run relation between real money balances, output and the opportunity cost of holding money that is used to over-identify the cointegrating space. The over-identifying restrictions suggest that the representative household is equally averse to variations in consumption and money holdings. Output and the opportunity cost are found to be strongly and super exogenous for the parameters of the money demand equation. Finally, a forward looking interpretation of the short-run dynamics, assuming quadratic adjustment costs, cannot be rejected by the data. This interpretation is, however, found to be problematic as the forecasting equations used to generate future expectations are found to be unstable.

Key words: Money demand, cointegration, error correction models, super exogeneity, forward looking behaviour
JEL Classifications: E41, C32, C33

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1. Introduction

The modelling of the demand for money has been a major focus of interest in macroeconometrics since the early 1970s. This is not surprising considering its importance for monetary policy and its role in modern economies. The existence of a stable demand function, the link between money and prices, and the endogeneity or exogeneity of money are all recurring issues in the literature. Obtaining constant empirical money demand equations has, however, proven elusive, leading many prominent monetary economists to declare that no such constant relation exists (see, for example, Goldfeld and Sichel, 1990 and Fase, 1994). Recently, however, a number of empirical studies of money demand have been successful in finding stable money demand relations. Among them are Baba *et al.* (1991) for the U.S., Hendry and Ericsson (1991) for the U.K. and Hoffman *et al.* (1995) for the U.S., Japan, Canada, the U.K., and West Germany.[1]

The purpose of this paper is to analyse the demand for broad money (M3) in Iceland using annual data for the period 1962 to 1995. A simple money demand model is derived from an optimization problem of a representative household with liquidity services directly in the utility function. The use of an explicit optimizing model in the cointegrating analysis should facilitate economic interpretation of the long-run parameters of the money demand equation.[2]

The dynamic adjustment towards the long-run equilibrium is assumed to be characterized by a contingent plan model of the error correction (ECM) form. Since some of the earlier studies of Icelandic money demand have suffered from instability problems, that can be explained by too restrictive dynamics, the approach here is to estimate the short-run dynamics freely, using the general-to-specific approach.[3] These short-run dynamics could, however, in principle be derived explicitly assuming, for example, quadratic adjustment costs. To that end, a forward looking, rational expectations specification of the short-run adjustment dynamics is examined.

The remainder of the paper is organized as follows. Section 2 describes the

[1] For surveys of empirical money demand equations, see for example Goldfeld and Sichel (1990), Fase (1994) and Browne *et al.* (1997).

[2] This approach has two important advantages (cf. Pesaran, 1997). First, it ensures that the model embodies theory-consistent steady state relations from the outset. Second, to the extent that the theory implies over-identifying restrictions on the long-run relations, it allows the model's long-run properties to be evaluated empirically.

[3] Among earlier studies of money demand in Iceland are Eggertsson (1982), Cornelius (1990) and Gudmundsson (1986). The first two use a partial adjustment framework, whereas the last one uses a conditional ECM. The demand equations in the first two papers break down in 1979, whereas Gudmundsson, by allowing a richer set of explanatory variables and more flexible dynamics inherent in the ECM, finds no such problems. This suggests a potential benefit of estimating the short-run dynamics unrestrictetely.

underlying theoretical model of money demand. The specification of the short-run dynamic adjustment path is also discussed and how it relates to the long-run equilibrium demand for money, which constitutes the econometric model studied in this paper. Section 3 describes the data and necessary transformations due to financial innovations in the domestic financial system in the last decade. The fourth section presents the estimation results for the simple monetary system analysed. The fifth section discusses the single equation demand for money and the forward looking, rational expectations interpretation in more detail. The final section concludes.

2. Theoretical Specification

2.1. Steady state analysis

To model households' demand for money it is assumed that money provides direct utility from liquidity services using a standard money-in-the-utility-function (MIUF) model. This model allows the utility function to capture the liquidity services of money and has the advantage of being easily tractable and giving log-linear steady state solutions, therefore being easily estimable. Furthermore, Feenstra (1986) has shown that models which explicitly model the transaction services of money can be approximated as MIUF models. This is important since existing models which explicitly model the transaction services of money, such as cash-in-advance models, quickly become cumbersome and often analytically intractable.[4]

The representative household is assumed to solve the following infinite horizon problem

$$\max_{\{C_{t+i}, M_{t+i}\}_{i=0}^{\infty}} \mathsf{E}\left\{ \sum_{i=0}^{\infty} \delta^i \left[u(C_{t+i}) + \zeta v\left(\frac{M_{t+i}}{P_{t+i}}\right)\right] \middle| \mathcal{I}_t \right\} \tag{2.1}$$

where C_t is real valued consumption and $u(\cdot)$ and $v(\cdot)$ are twice differentiable concave instantaneous utility functions. $\mathsf{E}(\cdot \mid \mathcal{I}_t)$ denotes expectations conditional on information at time t, δ is the discount factor and ζ measures the relative weight of consumption goods and real money balances in the overall utility function.

The representative household allocates its real income, I_t, along with accrued capital earnings among consumption goods C_t, bonds B_t, and real money balances $\frac{M_t}{P_t}$, where P_t is the price of consumption at time t. B_t denotes the real value of bonds denominated in unit of time t consumption. These bonds pay a gross real return of $(1 + r_t)$ from time t to $t + 1$. Real money balances pay the gross return

[4] See Ripatti (1996) and Poterba and Rotemberg (1984) for empirical applications of MIUF models. See Lucas (1988) for an empirical application of a cash-in-advance model.

$(1+d_t)\frac{P_t}{P_{t+1}}$, where $(1+d_t)$ is the gross own rate of money from time t to $t+1$. The households budget constraint is then

$$C_t + B_t + \frac{M_t}{P_t} \leq I_t + (1+d_{t-1})\frac{M_{t-1}}{P_t} + (1+r_{t-1})B_{t-1} \tag{2.2}$$

The first order conditions for this optimization problem are given by the standard Euler conditions

$$(1+r_t) = \left[\mathsf{E}\left(\delta\frac{u'(C_{t+1})}{u'(C_t)}\middle|\mathcal{I}_t\right)\right]^{-1} \tag{2.3}$$

$$1 = (1+d_t)\mathsf{E}\left\{\frac{P_t}{P_{t+1}}\frac{\delta u'(C_{t+1})}{u'(C_t)}\middle|\mathcal{I}_t\right\} + \zeta\frac{v'\left(\frac{M_t}{P_t}\right)}{u'(C_t)} \tag{2.4}$$

The first Euler equation states that along an optimal path the representative household cannot increase its expected utility by forgoing one unit of consumption in period t, investing its value in bonds, and consuming its proceedings at time $t+1$. The second Euler equation states that expected utility cannot be increased by holding one unit less of money at time t, investing it in bonds, and consuming its proceedings at time $t+1$.

To solve this model explicitly two simplifying assumptions are made. First, it is assumed that the Fisher parity holds[5]

$$(1+i_t) = (1+r_t)\frac{\mathsf{E}(P_{t+1} \mid \mathcal{I}_t)}{P_t} \tag{2.5}$$

where i_t is the nominal bond rate. Second, it is assumed that

$$\mathsf{cov}\left(\frac{P_t}{P_{t+1}}, \frac{\delta u'(C_{t+1})}{u'(C_t)}\middle|\mathcal{I}_t\right) = 0 \tag{2.6}$$

This covariance restriction implies that the real stochastic discount factor is independent of inflation, i.e that inflation and the intertemporal marginal rate of substitution are independent. This could be interpreted as a neutrality of money condition (see Ripatti, 1996).[6]

[5] One could also assume that there exists an asset with indexed returns.

[6] Both these assumptions follow directly from a perfect capital market assumption and imply that the short term bond rate, i_t, will include all the information on the return on alternative assets available to households. This is, of course, a strong assumption and is subject to empirical testing. More discussion on this assumption and empirical testing of the validity of some of its implications is contained below. See Gibbons and Ramaswamy (1993) for an application of this assumption for testing real term structure models. Lucas (1988) obtains a similar result from a cash-in-advance model where the critical assumption is that all agents engage in securities trading at the same time, with the same fixed period.

The first order condition for real money balances can now be written as

$$\zeta \frac{v'\left(\frac{M_t}{P_t}\right)}{u'(C_t)} = \left(\frac{1+d_t}{1+i_t} - 1\right) \tag{2.7}$$

Finally, the instantaneous utility functions are parameterized within the standard constant relative risk aversions (CRRA) class as follows[7]

$$u(C_t) = \frac{C_t^{1-\omega}}{1-\omega} \tag{2.8}$$

$$v\left(\frac{M_t}{P_t}\right) = \frac{\left(\frac{M_t}{P_t}\right)^{1-\psi}}{1-\psi} \tag{2.9}$$

where ω and ψ are the constant relative risk aversion coefficients. In the case $\omega = \psi = 1$ these utility functions become $u(C_t) = \log C_t \equiv c_t$ and $v(\frac{M_t}{P_t}) = \log \frac{M_t}{P_t} \equiv (m-p)_t$.

Inserting the given utility functions into the first order condition, taking logs and approximating gives

$$(m-p)_t = \frac{1}{\psi}\log\zeta + \frac{\omega}{\psi}c_t - \frac{1}{\psi}R_t \tag{2.10}$$

where lower case letters denote logs and $R_t \equiv (i_t - d_t)$ is the net opportunity rate. In stationary equilibrium $M_t = M^d$, $C_t = C$, $R_t = R$ and $P_t = P$. The stationary equilibrium for real money balances is therefore

$$(m^d - p) = \kappa + \eta c - \theta R \tag{2.11}$$

where $\kappa \equiv \frac{1}{\psi}\log\zeta$, $\eta \equiv \frac{\omega}{\psi}$ and $\theta \equiv \frac{1}{\psi}$. This is a standard functional form used in empirical studies on money demand. However, it is more common to use some measure of aggregate income as the scale variable rather than private consumption. This is also done here as the study uses a broad measure of money that not only includes the money holdings of households but also of the corporate sector. The use of a more broad expenditure variable than private consumption is therefore appropriate. The steady state money demand relation analysed in this paper is therefore the following[8]

$$(m^d - p) = \kappa + \eta y - \theta R \tag{2.12}$$

[7] Addilog preferences are commonly used in cointegration analysis of preference parameters. See, for example, Ogaki (1992) and Clarida (1994).

[8] This implies that ζ cannot be identified since a change of scale variable will influence the estimate of ζ.

Theory is not unified on the size of the long-run income elasticity, η. In the transaction model of Baumol (1952) and Tobin (1956) η equals $\frac{1}{2}$, whereas in the precautionary model of Miller and Orr (1966) it equals $\frac{1}{3}$. The quantity theory of Friedman (1956) predicts, however, a value of unity. The size of η is therefore left open as an empirical question. A unit scale elasticity further implies in this model that $\omega = \psi$, i.e. that preferences are homothetic. This also implies that households are equally risk averse to variations in consumption and real money balances.

2.2. Dynamic adjustment

The previous section led to an expression for the demand for money when households face no adjustment costs. It is natural to interpret this money demand relation as a long-run equilibrium relation. Dynamic adjustment towards this long-run equilibrium can be motivated in several ways. The simplest way is to assume that dynamic adjustment can be characterized by a contingent planning model of the following ECM form (see Hendry and Ericsson, 1991)

$$\rho_0(L)\Delta m_t = \rho_1(L)\Delta p_t + \rho_2(L)\Delta y_t + \rho_3(L)\Delta R_t + \rho_4(m - m^d)_{t-1} + \varepsilon_{mt} \quad (2.13)$$

where $L^n x_t \equiv x_{t-n}$ is the lag operator, $\Delta \equiv (1 - L)$, $\rho_i(L)$ $(i = 0, 1, 2, 3)$ are finite order lag polynomials in L and ε_{mt} denotes deviations of outcome from plan. For the model to be stable ρ_4 should be less than zero (in the single-equation case), which again indicates that the variables in (2.12) cointegrate, see Engle and Granger (1987). For (2.13) to be interpretable as a demand function it should also hold that $\rho_1(1), \rho_2(1) \geq 0$ and $\rho_3(1) \leq 0$.

Economically, this model could be related to the target-threshold model (or buffer stock model) of money demand of Akerlof (1979) and Milbourne (1983). According to these models, money holdings are accumulated passively until some threshold is reached. At that time, they are restored to its long-run target value. Money balances inside the bands are therefore determined by the short-run dynamics in (2.13), whereas the levels of the bands are determined by the long-run factors, cf. the (S, s) model. See, for example, Hendry (1994).

The ECM in (2.13) can simply be interpreted as a parsimonious method of representing lag responses. It can, however, also be derived explicitly from a cost minimization problem, where households are penalized if actual money holdings deviate from the long-run target and for adjusting their actual money holdings

$$\mathcal{K}_t = \phi(m_t - m_t^d)^2 + (\Delta m_t)^2 \quad (2.14)$$

where m_t^d is the desired money balances when households face no adjustment costs, as derived from the previous section. The problem with this approach is that it assumes myopic behaviour on behalf of households as they overlook the effects of current decisions on future costs. A more appropriate approach is to assume that they minimize the expected present value of $\mathcal{K}_t$

$$\mathcal{L}_t = \mathsf{E}\left\{\sum_{i=0}^{\infty} \delta^i \mathcal{K}_{t+i} \mid \mathcal{I}_t\right\} \tag{2.15}$$

In this case money demand will depend on expected future income and interest rates.[9] This forward looking interpretation of the money demand equation is tested below.

3. The Data

This paper analysis the demand for money in Iceland, using annual data for the period 1962 to 1995. As a measure of money, broad money, M3, is used (mainly due to its more stable definition throughout the estimation period than narrower measures of money). As a scale variable gross domestic output (GDP) is chosen and the implicit GDP price deflator as a measure of prices. Both variables are measured as annual averages and output is measured at constant, 1990 prices.[10] The final variable in the model measures the opportunity cost of money. Since interest rates in Iceland were not market determined for the main part of the period analysed here and alternative financial assets were generally not available to individual investors, it is not clear how the opportunity cost of money holdings should be defined.

Interest rates became partially market determined in 1984 when banks were allowed to determine their own rates. A market for short term bonds did not, however, develop until 1987 when trading with treasury bills initiated. At the same time, a secondary market for government bonds developed and it is really not until then that short term, low risk interest bearing marketable assets become available for individual investors.

Until 1987 individual investors had few available asset choices other than money or real assets, such as real estate or consumer durables. The rate of inflation is therefore used as a proxy for the rate of return on these alternative real assets. After 1987, however, new financial instruments gradually became available and individual investors were able to invest their wealth in more liquid assets such as treasury bills and banking bills.

[9] See, for example, Cuthbertson and Taylor (1987), who estimate such a money demand equation.

[10] For a more detailed discussion of these and other measures of money and scale variable, see Pétursson (1998a).

As these new assets are introduced, individual asset holders must learn about their existence and characteristics. Therefore it is assumed that the introduction of liquid, short-term assets in 1987 only affected investment behaviour gradually. Hence, the rate of return on alternative assets is calculated using a "learning adjusted" rate[11]

$$Ra_t = \Delta p_t + w_t(Rtb_t - \Delta p_t) \tag{3.1}$$

where Δp_t is inflation, Rtb_t is the interest rate on treasury bills (used to proxy liquid, short-term assets) and $0 \leq w_t \leq 1$ is the weight at time t, reflecting knowledge of the characteristics on the newly introduced asset, Rtb_t (cf. Baba *et al.,* 1991 and Hendry and Ericsson, 1991 for similar ideas). The learning curve, w_t, is given by the logistic function

$$w_t = \left[1 + \exp(a - b(t - t_0 + 1))\right]^{-1}; \quad \text{for } t \geq t_0 \tag{3.2}$$

where $t_0 = 1988$ is the first full year of treasury bills trading. a corresponds to the initial knowledge of investors and b to the rate of learning. $w_t = 0$ till t_0 when the new asset is introduced. The values of $a = 5$ and $b = 1.2$ are chosen so as w_t is about a half in 1991 and unity in 1995. These values also correspond to the values used by Hendry and Ericsson (1991).[12]

The learning adjusted rate, Ra_t, is therefore simply the inflation rate in the period 1962 to 1987, but a weighted average of the inflation rate and the treasury bills rate in the period 1988 to 1995, with the weight on the treasury bill increasing, corresponding to increased knowledge and trade in treasury bills.

There are four potential shortcomings of this measure of the opportunity cost of money. First, inflation may include some independent information on the cost of holding money, not reflected in the nominal interest rate, cf. Hendry and Ericsson (1991). However, due to the definition of the opportunity cost in this analysis, it is not possible to have the inflation rate as an independent explanatory variable. It is, however, important to note that inflation *does* play an important direct role throughout the period through Ra_t, although its role declines with w_t.[13] Second,

[11] Using a learning adjusted rate as above obviously imposes restrictions on the short-run dynamics of the model. However, since the data set is rather small there is a strong case for trying to keep the dimension of the system as low as possible. Some of the imposed restrictions are tested below and not rejected.

[12] The main results remain largely unchanged when $w_t = (0, 1)$, i.e. when $Ra_t = \Delta p_t$ for the whole period or when $Ra_t = Rtb_t$ immediately after 1987. Other values of a and b were also tried with only small changes in the results.

[13] When tested whether the coefficient on inflation was significantly different from the interest rate coefficient after 1988, by using a dummy variable, the coefficient was found insignificant, $F(1, 27) = 2.7$ ($\mathsf{p} = 0.11$).

many economists argue that the exchange rate and foreign interest rates may have an important independent explanatory power for domestic money demand in a small open economy, cf. Hakkio and Domowitz (1990) and Bårdsen (1992). This was tested by adding the effective nominal exchange rate and a similarly weighted foreign interest rate. In both cases these variables were found insignificant, which is not surprising considering that domestic investors did not have many opportunities to invest abroad for most part of the period analysed here. Third, it is often argued that some measure of risk is an important factor of money demand (cf. Baba *et al.*, 1991). This was tested by testing the significance of the conditional variance of inflation (measured with an ARCH model). This measure of risk was not found significant. The final potential shortcoming of the measurement of the opportunity cost used here is that it lacks a long term yield, as some components of M3 and government bonds should be competing asset forms. Even though government bonds became available in the early 1960s, it would not be appropriate to use these rates, as trading of government bonds on the primary market was not continuous and the corresponding rate of returns administratively fixed for long periods. It was not until 1987, when an organized secondary market starts to operate, that interest rates on government bonds become fully market determined. A similar learning adjusted rate on government bonds would therefore be needed, thus making it impossible to use both learning adjusted rates simultaneously.[14]

Therefore the opportunity cost measure used in this paper is given by

$$R_t = Ra_t - Rm_t \tag{3.3}$$

where Rm_t is the own rate of money (calculated as a weighted average deposit rate).

Figure 1 plots the data. Two things should be noted. First, a positive long-run relation between velocity of money, $v_t \equiv y_t - (m-p)_t$, and the opportunity cost of money, R_t, seems evident. Both velocity and the opportunity cost rise considerably until 1979, when widespread financial indexation was introduced. After that, velocity falls as the opportunity cost of holding money falls.

Second, it seems obvious that money and prices are not stationary. The smoothness of m_t and p_t in Figure 1a would further suggest that these processes are close to being $\mathsf{I}(2)$, but the more erratic nature of $(m-p)_t$ in Figure 1b implies that $(m-p)_t$ is $\mathsf{I}(1)$. This indicates that money and prices cointegrate, with the cointegrating vector $(1, -1)$. This again implies a long-run price elasticity of unity, which will be imposed throughout the paper. The model is therefore estimated for real rather than nominal money, as suggested by the theoretical model

[14]The bond rate was tried instead of the treasury bill rate to calculate the learning adjusted rate, Ra_t. The results were somewhat inferior to the ones reported here.

in the previous section. Output in Figure 1b, the rate of returns, Ra_t and Rm_t, in Figure 1c and velocity in Figure 1d all seem to be I(1). The opportunity cost measure, R_t, also seems to be I(1), even though it is a net rate (or a real rate) which might be expected to be stationary. From the figure, it is however evident that it is more appropriate to treat this series as difference stationary rather than level stationary. It seems therefore appropriate to treat $(m-p)_t$, y_t and R_t as I(1) series, implying that their growth rates are stationary.

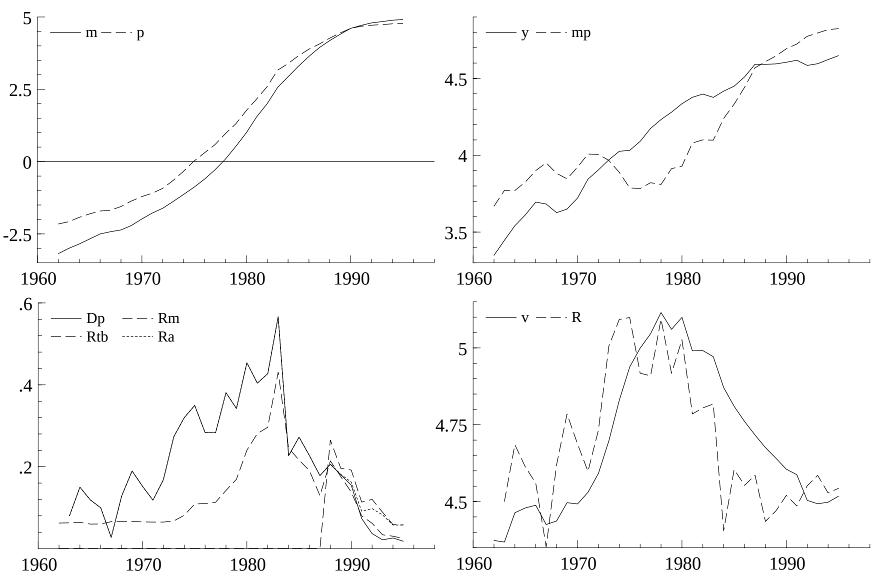

Figure 1. The data

4. Estimation Results

4.1. A general VAR model

The first step towards obtaining the final money demand equation is to estimate the joint data density with an unrestricted vector autoregressive (VAR) model. Having obtained a stable and congruent VAR, the system can be analysed for cointegration relations and further structural interpretations obtained from the theoretical model in the previous section. The next step is to test weak exogeneity restrictions so as to examine whether the joint data density can be conditioned further. The important benefit of the approach used here is that conditioning hypotheses can be tested within a consistent framework, thus avoiding any invalid conditioning of the joint density. This approach is consistent with the progressive reduction procedure advocated by the general-to-specific school, generalized to system analysis, as applied by Hendry and Mizon (1993), Clements and Mizon (1991) and Hendry and Doornik (1994).

The unrestricted, Gaussian VAR(k) system for the p-dimensioned vector, $\{\mathbf{X}_t\}$ is

$$\mathbf{X}_t = \sum_{i=1}^{k} \mathbf{\Pi}_i \mathbf{X}_{t-i} + \mathbf{\Upsilon D}_t + \boldsymbol{\varepsilon}_t; \qquad \boldsymbol{\varepsilon}_t \sim \mathsf{IN}_p(\mathbf{0}, \mathbf{\Omega}) \tag{4.1}$$

where the initial values $\{\mathbf{X}_{-k+1}, ..., \mathbf{X_0}\}$ are taken as given and $\mathbf{D}_t$ is a $s \times 1$ vector of deterministic variables, such as constant and dummy variables. The vector includes one such dummy variable for the year 1984, d_{84t}, corresponding to a structural change in the Icelandic financial system, due to the liberalization of interest rate determination.[15] It would have been preferred to model this structural break explicitly, but as the dummy variable is not found significant in the money demand equation, which is of primary interest, this is not pursued here.

The lag length of two was chosen for the VAR. A likelihood ratio (LR) test for lag 2 against lag 3 gave a test statistic of $F(9, 41) = 0.96$ ($\mathsf{p} = 0.49$), using Rao's F-approximations of the LR test (see Doornik and Hendry, 1995).

As the parameters of the unrestricted VAR are not of interest by themselves, the residual analysis of the system is only reported in Table 1. The first part reports the residual correlations of the system. There are large correlations between real balances and the opportunity cost and real output, respectively, that need further modelling. At the same time the correlation between the output and opportunity cost residuals is close to zero. The second part reports tests for normally distributed homoscedastic innovation errors for each individual equation and the system as a whole. As can be seen, these tests suggest no serious specification problems at this stage: the tests do not reject that the residuals of each equation are normally distributed white noise processes. The same applies for the system.

Another important part of system congruency is that the initial general system displays constancy. This can be tested by estimating the VAR with recursive methods. The problem here is the relatively few number of observations available. Estimation with recursive methods suggests no evidence of non-constancy, although the results must be interpreted with caution.[16]

Finally, the long-run impact matrix, $\mathbf{\Pi} \equiv \sum_{i=1}^{k} \mathbf{\Pi}_i - \mathbf{I}$, has one eigenvalue with modulus equal to 0.54 and two eigenvalues equal to 0.05. This suggests that $\mathbf{\Pi}$ has

[15] The dummy variable takes the value unity in 1984 and zero otherwise. Due to the definition of R_t, d_{84t} could also be interpreted as a structural break in the price process, when inflation fell from 60% in 1983 to 20% in 1984. This structural break is fully explained by changes in wages and import prices, see Pétursson (1998b).

[16] The results are available from the author upon request.

Table 1. Residual analysis of the VAR(2) system

Residual correlations

	$(m-p)_t$	y_t	R_t
$(m-p)_t$	1.00	–	–
y_t	0.52	1.00	–
R_t	-0.71	0.01	1.00

Residual diagnostics

	$(m-p)_t$	y_t	R_t	VAR model	
$\widehat{\sigma}$	5.66%	3.62%	4.95%	$F_{ar1-2}(18,42)$	0.46
$F_{ar1-2}(2,21)$	0.56	1.64	0.45	$F_{het}(72,33)$	0.68
$F_{arch1}(1,21)$	0.43	0.64	0.90	$\chi^2_n(6)$	8.75
$F_{het}(12,10)$	1.86	0.43	0.76		
$\chi^2_n(2)$	0.34	1.66	1.40		

Note: $\widehat{\sigma}$ denotes the standard error of the equation. F_{ar1-2} denotes an F-test for no serial correlation, against a second order autocorrelation. F_{arch1} denotes the Engle (1982) for no autoregressive conditional heteroscedasticity, against a first order ARCH effect. F_{het} denotes the White (1980) test for no heteroscedasticity. χ^2_n denotes the Doornik and Hansen (1993) normality test. The table also shows analogous system tests.

reduced rank, implying one cointegrating combination and two common stochastic trends. The estimation and identification of this cointegrating relationship is the subject of the next section.

4.2. Cointegration

A useful reformulation of the unrestricted VAR model in (4.1) is the vector ECM (VECM) representation of the model

$$\Delta \mathbf{X}_t = \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta \mathbf{X}_{t-i} + \boldsymbol{\alpha}\boldsymbol{\beta}^{'} \mathbf{X}_{t-1} + \mathbf{\Upsilon} \mathbf{D}_t + \boldsymbol{\varepsilon}_t \tag{4.2}$$

where $\mathbf{\Gamma}_i \equiv -\sum_{j=i+1}^{k} \mathbf{\Pi}_j$ and $\mathbf{\Pi}$ has been factorized into two $p \times r$ matrices with full rank $\boldsymbol{\alpha}\boldsymbol{\beta}^{'}$, where $\boldsymbol{\alpha}$ is the matrix of adjustment coefficients (the factor loadings) and $\boldsymbol{\beta}$ is the matrix of cointegrating relations. There are therefore r cointegrating I(0) relations inducing the restricted, reduced rank I(0) specification (see Johansen, 1988) in equation (4.2).

Before (4.2) is estimated with the full information maximum likelihood (FIML) method of Johansen (1988, 1991), the nature of the deterministic variables needs to be determined and how they enter the model. The intercept cannot be restricted *a priori* to lie in the cointegration space as a deterministic trend in output cannot be ruled out, cf. Figure 1. There should, however, be no trend in

Table 2. Cointegration tests

H_0 hypotheses	Eigen-values	χ^2_{max}	95% critical values	χ^2_{trace}	95% critical values
$r = 0$	0.583	27.12	21.0	30.76	29.7
$r \leq 1$	0.078	2.51	14.1	3.64	15.4
$r \leq 2$	0.036	1.13	3.8	1.13	3.8

Standardized eigenvectors $\widehat{\boldsymbol{\beta}}'$

	$(m-p)_t$	y_t	R_t
1	1	-0.877	3.846
2	0.487	1	1.844
3	-18.94	12.52	1

Standardized factor loadings $\widehat{\boldsymbol{\alpha}}$

	1	2	3
$(m-p)_t$	-0.093	0.011	0.002
y_t	0.080	-0.011	0.001
R_t	-0.097	-0.024	-0.001

Note: r is the number of cointegrating relations. The LR tests, χ^2_{max} and χ^2_{trace}, are derived in Johansen and Juselius (1990). The critical values are taken from Osterwald-Lenum (1992).

the interest rate. The intercept is therefore included unrestrictedly. The dummy variable, d_{84t}, is restricted to have only short-run effects and no effects on the long-run relations.[17]

Table 2 reports the results of the reduced rank regressions and the Johansen (1988) tests for the number of cointegrating vectors. The long-run matrix has one quite large eigenvalue and two small, matching earlier results.[18]

Figure 2 plots the three estimated relations, $\widehat{\boldsymbol{\beta}}'\mathbf{X}_t$, together with the fitted and actual values of $\mathbf{X}_t$, where x_{it} are the actual values and $-\sum_{j\neq i}\widehat{\beta}_{ij}x_{jt}$ are the fitted values. The figure also plots the recursively estimated eigenvalues (μ_i, $i = 1, 2, 3$), using the switching algorithm of Hansen and Johansen (1992), after having partialled out the full-sample short-run dynamics and unrestricted variables. The first relation seems stationary and, from Table 2, looks like a money demand relation, with a positive income elasticity and a large negative interest rate semi-elasticity. The other two components are distinctly not stationary. Accordingly, the largest eigenvalue is quite stable and always larger than zero, whereas the other two are almost zero all the time.[19] Finally, the cointegrating vector seems

[17] Trend stationarity of the data was also checked by allowing a linear time trend in the cointegrating space. The trend was found insignificant, $\chi^2(1) = 1.03$ ($\mathsf{p} = 0.31$).

[18] As the critical values are more suited to larger data sets than used here, the formal test statistics must be interpreted with caution. The small sample corrections suggested by Reimers (1992) were not used since the results in Kostial (1994) indicate a tendency for the critical values to underestimate the dimension of the cointegrating space even when unadjusted.

[19] Again, it must be kept in mind that relatively few observations are used in the recursive estimates.

to have reasonable explanatory power for real money balances but very little for output and the opportunity cost.

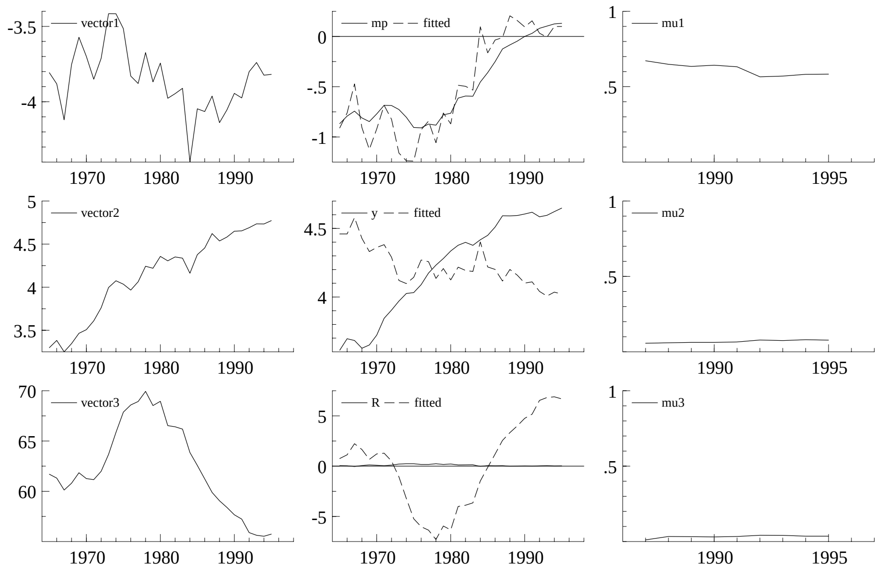

Figure 2. Cointegration analysis

4.2.1. The restricted cointegration vector

As it stands, the theoretical money demand model in (2.12) imposes no over-identifying restrictions on the cointegrating space. However, standard models of money demand suggest various values for the long-run income elasticity, η. Therefore it would be interesting to test these restrictions.

The structural hypotheses are formulated as tests about the cointegrating space, i.e. whether the stationary part of the space spanned by the non-stationary variables includes the theoretical long-run relation from the previous section. Table 3 reports the results. As seen in the first part of Table 3, income elasticity of unity is not rejected, whereas values of $\frac{1}{2}$ and $\frac{1}{3}$ are strongly rejected, therefore rejecting the transaction model of Baumol (1952) and Tobin (1956) and the precautionary model of Miller and Orr (1966) in this data set.

The second part in Table 3 reports tests of weak exogeneity of the forcing variables, y_t and R_t, for the long-run parameters. The LR tests indicate that y_t and R_t are weakly exogenous for the long-run parameters in the money demand equation, which is a necessary condition for valid conditioning on these variables (see Engle *et al.*, 1983 and Johansen, 1992). When the restrictions on $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are tested jointly, the LR-test gives $\chi^2(3) = 6.8$ ($\mathsf{p} = 0.08$).

Table 3. Testing restrictions on $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$

Testing restrictions on $\boldsymbol{\beta}$		
Restrictions on $\eta = \frac{\omega}{\psi}$	Test statistic	p-value
H_0: $\eta = 1$	$\chi^2(1) = 2.95$	0.09
H_0: $\eta = 1/2$	$\chi^2(1) = 16.57$	0.00
H_0: $\eta = 1/3$	$\chi^2(1) = 19.66$	0.00
Testing restrictions on $\boldsymbol{\alpha}$		
Weak exogeneity restrictions	Test statistic	p-value
H_0: $\alpha_y = 0$	$\chi^2(1) = 2.10$	0.15
H_0: $\alpha_R = 0$	$\chi^2(1) = 1.62$	0.20
H_0: $\alpha_y = \alpha_R = 0$	$\chi^2(2) = 3.79$	0.15
Joint test:		
H_0: $\eta = 1, \alpha_y = \alpha_R = 0$	$\chi^2(3) = 6.84$	$\mathsf{p} = 0.08$

Note: The LR tests on the cointegrating space and the factor loadings are derived in Johansen and Juselius (1990).

The cointegrating vector used in this study therefore imposes a unit long-run income elasticity, thus implying equal values of the risk aversion parameters, and weak exogeneity of y_t and R_t with respect to the long-run parameters.[20] The resulting long-run relation is

$$(m^d - p)_t = y_t - \underset{(0.41)}{4.030}\ R_t + const \tag{4.3}$$

The number in parenthesis is a standard error (see Johansen, 1991). This relation implies a positive long-run relation between velocity and the opportunity cost of money.[21]

The long-run income elasticity of money demand is found to be unity, suggesting homothetic preferences. This also implies that households are equally averse to variations in consumption and real money balances. The implied value of $\widehat{\omega} = \widehat{\psi}$ is 0.25 with a standard error of 0.021 (calculated using Cramér's rule). These estimates are in the lower region of cross country estimates from consumption-based

[20] A LR test for the null hypothesis of stationarity is strongly rejected for all series, including velocity. This matches the conclusions drawn from Figure 1. The results are available upon request.

[21] It is also of interest to test whether the data rejects equal long-run semi-elasticities of Ra_t and Rm_t. This can be done by estimating a VAR model for $[(m-p)_t, y_t, Ra_t, Rm_t]$ and testing whether Ra_t and Rm_t have equal coefficients but with opposite signs in the cointegration analysis. The LR test gives $\chi^2(1) = 0.59$ ($\mathsf{p} = 0.44$), therefore not rejecting equal long-run semi-elasticities. This implies that the opportunity cost, R_t, can be used instead of the two interest rates separately in the cointegrating analysis to reduce the dimensionality of the system.

asset pricing models. They typically find estimates of the risk aversion parameters lying in the region 0.5 to 4 for different countries. Further, when the set of assets is augmented with stocks, the risk aversion parameter is typically higher, lying in the region 2 to 6, see Roy (1995). Allowing for habit persistence in utility widens the estimation region further to 0.35 to 12, see Braun *et al.* (1993).

5. A Single-Equation Money Demand Model

5.1. A conditional ECM for real money balances

This final section focuses on the conditional density of real money balances, given y_t and R_t and the past history of the process. Therefore, the VAR model is partitioned into the conditional and marginal models. The data vector is partitioned as $\mathbf{X}_t = [(m-p)_t : \mathbf{Z}_t]$, where $\mathbf{Z}_t \equiv (y_t, R_t)'$ are the variables conditioned upon. The cointegrated VAR(2) with one cointegrating vector can thus be rewritten as

$$\begin{aligned} \Delta(m-p)_t &= \mathbf{\Gamma}_{m1}\Delta\mathbf{X}_{t-1} + \alpha_m \boldsymbol{\beta}'\mathbf{X}_{t-1} + \mathbf{\Upsilon}_m\mathbf{D}_t + \varepsilon_{mt} && (5.1)\\ \Delta\mathbf{Z}_t &= \mathbf{\Gamma}_{\mathbf{Z}1}\Delta\mathbf{X}_{t-1} + \boldsymbol{\alpha}_{\mathbf{Z}}\boldsymbol{\beta}'\mathbf{X}_{t-1} + \mathbf{\Upsilon}_{\mathbf{Z}}\mathbf{D}_t + \boldsymbol{\varepsilon}_{\mathbf{Z}t} && (5.2) \end{aligned}$$

where $\mathbf{\Gamma}_1 = (\mathbf{\Gamma}_{m1} : \mathbf{\Gamma}_{\mathbf{Z}1})$, $\boldsymbol{\alpha} = (\alpha_m : \boldsymbol{\alpha}_{\mathbf{Z}})$, $\mathbf{\Upsilon} = (\mathbf{\Upsilon}_m : \mathbf{\Upsilon}_{\mathbf{Z}})$ and $\boldsymbol{\varepsilon}_t = (\varepsilon_{mt} : \boldsymbol{\varepsilon}_{\mathbf{Z}t})$ have been partitioned accordingly.

The conditional model for $\Delta(m-p)_t$ given the past and $\Delta\mathbf{Z}_t$ is given by

$$\Delta(m-p)_t = \mathbf{\Gamma}^*_{m1}\Delta\mathbf{X}_{t-1} + \alpha^*_m\boldsymbol{\beta}'\mathbf{X}_{t-1} + \boldsymbol{\xi}\Delta\mathbf{Z}_t + \mathbf{\Upsilon}^*_m\mathbf{D}_t + \varepsilon^*_{mt} \qquad (5.3)$$

where $\mathbf{\Gamma}^*_{m1} \equiv (\mathbf{\Gamma}_{m1} - \boldsymbol{\xi}\mathbf{\Gamma}_{\mathbf{Z}1})$, $\alpha^*_m \equiv (\alpha_m - \boldsymbol{\xi}\boldsymbol{\alpha}_{\mathbf{Z}})$, $\mathbf{\Upsilon}^*_m \equiv (\mathbf{\Upsilon}_m - \boldsymbol{\xi}\mathbf{\Upsilon}_{\mathbf{Z}})$, $\varepsilon^*_{mt} \equiv (\varepsilon_{mt} - \boldsymbol{\xi}\boldsymbol{\varepsilon}_{\mathbf{Z}t})$ and $\boldsymbol{\xi} \equiv \mathbf{\Omega}_{m\mathbf{Z}}\mathbf{\Omega}^{-1}_{\mathbf{ZZ}}$, where the covariance matrix has been partitioned conformably. In general a full system analysis is necessary since the factor loadings from the marginal model, $\boldsymbol{\alpha}_{\mathbf{Z}}$, also enter the conditional model. Estimation of the conditional model alone would therefore result in a loss of information. However, if $\boldsymbol{\alpha}_{\mathbf{Z}} = \mathbf{0}$ the forcing variables are weakly exogenous, in the sense of Engle *et al.* (1983), and efficient estimates of parameters of interest can be obtained from estimating the conditional model alone, see Johansen (1992). The results from the weak exogeneity tests in Table 3 therefore indicate that the analysis can be further simplified to analysing the conditional density of real money balances.

Estimating the conditional VAR amounts to estimating a single-equation money demand model. To estimate the conditional ECM, the long-run money relation from (4.3) is used. The final model is

$$\Delta\widehat{(m-p)}_t = \underset{(0.12)}{0.769}\,\Delta y_t - \underset{(0.09)}{0.755}\,\Delta R_t + \underset{(0.02)}{0.174}\,(v - 4.030R)_{t-1} - \underset{(0.09)}{0.752} \qquad (5.4)$$

OLS, 1964-1995 ($T = 32$), $\overline{\mathsf{R}}^2 = 0.80$, $\widehat{\sigma}_\varepsilon = 2.74\%$, $F_{enc}(5,22) = 0.81$, $F_{ar1-2}(2,26) = 0.11$, $F_{arch1}(1,26) = 0.67$, $\chi^2_n(2) = 5.62$, $F_{het}(6,21) = 0.93$, $F_{fn}(9,18) = 0.83$, $F_{reset}(1,27) = 0.30$, $F_{for}(5,23) = 1.34$, $F_{chow79}(12,12) = 0.51$, $VS = 0.11$, $JS = 0.65$

where the numbers in parentheses are White's (1980) heteroscedasticity adjusted standard errors. $\overline{\mathsf{R}}^2$ is the degrees of freedom adjusted coefficient of determination, F_{enc} tests for parsimonious encompassing, testing the validity of the simplifying restrictions on an unrestricted distributed lag model with two lags of each variable, ADL(2). The ECM imposes five over-identifying restrictions on the ADL(2) which are easily accepted.[22]

F_{fn} and F_{reset} are the White (1980) and Ramsey (1969) tests for no specification error. F_{for} is the Chow test for no forecast instability for the last five years. F_{chow79} is a Chow test for a structural break in 1979. VS and JS are the Hansen (1992) tests for no in-sample variance instability and joint variance and parameter instability, respectively. Other tests have previously been explained.

All the tests have p-values above 0.2,[23] implying that the hypothesis of normally distributed, homoscedastic innovation errors is not rejected. There is no evidence of instability in- or outside the sample. Finally, there is no evidence of functional mis-specification.[24] The model explains 80% of the growth in real money balances with a standard error of 2.7%. All parameters are well determined (with t-values above 6.5) and with correct signs, allowing (5.4) to be interpreted as a money demand equation.

Model stability is another important factor of data congruency. The stability tests reported above are not able to reject a constant money demand equation. Further, when estimated by recursive methods, no instability is detected. Figure 3 reports the results.

The four first figures show the recursive parameter estimates along with their $\pm 2\widehat{\sigma}$ bands. The fifth figure shows the recursive one-step residuals with $\pm 2\widehat{\sigma}$ bands, and the final figure shows the recursive one-step Chow tests scaled by their 1% significance values. There appears no evidence of structural instability. The recursive parameters are constant and significant from zero for the whole

[22] This test is similar to the Hendry and Mizon (1993) over-identifying test for encompassing the VAR.

[23] Except the Doornik and Hansen (1993) normality test, which gives $\chi^2_n(2) = 5.6$ ($\mathsf{p} = 0.06$). The hypothesis of normally distributed residuals is therefore close to being rejected at the 5% critical level.

[24] The impulse dummy, d_{84t}, is found insignificant, with a F statistic of $F(1,28) = 1.2$ ($\mathsf{p} = 0.29$). It was also tested whether the short-run semi-elasticities of the own rate and the alternative rate were significantly different. The test statistic was $F(1,26) = 0.03$ ($\mathsf{p} = 0.86$). The data does therefore not reject equal short-run semi-elasticities.

period, and the Chow tests are nowhere close to their 1% significant level. The model thus displays considerable stability, in spite of large fluctuations in the conditional variables, financial innovations in the domestic financial market, and changes in the monetary policy regime.

The results therefore imply that the conditional model is data congruent, with constant parameters, a well defined long-run equilibrium, and homoscedastic innovation errors. Further, its explanatory variables constitute a near orthogonal parameterization.[25]

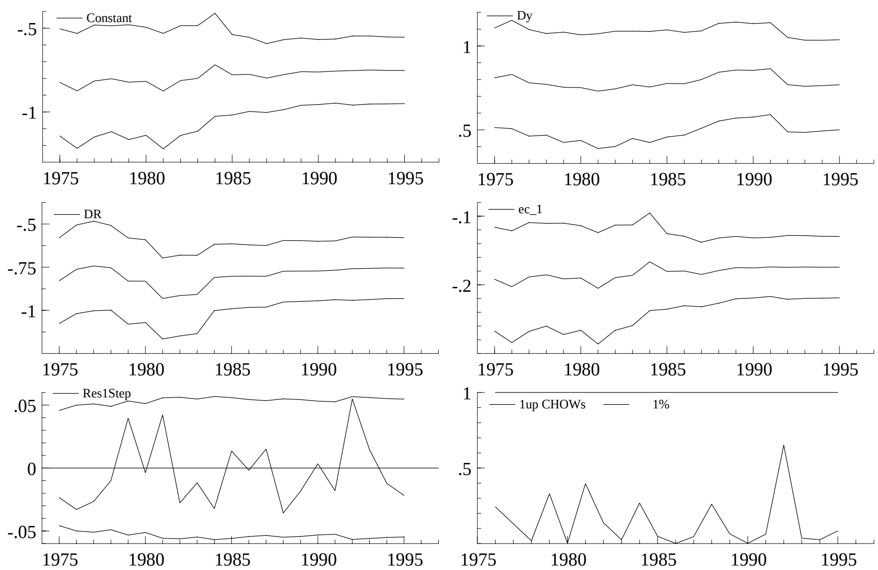

Figure 3. Recursive statistics

5.2. Money demand and super exogeneity

The single-equation money demand equation is estimated assuming that output and the interest rate are weakly exogenous for the parameters of interest. If, in addition, the parameters of interest are invariant to the class of interventions occurring during the sample period, the variables are also super exogenous for the parameters of interest, see Engle, *et al.* (1983). This implies that policy analysis can be performed by suitably changing the processes driving these variables.

To test for super exogeneity, the invariance test of Engle and Hendry (1993) is used. This amounts to estimating the marginal processes for the conditional variables. If these conditional variables are super exogenous, the parameters of the conditional model should be invariant to the parameters of the marginal models.

[25]The correlation between Δy_t and ΔR_t is -0.08, the correlation between Δy_t and the cointegration term is 0.38, and the correlation between ΔR_t and the cointegration term is -0.22.

Table 4. Estimated marginal equations

$$\widehat{\Delta y_t} = \underset{(0.009)}{0.019} + \underset{(0.16)}{0.461}\ \Delta y_{t-1}$$
$$\mathsf{R}^2 = 0.23,\ \widehat{\sigma} = 3.55\%$$

$$\widehat{\Delta R_t} = -\underset{(0.01)}{0.002} - \underset{(0.18)}{0.228}\ \Delta R_{t-1}$$
$$\mathsf{R}^2 = 0.05,\ \widehat{\sigma} = 5.59\%$$

$$\widehat{\Delta p_t} = \underset{(0.03)}{0.032} + \underset{(0.11)}{0.836}\ \Delta p_{t-1}$$
$$\mathsf{R}^2 = 0.66,\ \widehat{\sigma} = 8.44\%$$

Wu-Hausman test: $F(3, 24) = 0.82$ ($\mathsf{p} = 0.50$)

Thus, the determinants of the marginal models instability should be insignificant if added to the conditional model. This can be tested by applying a Wu-Hausman type of test by testing the significance of the residuals from the marginal equations in the conditional model.

The marginal processes for Δy_t, ΔR_t and Δp_t (included due to its affect on money demand through $(m-p)_t$ and R_t) are therefore estimated. They are approximated with univariate fourth-order autoregressive processes and in all cases these could be simplified to a first order autoregressive processes.[26] Table 4 reports the resulting estimates.

The invariance test for super exogeneity indicates that the conditional variables are super exogenous for the parameters of the money demand equation. Furthermore, the Δp_t and ΔR_t equations break down in 1984, matching the structural break captured by d_{84t}, discussed above. This also implies super exogeneity according to Hendry's (1988) constancy test. He shows that if the conditional model is constant but the marginal models are not, the conditional parameters cannot depend on the marginal processes and the conditional parameters are therefore super exogenous.

To summarize, the results from the super exogeneity tests indicate that output, the opportunity cost and inflation are super exogenous for all parameters of the money demand equation. Policy experiments can therefore be conducted with the money demand equation, conditional on these variables, as changes in the marginal processes will not affect the parameters of the money demand equation. As super exogeneity implies weak exogeneity for all the parameters of the money demand equation, these results also support previous findings. This further

[26] Hendry (1988) shows that these tests have power even when the marginal processes are incorrectly specified.

facilitates identification of (5.4) as a money demand equation.

5.3. Testing the forward looking model

In this final section the forward looking interpretation of the money demand equation in (2.13) is tested. Under this interpretation households minimize the expected, discounted present value of a quadratic cost function given in (2.15).[27] The minimization problem is a standard, discrete time, calculus of variation problem of which the solution method is given in Sargent (1979) and Hansen and Sargent (1980).

The Euler equation obtained from the minimization problem can be written in the following way

$$\Delta(m-p)_t = \delta \mathsf{E}(\Delta(m-p)_{t+1} \mid \mathcal{I}_t) - \phi(m-m^d)_t \tag{5.5}$$

The Euler equation can be solved further to give a forward looking ECM

$$\Delta(m-p)_t = (1-\lambda)\sum_{i=0}^{\infty}(\lambda\delta)^i \boldsymbol{\beta}'_{\mathbf{Z}} \mathsf{E}(\Delta \mathbf{Z}_{t+i} \mid \mathcal{I}_t) - (1-\lambda)\,\boldsymbol{\beta}'\mathbf{X}_{t-1} \tag{5.6}$$

where the cointegrating vector has been partitioned as $\boldsymbol{\beta} = (1 : -\boldsymbol{\beta}_{\mathbf{Z}})$ and λ is the stable root of the quadratic equation $\delta l^2 - (1+\delta+\phi)l + 1 = 0$ from the Euler equation.

To obtain a closed form solution to this model some assumptions must be made concerning the data generating process for the forcing variables $\Delta\mathbf{Z}_t$. The obvious choice is the marginal VAR model in (5.2) (abstracting from deterministic terms to simplify the notation)

$$\Delta\mathbf{Z}_t = \boldsymbol{\Gamma}_{\mathbf{Z}1}\Delta\mathbf{X}_{t-1} + \boldsymbol{\alpha}_{\mathbf{Z}}\boldsymbol{\beta}'\mathbf{X}_{t-1} + \boldsymbol{\varepsilon}_{\mathbf{Z}t}$$

The previous analysis indicates that the forcing variables are weakly exogenous for the long-run parameters, i.e. $\boldsymbol{\alpha}_{\mathbf{Z}} = \mathbf{0}$. Furthermore, $\Delta(m-p)_t$ is not found to Granger (1969) cause Δy_t or ΔR_t, $\chi^2(2) = 0.92$ ($\mathsf{p} = 0.63$). Thus, real money holdings are strongly exogenous in the sense of Engle *et al.* (1983) with respect to the parameters of interest in the money demand equation.

The data generating process for $\Delta\mathbf{Z}_t$ can therefore be approximated by

$$\Delta\mathbf{Z}_t = \boldsymbol{\Gamma}_{\mathbf{ZZ}1}\Delta\mathbf{Z}_{t-1} + \boldsymbol{\varepsilon}_{\mathbf{Z}t} \tag{5.7}$$

[27]The cost minimization is in terms of real money balances rather than nominal money balances as in (2.15), as the growth rate of real money balances is stationary whereas the growth rate of nominal money balances is not. This has the unappealing implication that adjustments of nominal money holdings are costless if prices adjust by the same magnitude, see Goodfriend (1990).

where $\mathbf{\Gamma_{Z1}} = (\mathbf{\Gamma_{Z}}_{m1} : \mathbf{\Gamma_{ZZ1}})$, and $\mathbf{\Gamma_{Z}}_{m1} = \mathbf{0}$ as $\Delta(m-p)_t$ does not Granger cause Δy_t or ΔR_t.

From (5.7) it is simple to generate future expectations of $\Delta\mathbf{Z}_t$ using the law of iterated expectations

$$\mathsf{E}(\Delta\mathbf{Z}_{t+i} \mid \mathcal{H}_t) = \mathbf{\Gamma}^i_{\mathbf{ZZ}1}\Delta\mathbf{Z}_t \tag{5.8}$$

where $\mathcal{H}_t = \{\Delta\mathbf{Z}_t, \Delta\mathbf{Z}_{t-1}, ...\} \subseteq \mathcal{I}_t$ is the information set available to the econometrician.

Inserting this optimal projection into (5.6) gives

$$\Delta(m-p)_t = const + (1-\lambda)(\mathbf{I} - \lambda\delta\mathbf{\Gamma_{ZZ1}})^{-1}\boldsymbol{\beta}'_{\mathbf{Z}}\Delta\mathbf{Z}_t - (1-\lambda)\,\boldsymbol{\beta}'\mathbf{X}_{t-1} + e_t \tag{5.9}$$

The restrictions imposed on the unrestricted ECM in (5.4) are therefore $\mathbf{\Gamma}^*_{m1} = \mathbf{0}$, $\alpha^*_m = -(1-\lambda)$ and $\boldsymbol{\xi} = (1-\lambda)(\mathbf{I}-\lambda\delta\mathbf{\Gamma_{ZZ1}})^{-1}\boldsymbol{\beta}'_{\mathbf{Z}}$. The error term e_t is given by the difference between the expected future path of the forcing variables conditional on the information set available to households $\mathcal{I}_t$, and the information set available to the econometrician $\mathcal{H}_t$, i.e. $e_t \equiv (1-\lambda)\sum_{i=\mathbf{0}}^{\infty}\boldsymbol{\beta}'_{\mathbf{Z}}\,[\mathsf{E}(\Delta\mathbf{Z}_{t+i} \mid \mathcal{I}_t) - \mathsf{E}(\Delta\mathbf{Z}_{t+i} \mid \mathcal{H}_t)]$.[28]

The forward looking interpretation of the money demand equation is estimated simultaneously with the expectations generating process for $\Delta\mathbf{Z}_t$ in (5.7). As the cointegrating vector $\boldsymbol{\beta}$ is estimated super-consistently, it can be treated as fixed when estimating the money demand equation, just as when estimating the conditional, backward looking ECM above. Furthermore, as is standard in the rational expectations literature, the discount factor δ is taken as given. The value of δ is chosen as 0.96 which corresponds approximately to a 4% annual rate of time preferences. Thus, only λ and $\mathbf{\Gamma_{ZZ1}}$ are estimated. Finally, the super exogeneity analysis above indicated that the marginal equations for Δy_t and ΔR_t can be approximated by univariate first order AR equations. Therefore the off-diagonal elements of $\mathbf{\Gamma_{ZZ1}}$ will be set to zero in the following analysis. The resulting estimates are reported in Table 5.

The restricted system estimate of the forward looking model implies a error correction coefficient of $(1-\widehat{\lambda}) = 0.175$, which is almost identical to the estimate from the backward looking model from above.[29] The autoregressive coefficients in the marginal equations are also almost identical to the unrestricted estimates in Table 5. The standard error of the money demand equation is 3.7%, which is about 1% point higher than in the unrestricted backward looking model. The

[28] For a more general version of these non-linear cross-equation restrictions see the analysis in Pétursson (1998b).

[29] The implied coefficients on Δy_t and ΔR_t in the forward looking model are 0.24 and -0.60, respectively, which are quite close to the unrestricted estimates in (5.4), although the short-run income elasticity is somewhat smaller.

Table 5. Estimating the forward looking money demand system

Estimated system

$$\Delta(m-p)_t = const + \left(\frac{1-\lambda}{1-\lambda\delta\gamma_1}\right)\Delta y_t - \left(\frac{(1-\lambda)\theta}{1-\lambda\delta\gamma_2}\right)\Delta R_t + (1-\lambda)\,(v-\theta R)_{t-1} + e_t$$

$$\Delta y_t = const + \gamma_1 \Delta y_{t-1} + \nu_{1t}$$

$$\Delta R_t = const + \gamma_2 \Delta R_{t-1} + \nu_{2t}$$

Estimated parameters

$\underset{(0.042)}{\widehat{\lambda} = 0.825}$ $\quad \underset{(0.122)}{\widehat{\gamma}_1 = 0.358}$ $\quad \underset{(0.164)}{\widehat{\gamma}_2 = -\,0.232}$ $\quad \widehat{\sigma}_e = 3.7\%$ $\quad \widehat{\sigma}_{\nu_1} = 3.6\%$ $\quad \widehat{\sigma}_{\nu_2} = 5.7\%$

Likelihood ratio test for parameter restrictions

$\chi^2_{or}(2) = 3.68 \quad (\mathsf{p} = 0.16)$

standard errors of the marginal equations are, however, almost the same as in the unrestricted estimates. Finally, the LR test for the over-identifying cross-equation restrictions does not reject.[30]

The forward looking, rational expectations interpretation of the money demand equation is therefore not rejected and the restricted estimates of the dynamic adjustment coefficients are very similar to the estimates from the unrestricted estimates of the contingent planning equation. The problem with this forward looking interpretation, however, is that in the previous section it was found that one of the marginal equations is unstable, as the ΔR_t equation breaks down in 1984. One should therefore be careful in interpreting this equation as an expectations generating process, for it would not be sensible for households to use unstable expectations generating processes. Furthermore, as Hendry (1988) points out, constancy of the conditional model and non-constancy of the marginal model precludes a forward looking interpretation of the conditional model. Cuthbertson (1991) shows, however, that these tests may have low power in small samples.

It is therefore not clear whether a forward looking interpretation of the money demand equation, although statistically not rejected, is valid.

6. Conclusions

This paper analysis the demand for broad money in Iceland in the period 1962 to 1995. A model of a representative household with money-in-the-utility-function

[30] No serial correlation in e_t was detected. The Ljung-Box Q test for second order serial correlation gave $Q = 2.39$ ($\mathsf{p} = 0.30$).

preferences is applied to derive the steady state solution for money demand used in the empirical analysis.

A multivariate cointegrated VAR for real money balances, real output and a measure of the opportunity cost of money is then estimated. From the cointegration analysis a single cointegrating relation is found. Testing over-identifying restrictions on the cointegrating space suggests a unit long-run income elasticity which implies a long-run relation between velocity and the opportunity cost of money holdings. This implies homothetic preferences and that households are equally averse to variations in real consumption and real money balances. The estimated value of the risk aversion parameters is 1/4 which is in the lower region of estimates found in international studies.

The results from the cointegrating VAR imply that output and the opportunity cost are weakly exogenous for the long-run parameters. A conditional model, amounting to a single-equation money demand equation, can therefore be estimated. This conditional model implies that real money balances are affected by current changes in output and the opportunity cost of money holdings, and lagged deviations of actual real balances from the long-run target. The results closely match many other recent empirical money demand studies and can be interpreted within the class of target-threshold money demand literature.

The money demand equation shows remarkable stability, in spite of large fluctuations in the conditional variables, substantial changes in the institutional framework in the domestic financial market, such as the introduction of financial indexation in 1979, interest rate liberalization in 1984 and the introduction of a secondary market with financial instruments in 1987. The model explains a large proportion of changes in the growth rate of real balances, e.g. the large decline of real balances between 1972 and 1976, the large fall in velocity from 1981 to 1984, and the large increase in real balances from 1984 to 1987.

The validity of using the money demand equation for forecasting and policy simulations is also analysed. The results of the strong and super exogeneity tests imply that forecasting and policy experiments with the money demand equation can in fact be conducted, conditional on the forcing variables.

Finally, a forward looking interpretation of the ECM is analysed. The over-identifying cross-equation restrictions implied by the forward looking interpretation are not rejected. The money demand equation could therefore be interpreted as a forward looking ECM. However, non-constancy of the expectations generating processes makes this interpretation problematic.

At least three policy implications follow from the analysis. First, money seems to be endogenously determined by the private sector, whereas the monetary policy authority targets the interest rate. Secondly, inverting the money demand equation to obtain models explaining inflation or interest rates is invalid. Finally, a forward looking interpretation of the short-run adjustment dynamics is not rejected, implying that households determine their desired money holdings in a forward looking fashion. However, the non-constancy of the expectations generating processes makes this interpretation problematic, as it would not seem sensible for households to use unstable expectations generating processes if alternative stable rules exist.

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