# FLOW CONTROLLER ASSUMPTIONS IN PHYSICAL MODELS OF GEOTHERMAL DISTRICT HEATING SYSTEMS

**Dr. Páll Valdimarsson**

**University of Iceland, Hjarðarhagi 6, IS-101 Reykjavik, Iceland**

**Key words: district** heating, modelling, Iceland

## ABSTRACT

Geothermal district heating systems have been modelled in order to evaluate the influence of the outdoor temperature on the system flow. The district heating system was lumped into a single building, and the models were based on knowledge of its thermodynamics. The results were compared with data from the Reykjavík Municipal District Heating Service, Hitaveita Reykjavíkur. The building thermodynamics can not describe the flow controller element, which defines the relation between the indoor temperature and the district heating water flow. The flow controller is a combination of the radiator control system and the behaviour of the residents. Three flow controllers were studied. The ideal controller, giving the correct flow to keep the indoor temperature at the set point, results in a steady state model for the system consumption. The P-controller (proportional) results in a dynamic model with one state, the system flow. The PI-controller (proportional and integrating) results in a dynamic model with two states, the indoor temperature and the system flow. A linear model based on the PI-controller gave the best results of the models tested.

## 1. INTRODUCTION

The district heating system is spread out over the city to be heated. In a macroscopic model, the whole system is lumped into one model block, relating the output signals to relevant inputs. In each macroscopic model, the entire distribution system and all of the consumers are considered as seen from the district heating water supply station. In each model, use is made of either black box methods to relate the input signal to the output signal, or physical knowledge of the process involved. Parameter estimation techniques are used to obtain estimates of the unknown values of the various parameters. The macroscopic models treated in the present work are physical models, with their parameters estimated by statistical methods.

## 2. PHYSICAL MODELS

The basis for physical district heating models is knowledge of the thermal behaviour of buildings. The logical choice of influence factors is to take weather as the input signal, and system water supply temperature as a control signal. The model output signals are then the water mass flow to the district heating system, and the return water temperature.

A macroscopic model treats the district heating system as one entity. The whole system is then lumped into a single "equivalent consumer". The system is then treated in the same manner as a single building would be. Like for a single building, the indoor temperature plays a central role. The building internal energy (stored in the building itself) is directly proportional to this temperature. The indoor temperature could be treated as an output signal, but is more akin to an internal state variable, as there is no way of measuring it directly. As a refinement, the model of one equivalent building can be serially coupled to a pipeline cooling model, to account for the heat loss in the distribution system.

Basically, a building model is composed of four elements, a building energy storage element, a heat loss element, a radiator element and a flow controller element. The building energy storage element describes the building thermal storage effect, that is the reaction of the indoor temperature to the net heat flow into the building. The heat loss element describes the heat lost to the surroundings as a function of the weather and of the indoor temperature. The radiator element describes how heat is transferred from the district heating water to the building as a function of water mass flow, building water supply temperature and indoor temperature. As an additional output signal the water return temperature is calculated The flow controller element describes how the indoor temperature controls the district heating water flow.

The flow controller is a combination of the behaviour of the people living in the building, and of the radiator control system, and cannot be determined theoretically. The radiator control systems have known characteristics, but they are of different types from one building to another. The residents have almost an unpredictable behaviour. Their tolerance to variations in the indoor temperature is very individual.

A block diagram for a physical district heating model is shown in Figure 1.

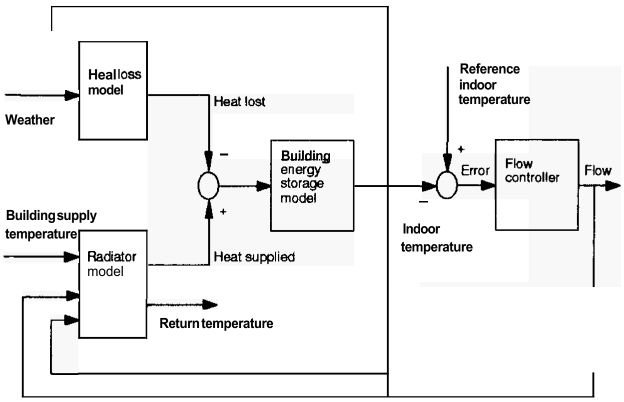



Figure 1. Block diagram of a lumped district heating model.

### 2.1 Building Heat Loss Model

The heat lost is a function of the outdoor weather conditions and the indoor temperature. The heat is lost by heat transfer through the building surfaces, and by exchange of *air* between the heated space and the building surroundings. The heat loss is mainly a function of the outdoor *air* temperature. By taking the outdoor temperature as a primary influencing factor for the weather, the heat loss model becomes:

$$Q_{loss} = k_l\,(T_i - T_a) \tag{1}$$

where

| | | |
|---|---|---|
| $Q_{loss}$ | : Building heat loss | [W] |
| $k_l$ | : Building heat loss factor | [W/°C] |
| $T_i$ | : Indoor temperature | [°C] |
| $T_a$ | : Outdoor air temperature | [°C] |

### 2.2 Radiator Model

The radiator transfers heat from the district heating water to the indoor air. The heat transferred from the water is written as:

$$Q_{supp} = m\,c_p\,(T_s - T_r) \tag{2}$$

where

| | | |
|---|---|---|
| $Q_{supp}$ | : Heat supplied | [W] |
| $m$ | : Water flow | [kg/s] |
| $c_p$ | : Water heat capacity | [J/(kg °C)] |
| $T_s$ | : Building supply water temperature | [°C] |
| $T_r$ | : Building return water temperature | [°C] |

The cooling of the district heating water is a non-linear function of the operational and design parameters, and can be written as

$$T_r = f\left(T_s, T_i, m, T_{so}, T_{io}, m_o, T_{ro}\right) \tag{3}$$

where

| | | |
|---|---|---|
| $T_{so}$ | : Supply water temperature at ref. condition | [°C] |
| $T_{io}$ | : Indoor temperature at reference condition | [°C] |
| $m_o$ | : Water flow at reference condition | [kg/s] |
| $T_{ro}$ | : Return water temperature at ref. condition | [°C] |

This non-linearity has not a great effect on the model performance, and can barely be extracted from the available operational data. See Valdimarsson (1993) for closer treatment of the identification of the return water temperature.

### 2.3 Building Energy Storage Model

By assuming all heated parts of the building to be at uniform indoor temperature at all times, the building can be modelled as a single heat capacity element. A differential equation is then written relating the net heat flow to the building to the time derivative of the indoor temperature and the building heat capacity. Then the building energy storage becomes as described in Equation (4).

$$\frac{dT_i}{dt} = \frac{1}{C} Q_{net} = \frac{1}{C}\left(Q_{supp} - Q_{loss}\right) \tag{4}$$

where

| | | |
|---|---|---|
| $C$ | : Building heat capacity | [J/°C] |
| $Q_{net}$ | : Net heat transferred to the building | [W] |

The heat supplied is an output signal from the radiator model, and the lost heat is calculated in the heat loss model.

### 2.4 Flow Controller Model

The flow controller is unknown, there being no direct physical relation between the indoor temperature and the water flow. Buildings have different regulating systems, and additionally its inhabitants have different tolerances to changes in the indoor temperature. The relation used between the indoor temperature and the water flow has to represent some average of all consumers in the system.

One way to treat this problem is to study typical controller characteristics. A P-controller (Proportional controller) is defined by Equation *(5)*.

$$m = k_p(T_{iref} - T_i) + \overline{m} \tag{5}$$

where

| | | |
|---|---|---|
| $k_p$ | : Flow controller gain | [(kg/s)/°C] |
| $T_{iref}$ | : Indoor temperature set point | [°C] |
| $\overline{m}$ | : Average flow | [kg/s] |

The proportional controller adjusts the water flow as a linear function of the deviation of the indoor temperature from the desired value. Here the water flow is assumed to be at its average value when the indoor temperature is at the desired level. The indoor temperature will not be at the desired value for any other situation.

A PI-controller (Proportional and Integral) uses, in addition to this, the integral of the temperature error, which is defined by Equation *(6)*

$$m = k_p(T_{iref} - T_i) + k_i \int_0^t (T_{iref} - T_i)dt \tag{6}$$

where

| | | |
|---|---|---|
| $k_i$ | : Flow controller integration factor | [(kg/s)/(°C s)] |
| $t$ | : Time | [s] |

### 2.5 Macroscopic Steady State Model

A steady state model assumes that the system is memoryless, i.e. that any previous history can be discarded. Thus the output signals are only dependent on the input and the control signals at the same point in time. This implies that there are no state variables, the input temperature is a constant. That also means that the flow controller is ideal, i.e. it will be able to adjust the flow such that the heat lost will be supplied to the building exactly. Recalling Equations (1) and (2) this implies that:

$$Q_{supp} = Q_{loss}; \quad mc_p(T_s - T_r) = k_l(T_i - T_a) \tag{7}$$

The steady state model for the flow can then be obtained directly as:

$$m = \frac{k_l(T_i - T_a)}{c_p(T_s - T_r)}$$

or

$$m = -\frac{k_l}{c_p(T_s - T_r)}T_a + \frac{k_l}{c_p(T_s - T_r)}T_i \tag{8}$$

This is the steady state model, relating the input signal $T_a$ to the flow $m$ for constant indoor temperature $T_i$.

### 2.6 Macroscopic Dynamic P-Controller Model

A model with a P-controller can be written as a first order differential equation by differentiating Equation *(5)*:

$$\frac{dm}{dt} = -k_p \frac{dT_i}{dt} \tag{9}$$

Additionally Equation *(5)* can be solved for $T_i$ to obtain

$$T_i = T_{iref} - \frac{m - \overline{m}}{k_p} \tag{10}$$

and the building model for a building with a P-controller is then obtained by combining Equations (1), (2), (4), (9) and (10) as

$$\frac{dm}{dt} = -\frac{k_p}{C}\left(c_p(T_s - T_r) + \frac{k_l}{k_p}\right)m - \frac{k_p k_l}{C}T_a + \frac{k_p k_l}{C}\left(T_{iref} + \frac{\overline{m}}{k_p}\right) \tag{11}$$

This is the final form of the P-controller building model.

### 2.7 Macroscopic Dynamic PI-Controller Model

The model of a building with a PI-controller is a second order model, and is obtained by first differentiating Equation (6):

$$\frac{dm}{dt} = -k_p \frac{dT_i}{dt} + k_i(T_{iref} - T_i) \tag{12}$$

By combining Equations (1), (2), and (4) the first state equation for the model becomes:

$$\frac{dT_i}{dt} = -\frac{k_l}{C}T_i + \frac{c_p}{C}(T_s - T_r)m + \frac{k_l}{C}T_a \tag{13}$$

By inserting Equations (10) and (13) into Equation (12) the second state equation becomes

$$\frac{dm}{dt} = \left(k_p \frac{k_l}{C} - k_i\right)T_i - k_p \frac{c_p}{C}(T_s - T_r)m - k_p \frac{k_l}{C}T_a + k_i T_{iref} \tag{14}$$

No measurements are available for the indoor temperature, and it is also only a measure of the stored energy in the hot mass of the buildings. A measurement equation relates the two states to the available output signal.

The classical state form for Equations (13) and (14) together with the measurement equation is:

$$\begin{bmatrix} \frac{dT_i}{dt} \\ \frac{dm}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{k_l}{C} & \frac{c_p(T_s - T_r)}{C} \\ \frac{k_p k_l}{C} - k_i & -\frac{k_p c_p(T_s - T_r)}{C} \end{bmatrix} \begin{bmatrix} T_i \\ m \end{bmatrix} + \begin{bmatrix} \frac{k_l}{C} & 0 \\ -\frac{k_p k_l}{C} & k_i T_{iref} \end{bmatrix} \begin{bmatrix} T_a \\ 1 \end{bmatrix}$$
$$m = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} T_i \\ m \end{bmatrix} \tag{15}$$

This is the final form of the PI-controller building model.

### 2.8 Identifiability

It is not certain that all parameters needed in a model can be identified. Some of the parameters may be related through the model structure, or they may not be separable in the real world. The former case can be tested for by calculation,, but the latter case only by studying the model behaviour.

Calculation of the identifiability of a certain model structure involves calculating the transfer functions from the input signals to the output signals, counting the number of independent parameters, and ensuring that there are not any common factors cancelling out, see Ljung (1987). This number is the maximum number of parameters available for identification for this given model structure. Ljung also mentions that it is possible to show that the number of identifiable parameters is three times the number of states.

A maximum of six parameters can thus be estimated in the PI-controller model. Furthermore, since the second input in Equation *(15)* is constant one of these parameters cannot be estimated, because there is no time dependency in the signal. Therefore only five parameters in Equation (15) can be estimated for this model structure.

Equation (15) contains eight physical parameters, C, $c_p$, $k_i$, $k_l$, $k_p$, $T_{iref}$, $T_s$, and $T_r$. One is given, in that the heat capacity of water $c_p$ is known. The difference between the building supply and return temperatures goes into the model, so only their difference $T_s$ - $T_r$ can be estimated. The indoor temperature set point can be estimated to be close to 20°C, and to remain constant at that value. This brings the number of individual parameters down to five, which allows them all to be estimated. The underlying physics of the system, however, show a coupling between the parameters $k_l$ and $T_s$ - $T_r$, so only the ratio between them can be estimated.

## 3. STATISTICAL METHODS

### 3.1 Score Functions

A score function can be used to define the quality of the model, when calculated and measured values are compared. The most common type of score function used is the quadratic error measure. If N measurements are available for evaluation of the model quality, and the error can be assumed independent of time, the quadratic error function is:

$$V = \frac{1}{N}\sum_{i=1}^{N}(\hat{y} - y_i)^2 = \frac{1}{N}\sum_{i=1}^{N} e_i^2 \qquad (16)$$

where

V : Value of the score function

$N$ : Number of data points [-]

Y : Model estimate of the output y

$e_i$ : The error at time point $i$

This function can be minimised to obtain the model parameter values. It is not helpful when deciding the model degree, as it will always be lower for the higher model degree. When the number of model parameters approaches the number of data points, the score function will approach zero, "parrot" learning of the model has occurred and the model is likely to perform badly on another set of data. The relative error is also of interest, and is defined by:

$$V_{rel} = \frac{|\hat{m} - m|}{m} \qquad (17)$$

where

$V_{rel}$ : Relative flow model error [-]

$m$ : Flow estimate [kg/s]

The root mean square relative error is defined by:

$$V_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\hat{m}_i - m_i}{m_i}\right)^2} \qquad (18)$$

where

$V_{rms}$ : Root mean **square** relative flow model error [-]

These two rclative error measures are included here, as they are easy to understand, and can be expressed as percentages, so they give an indication on the quality of otherwise incomparable models. They are not used here as criteria in the decision on which model to use. The *AIC* and *FPE* error measures are better suited to that task, as they take the number of parameters into account. Both error measures are based on the absolute error. *AIC* and *FPE* use the variance for the model error defined as:

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{t=1}^{N} e_t^2 \qquad (19)$$

where

$\sigma$ : Standard deviation of the estimation error

The Akaike Information Criterion, *AIC,* is an attempt to measure the model quality taking into consideration the number of parameters used in the model. The derivation of the *AIC* criterion is treated m Ljung (1987) and Olbjer (1985).

The *AIC* for a model with $n_i$ parameters, N data points and the error variance estimate $\hat{\sigma}^2$ is:

$$AIC = N \cdot \log(2\pi) + N \cdot \log(\hat{\sigma}^2) + 2 \cdot n_i \qquad (20)$$

The Final Prediction Error, *FPE,* is a measure similar to the ***AIC,*** taking into account the number of parameters in the model. The *FPE* reflects the error variance when the model is applied to a validation data set, see Ljung (1987). The ***FPE*** for a model with $n_i$ parameters, *N* data points and the error variance estimate $\hat{\sigma}^2$ is defined as:

$$FPE = \frac{1}{2}\cdot\frac{1}{N-1}\sum_{t=1}^{N} e_t^2 \cdot \frac{1+\frac{n_i}{N}}{1-\frac{n_i}{N}} = \frac{1}{2}\cdot\hat{\sigma}^2\cdot\frac{1+\frac{n_i}{N}}{1-\frac{n_i}{N}} \qquad (21)$$

### 3.2 Error Analysis

The model error should be white noise when the model has incorporated all available information from the data. The auto-correlation function of the error will show whether there exists a correlation within the error function. The autocorrelation function is defined as:

$$r_{ee}(\tau) = \sum_{t=1}^{N-\tau} e_t \cdot e_{t+\tau}\left(\sum_{t=1}^{N} e_t^2\right)^{-1} \qquad (22)$$

where

$\tau$ : Time shift

The autocorrelation is considered here significantly different from zero if it lies outside the 99% confidence interval around zero. The halfwidth of the interval is then

$$h_{ee} = 2.58 \cdot \hat{\sigma}_{auto} = 2.58 \cdot \frac{\sqrt{\frac{1}{N}\sum_{t=1}^{N} e_t^2}}{\sqrt{N}} \qquad (23)$$

assuming a normal distribution of the error and time-invariant variance.

Similarly the correlation of the error to the input signals has to be tested. **Assuming** a normal distribution of the error, and time-invariant variance, the crosscorrelation function between the error $e$ and the input $x$ is defined by:

$$r_{ex}(\tau) = \frac{\sum_{t=1}^{N}(e_t - x_t)(e_{t+\tau} - x_t)}{\sqrt{\sum_{t=1}^{N} e_t^2 \cdot \sum_{t=1}^{N} x_t^2}} \qquad (24)$$

Here the crosscorrelation is also considered significantly different from zero if it lies outside the 99% confidence interval around zero. The halfwidth of the interval is then

$$h_{ex} = 2.58 \cdot \hat{\sigma}_{cross} = 2.58 \cdot \frac{\sqrt{\frac{1}{N}\sum_{\tau=-\infty}^{m}\sum_{t=1}^{N}(e_t - x_t)(e_{t+\tau} - x_t)}}{\sqrt{N}} \qquad (25)$$

assuming as before a normal distribution of the error and time-invariant variance.

### 3.3 Model Validation

In the present work a model is considered valid if:

a. The *AIC* or *FPE* are at minimum for the current set of parameter values.
b. All parameters are significantly different from zero.
c. The error is not significantly correlated with itself.
d. The error is not significantly correlated with any of the inputs.
e. The model performs well on a validation data set.

This concludes the treatment of thc basis for the statistical models used in the presen' work.

## 4. COMPARISON WITH DATA

### 4.1 Data Used

The Reykjavik Municipal District Heating Service, Hitaveita Reykjavikur, has made available operational data for the Reykjavik district heating system for the years 1989 and 1990. Meteorological data for the city of Reykjavik were obtained from the Icelandic Meteorological Office.

The district heating water flow series for the suburbs Breidholt 1 and Breidholt 2 during the year 1990 is used here as a demonstration of the modelling. The weather series used for this purpose was the outdoor *air* temperature during 1990. The temperature series was averaged to show daily averages. The models used are heating models, so the tap water consumption had to be removed from the data. In Valdimarson (1993) the removal of the tap water consumption is treated, but here the water flow at 04:00 hrs in the morning is used as the heating flow for the day. The district heating water was pumped into the distribution system at 80°C, and the ground temperature is assumed to be 3°C. No return temperature series were available.

The models were used to make a "one step ahead" prediction. Using this prediction, the errors caused by incorrect parameter values will be similar for all data points. Alternatively one could simulate the whole period from a single starting value. Then the deviation of the model from the real measurements will accumulate, the further away from the starting point one gets. The "one step ahead" prediction is more stable, and will give a better estimate of the parameter values.

It has been shown, Ljung (1987), that the parameters obtained using the "one step ahead" prediction, describe the real system better than those obtained by multistep methods.

### 4.2 Steady State Model

In the steady state model, no account is taken of dynamic effects in the system. Using it, a steady state curve, relating the flow to the outdoor air temperature is obtained. The steady state model can be written as a linear model, assuming $(T_s - T_r)$ to be constant:

$$m = b \cdot T_a + c \qquad (26)$$

where:

$$b = -\frac{k_l}{c_p \cdot (T_s - T_r)}$$

$$c = -b \cdot T_i$$

The flow data for a given day can be related to the outside *air* temperature of the same day. The dynamic character of the system, however, gives reason to believe that the flow might have a better correlation with the temperature at some previous time. This can be tested by shifting the temperature series relative to the flow series. A shift of one day relates the outdoor air temperature yesterday to the flow today, a shift of two days relates the outdoor *air* temperature the day before yesterday to the flow today and so on.

The best results are relating the flow today to the flow yesterday. Following are the resulting model parameters:

b=-5.531 (kg/s)/°C, $\sigma_b$=0.0949 (kg/s)/°C, $T_i$ (calc)=21.62°C

The model performance parameters are:

*AIC*=2320.5, *FPE* –91.81, *V*=91.30 (kg/s)$^2$, max($V_{rel}$)=39.35%
$V_{rms}$= 11.24%

The results of the steady state model are shown in Figure 2 together with the model absolute error.

The model absolute error autocorrelation and the absolute error crosscorrelation with the outdoor temperature are then calculated. Figure 3 shows these correlation functions.

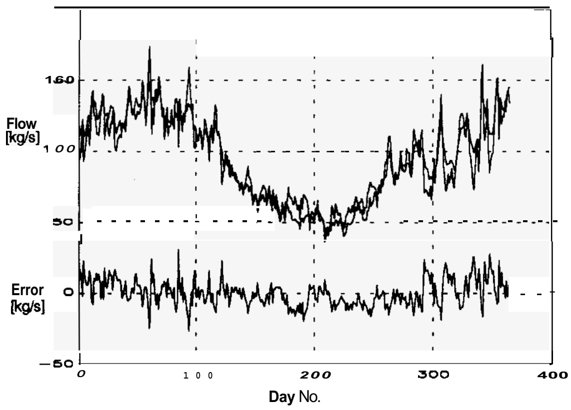


Figure 2 Steady state model results for Breidholt 1 and 2 flow series.

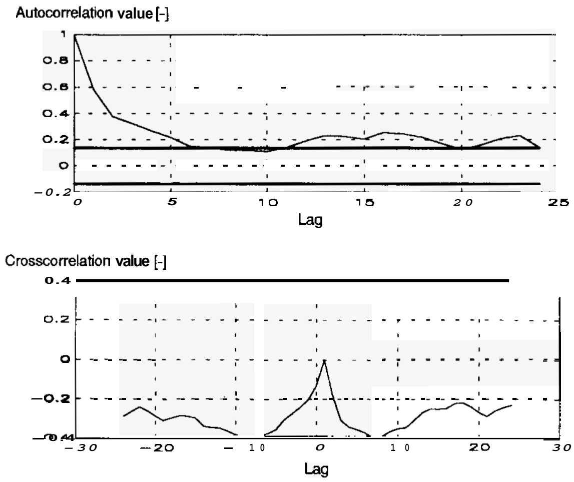


Figure 3 Error analysis for the steady state model.

The pair of solid horizontal lines drawn together with the correlation functions indicate the 2.58 standard deviations confidence interval around zero. This shows that there is information left in the error signal, since the autocorrelation function is over the upper limit. However the crosscorrelation function is not significantly different from zero, which implies that there is no further information to be extracted from the input signal.

The model does not give as good a performance when applied to a validation data set. The model compared to measurements when the year 1989 is used is shown in Figure 4.

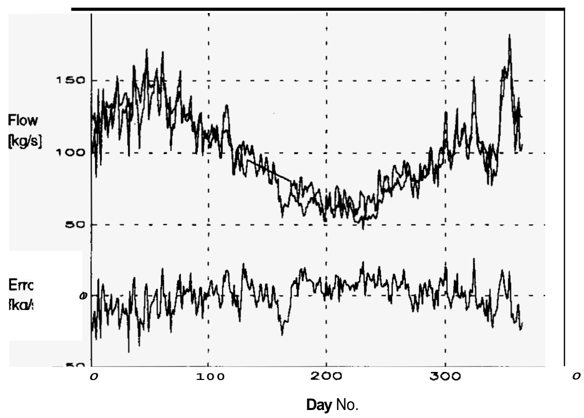


Figure 4 Validation of the steady state model for the year 1989.

### 4.3 Dynamic P-controller Model with Constant Coefficients.

The dynamic P-controller model can be rewritten to show which parameters can be identified. Here the water temperature decrease $T_s$-$T_r$ at the consumer is assumed to remain constant throughout the year, and is therefore simply included in one of the system parameters.

The P-controller model in Equation (11) then becomes:

$$\frac{dm}{dt} = a \cdot m + b \cdot T_a + c \qquad (27)$$

where

$$a = -\frac{k_p}{C} \cdot \left( c_p \cdot (T_s - T_r) + \frac{k_l}{k_p} \right) \qquad (28)$$

$$b = -\frac{k_p \cdot k_l}{C} \qquad (29)$$

$$c = -b \cdot \left( T_{iref} + \frac{\overline{m}}{k_p} \right) \qquad (30)$$

When this equation is integrated, the value of the flow at time $t+\Delta t$ is:

$$m_{t+\Delta t} = e^{a \cdot \Delta t} \cdot m_t + \left( b \cdot T_{a,t} + c \right) \cdot \int_t^{t+\Delta t} e^{a \cdot s} \, ds$$

or

$$m_{t+\Delta t} = e^{a \cdot \Delta t} \cdot m_t + \frac{1}{a}\left(e^{a \cdot \Delta t} - 1\right) \cdot \left(b \cdot T_{a,t} + c\right) \qquad (31)$$

The model parameters are:

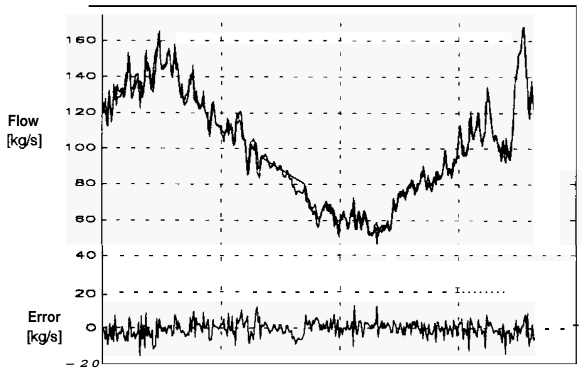
$a$=-0.3185 1/day, $b$=-1.922 kg/(s°C), $c$=38.92 kg/(s day)
$\sigma_a$=0.02403 kg/s$^2$, $\sigma_b$=0.1388 kg/(s°C), $\sigma_c$=2.885 kg/(s day)

The performance parameters are:

$AIC$=1684.0, $FPE$=16.05, $V$=15.79 (kg/s)$^2$, max($V_{rel}$) =14.71 %
$V_{rms}$=4.328 %

In this model, only three parameters are identified. However, a total of five physical parameters are needed.

The P-controller model with the absolute error for the Breidholt 1 and 2 validation flow series (1989) is shown in Figure 5.



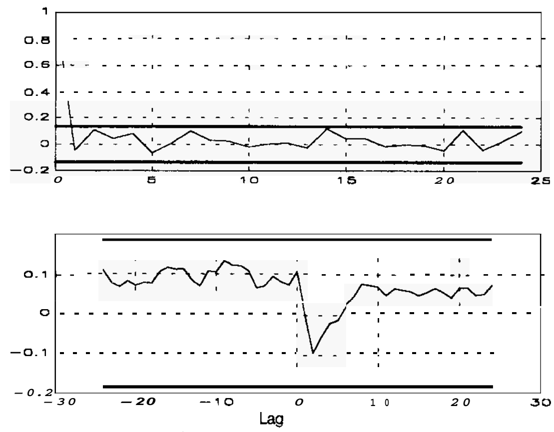




Figure 6 Error analysis for a linear dynamic P-controller state model.

It can be concluded that a dynamic F-controller model is a valid model of the district heating flow.

### 4.4 Dynamic PI-controller Model with Constant Coefficients.

The factor $T_{iref}$ can be assumed to be close to 20°C, and the radiator water cooling $T_s$-$T_r$ close to 45°C. The only available system data are the water flow, so the radiator water cooling can not be separated from the building heat loss coefficient $k_l$. For every value assumed for the radiator water cooling there will be a new pair of the thermal parameters C and $k_l$ resulting in the same model flow series.

For the analysis of the dynamic PI-controller model the above mentioned values for the indoor reference temperature and the radiator water cooling were taken as given. Then the building heat loss factor $k_l$, the building heat capacity $C$, the controller proportional gain $k_p$ and the integral factor $k_i$ were estimated from the model.

A Kalman filter was used to estimate the states for given parameter values, taking at any time into account the errors at the preceding time step. The score function minimum was found by searching through the parameter values with a minimisation algorithm.

The results show that the model performance and controller model parameters do not change when $T_s$-$T_r$ is varied from 40 to 50°C. Each set of the thermal parameters ($T_s$-$T_r$, $k_i$ and $C$) will result in the same time series for the estimate of the indoor temperature. The controller parameters define the relation between the indoor temperature and the flow, so they will not be dependent on which set of the thermal parameters is used.

The model performance parameters are:

$AIC$=1598.7, $FPE$=12.71, $V$=12.43 (kg/s)$^2$, max($V_{rel}$) =13.15 %
$V_{rms}$=3.894 %

The controller parameters are:

$k_p$ =4.446 kg/(s°C), $k_i$=0.7286 kg/(s°C day)
$\sigma_{kp}$=0.9412 kg/(s°C), $\sigma_{ki}$=0.1972 kg/(s°C day)

The controller time constant is found from:

$$\tau_c = \frac{k_p}{k_i} = 6.102 \text{ days} \qquad (32)$$

The parameters that are dependent on $T_s$-$T_r$ are shown in Table 1.

Table 1 Linear PI-controller model, parameters dependent of

| $T_s$-$T_r$ [°C] | $k_l$ [kW/°C] | $\sigma_{kl}$ [kW/°C] | $C$ [(kWday)/°C] | $\sigma_C$ [(kWday)/°C] |
|---|---|---|---|---|
| 40 | 1025.3 | 15.56 | 1344.4 | 422.8 |
| 42 | 1076.6 | 16.34 | 1411.6 | 444.0 |
| 44 | 1127.8 | 17.11 | 1478.8 | 465.1 |
| 46 | 1179.1 | 17.89 | 1546.0 | 486.3 |
| 48 | 1230.4 | 18.67 | 1613.3 | 507.4 |
| 50 | 1281.7 | 19.45 | 1680.5 | 528.5 |

All the dependent parameters were found to be linear functions of $T_s$-$T_r$, namely:

$$k_l = 25.63 \cdot (T_s - T_r)$$

$$\sigma_{k_l} = 0.3890 \cdot (T_s - T_r)$$

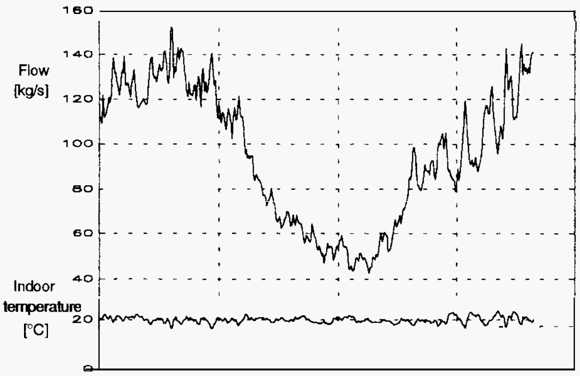
$$C = 33.61 \cdot (T_s - T_r)$$

$$\sigma_C = 10.57 \cdot (T_s - T_r)$$

The building time constant is found from:

$$\tau_b = \frac{C}{k_l} = 1.311 \text{ days} \tag{33}$$

The model output for the PI-controller model for Breidholt 1 and 2 is shown on Figure 7. Predicted values for the water flow and indoor temperature are shown.



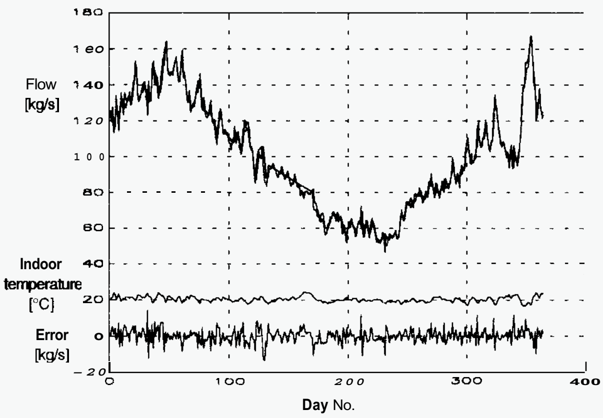




Figure 8 Validation of the PI-controller model for the year 1989.

## 5. CONCLUSION

Three assumptions on the type of flow controller have been tested against data from the geothermal district heating network in Reykjavik, Iceland, with the conclusion that the PI-controller assumption gives the best results, in addition to the most information on the physical parameters of the system. The steady state model based on the ideal controller was not found valid. Valdimarsson (1993) has shown that addition of the non-linear radiator cooling function does not improve the modelling, so it was not considered necessary to include non-linear buildings motel in the present study.

The PI-controller model estimates an average building time constant of 1.311 days, and a very large controller integrating time constant of 6.102 days.

## 6. REFERENCES

Ljung, L. (1987). ***System Identification - Theoryfor the User.*** Prentice-Hall Inc., Englewood Cliffs, New Jersey, **USA**
ISBN 0-13-881640-9

Olbjer, L. (1985). ***Tidserieanalys.*** Avdeling för matamatisk statistik, Lund Institute of Technology, Lund, Sweden

Valdimarsson, P. (1993). ***Modelling of Geothermal District Heating Systems.*** Háskólaútgáfan, University of Iceland, Reykjavik, Iceland
ISBN 9979-54-055-9