FUNDAMENTAL REASONS FOR MATHEMATICS EDUCATION IN ICELAND

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Abstract:
Iceland was predominantly a rural society under Danish rule until the 20th century. The paper discusses arguments and reasons for the presence and absence of mathematics education in Iceland through the centuries. The content of mathematics education traditionally concerned trade and prerequisites for university entrance until the 1960s. At that time the OECD channelled to Iceland radical ideas about the content and purpose of mathematics education. At that moment and others, when mathematical education was at crossroads, arguments brought up by influential individuals, referring to fundamental reasons for mathematics education, were of a great importance. The pros and cons of the dependence of Denmark in this respect are also discussed.

Keywords: Arguments and fundamental reasons for mathematical education; The OECD's impact on educational policy; Influential individuals; Iceland's dependence of Denmark.

FUNDAMENTAL REASONS FOR MATHEMATICS EDUCATION

Analysis of the history of education in Iceland reveals that for long periods there was only little emphasis on mathematics education. Cultural activities were concerned with a national heritage from medieval times, preserved in manuscripts written in the vernacular, in addition to European Latin influences. The medieval heritage included some original and translated observations and knowledge of mathematical nature, which was up to the date when written in the 12th to 14th century, but became outdated with time. From 1300–1800 neither original nor recent foreign mathematical knowledge is known to have been studied in Iceland, except for land surveying and map-making in the late 16th century and 17th century.

Can explanations for this fact be supplied? The Icelanders were extremely poor, but the educational elite never lost the sight of maintaining contact with the European culture. The history paints picture of a nation living on the boarder of the habitable world, putting pride in cultural activities, but selecting what it felt it could use from the European culture and leaving out other factors. Why did it leave out mathematics for long periods of time?

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In order to answer the question, the development will be measured by the fundamental reasons for mathematics education as identified by Prof. Mogens Niss:

> Analyses of mathematics education from historical and contemporary perspectives show that in essence there are just a few types of fundamental reasons for mathematics education. They include the following:
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> - contributing to the *technological and socio-economic development* of society at large, either as such or in competition with other societies/countries;
> - contributing to *society's political, ideological and cultural maintenance and development*, again either as such or in competition with other societies/countries;
> - providing *individuals with prerequisites which may help them to cope with life* in various spheres in which they live: education or occupation; private life; social life; life as a citizen.[2]

Did Icelandic society need mathematics education for its economical or cultural development, did it cultivate mathematics for its own sake, or did individuals need to be provided with mathematical prerequisites to cope with their private or professional lives? The fundamental reasons for mathematics education or its absence in Icelandic society at each particular period of time will be analysed.

ICELANDIC SOCIETY, ORIGIN AND STRUCTURE

Iceland is an island in the Northern-Atlantic slightly larger than Ireland. It was settled by Scandinavian tribes in the Viking Age. In the 10th century, the inhabitants established their own free state which survived for over three centuries. Iceland was under Denmark from the late 14th century. It was gradually releasing its bonds from Denmark from the 1870s when its parliament was granted legislative power, subject to the King's consent. It acquired Home Rule in 1904, sovereignty in 1918 and a republic was established in 1944. Cultural relationships with Denmark lasted still longer.

Due to harsh climate and difficult living condition the population did not grow markedly until the 20th century. It is estimated to have been 50,000–70,000 in the 11th century.[3] At its first census in 1703 it was 50,358 and by 1900 it was 78,203.[4] The culture was European, marked by the introduction of Christianity in the 11th century, the evangelic Lutheran reformation in 16th century, the Humanism in the 17th century and in the 18th century the Enlightenment, whose influence lasted into the 19th century.

From the adoption of Christianity the church ran Latin schools for boys in order to educate priests. In the early 19th century there was only one such school from which priests graduated. It was also a door to further education, usually sought at the University of Copenhagen where Icelandic students had some priory of grants for supporting themselves. All public education was built on home education under the supervision of parish priests by regulation of the 1740s.

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[2] Niss, M. (1996): 13
[3] Gunnar Karlsson (2000): 45
[4] Statistics Iceland, website

Cultural exchanges and other contacts with Europe, such as trade, depended on sailing. Sailing from Iceland to Europe was very common in the 11th and 12th century. However, when the Icelanders submitted to the rule of the Norwegian King in 1262, a part of their agreement with the King was that he would ensure that six ships sailed to Iceland each year. This indicates that the Icelanders themselves may not have had many ships at that time, presumable for lack of timber in an increasingly exploited country and colder climate than earlier.

The end of the middle ages marked a change in Iceland of an opposite kind to that which characterized most European countries. In Europe it generally meant the beginning of greatly increased trade. In the modern age, Iceland remained outside the mainstream of trade, under the trade monopoly of the Danish King, established in 1602.[5] The monopoly became a source of handsome income for the Danish crown but also a certain safety net for the Icelanders' contact with Europe.

There were several main harbours, where markets for import and export goods were located, but no towns grew up at these ports. Workers and servants of landowners were sent to the coast at fishing seasons to harness the precious export goods, and towns or villages of independent fishermen and boat-owners were not allowed to form. Trade within the country was so small that no infrastructure existed; neither roads nor bridges were needed for any major transport.[6] The lack of internal trade contributed to the persistence of an exclusively rural self-sustaining society in Iceland for centuries after towns were established in the other Nordic countries. Episcopal sees and the great estates were situated far inland, surrounded by rivers and mountains. The sees and cathedral schools established in the 11th and early 12th century remained relatively unchanged until the beginning of the 19th century. Icelandic society thus remained stagnant into the 19th century, while the complexity of trade increased in the neighbouring countries.

The 18th century was the most difficult period in Icelandic history. In 1707 a smallpox epidemic killed a large number of people, and by the middle of the century a series of cold years with pack-ice caused a famine. The situation reached its worst during the so called Haze Famine, following a massive volcanic eruption in 1783–1785, when toxic volcanic gases and ash poisoned the grass and killed the majority of all livestock, mostly from fluorosis.[7] From the end of 1783 to the end of 1786 the population decreased from 49,753 to 39,190 or just over 10,500, one fifth of the whole population. Following these calamities the Danish government gave up the trade monopoly of Iceland to become free to any citizen of the Danish crown, as the trade rendered no revenue, but also according to a plan on behalf of the government of organized urbanization of the country. By this plan, the first town began to form at Reykjavík, the present capital, in the late 18th century. [8]

GEOGRAPHY AND NAVIGATION IN THE 16TH CENTURY

Geography and navigation were two related aspects of mathematics, extremely important to the world of the sixteenth century.[9] Bishop Guðbrandur Þorláksson (1541/42–1627) was educated in mathematical subjects at the University of Copenhagen. He made land-

[5] Gunnar Karlsson (2000): 127, 138–142
[6] Thorsteinsson and Jónsson (1991): 136–137
[7] Björnsson (2006): 208
[8] Karlsson (2000): 181–182
[9] Katz, (1993): 360

surveying and produced a map of Iceland and thus worked on the same kind of tasks as mathematicians in the European world, contributing to the world's knowledge of Iceland's geography. His map was introduced to the European learned world in 1590 through the mediation of a Danish researcher, Andreas Sørensen Vedel. It remained a basis for Icelandic maps into the early 18th century.[10]

Bishop Guðbrandur Þorláksson was an adherent of Humanism and a proponent of education. He saw and utilized the new technology of the printing press as a prime channel for educating the people. The bishop's eagerly pursued theological activities and promotion of the Icelandic language in his publications, such as the translated Bible, may be attributed to his desire to ensure the power and influence of the Church and the independence of Icelanders. As long as they kept their own language they would retain some independence from foreign rule by Denmark. For both ends, the publication of theological works was useful and the printing press an excellent tool. Mathematical publications had no such purpose. Mathematical publications for the general public may not have been so widespread at the turn of the 16th century in Europe either, that there would be foreign models for that task, except for calendars, such as the *Calendarium*[11] published and presumably edited by the bishop. Other mathematical books might have aimed at merchants or university professors, professions not found in Iceland.

Mathematics was probably studied at the cathedral school during Bishop Guðbrandur Þorláksson's term in office. However, Latin was the main subject of the school. It was the *lingua franca* of the European world; it was the thread that kept Iceland in contact with the civilized world, and that had to be a priority.

INITIATION OF MATHEMATICS EDUCATION

In spite of hard times in the 18th century it saw a new dawn to education. The Enlightenment had considerable influence among Danes concerned with Icelandic affairs and the Icelandic elite, educated in Denmark. Among the products of the movement were a total of six arithmetic textbooks written in Icelandic in the period 1746–1841. All of them were primarily written as aids to trade, but three of them also as general textbooks in arithmetic. They witness an increase in trade, however small on European scale, but also, that the Icelanders had become aware of that they were behind the Danes and other European in mathematics education. There were no Icelandic merchants until late 19th century.

The adherents of the Enlightenment were interested in modernizing the structure of society. The episcopal sees and one of the cathedral schools were closed down and a new school and a new see were established in the growing capital Reykjavík around the turn of the 19th century. The other cathedral school had already broken down in an earthquake during the Haze Famine calamities. The idea was to establish a school in a modern style where the teachers would receive their salaries in cash, and the pupils would receive their alms in cash from the King's funds as well.

The experiment proved a disaster. The school building was poor, as was the teaching. When the old sees were abolished, an agreement was made that a certain amount of money should be allocated to cover the costs of the new see and a school for 40 pupils. The money declined in value during the Danish inflation years arising from the Napoleonic wars in the

[10] National and University Library of Iceland. Website of maps.
[11] Sæmundsson (1968)

early 19th century. The see, which had previously had huge assets, went bankrupt and in 1804 the pupils of the new school in Reykjavík could not even be adequately fed. They were sent home and the school was closed down. For a year there was no learned school in the country, while it was re-established in 1805. Then the number of pupils fell to 24.[12]

A part of this modernization in the Danish realm, inspired by the Enlightenment, was legislation in 1814 on public schools in Denmark.[13] The rural structure and lack of transport made it unthinkable to adjust such legislation to Iceland at that time. Legislation of 1880 prescribed instruction for children in arithmetic within the traditional home education prescribed by regulations of the 1740s. Legislation on public schools was first passed in 1907, then for children 10–14 years of age.

Minimum requirements of mid-18th century regulations about the ability to perform the four operations in whole numbers and fractions were not fulfilled at times in the Learned School around the year 1800.[14] For several years in the early 19th century, students from the Icelandic Learned School had to be exempted from new requirements in mathematical knowledge on behalf of the University of Copenhagen due to lack of mathematics teachers. In 1819 professors at the University of Copenhagen complained about the inadequacy of the mathematics education of the Icelandic students.[15]

By a stroke of luck a scion of the Enlightenment, Björn Gunnlaugsson, one of the finest examples of the home- and self-education tradition, had studied mathematics, merely on his own with the aid of land surveyors, and without ever being accepted at the Learned School, before he entered the University of Copenhagen. Yet this education enabled him to win a golden medal for a solution of a mathematical problem in his first year at the University of Copenhagen in 1818. After five years of study, Björn Gunnlaugsson's life was devoted to raising the level of mathematical teaching of the Learned School that had refused him earlier. Another of his great feat was his land surveying yielding a geodetic map of Iceland, the basis for maps of Iceland into the 20th century. However, Gunnlaugsson was the only Icelandic mathematician throughout the 19th century and he was isolated from the developments of mathematics in Europe. The intellectual environment and interests swayed him to philosophy and the didactics and applications of mathematics, such as land-surveying.[16]

Björn Gunnlaugsson presented his goals for mathematical education in his inauguration speech at the Learned School where he emphasized the utilitarian aspects of mathematics. It was a tool to explore nature, he said, while he also argued how mathematics could train people in logical thinking, as nowhere else was truth as easy to research and easily distinguished from falsehood.[17] Thus his personal attitude was based on acquaintance of the cultural values of mathematics. He may, however, have found it wise to emphasize the practical aspects to his fellow countrymen. His magnificent work in land surveying served as a tool for seafarers and a basis for future roads, bridges and harbours, and thus contributed to the technical development of Iceland.

[12] Ólafsdóttir (1961): 67–93
[13] Guttormsson (1990)
[14] Helgason, 1935; Helgason 1907–1915
[15] National Archives of Iceland: Skjalasafn kirkjustjórnarráðsins SK/4 (örk 23)
[16] Guðmundsson (2003)
[17] Gunnlaugsson (1993): 57–66

REGULATIONS OF 1877

Björn Gunnlaugsson was not succeeded by anyone his equal as mathematics teacher at the Learned School. Mathematics teaching at the school declined in quality after his days. At the time of his death in 1876, new regulations for Danish learned schools had divided them into a mathematics-science stream and a language-history stream. A board, established in 1875 to make proposals about Icelandic school affairs, suggested a combination of the two streams, similar to the previous structure of the school with increased emphasis on modern languages on the cost of the antique languages.[18] The number of pupils was so small at that time that a two-stream system was not feasible. Educational authorities opted for the language–history stream in 1877. The mathematics teaching was greatly reduced, while instruction in the Danish language was increased.

Letters between the Governor of Iceland and the Minister of Icelandic Affairs in Copenhagen, discovered at the National Archives of Iceland, explain why a language-history stream option was chosen in Iceland.[19] The resulting decision may presumably be attributed to lobbying on behalf of the headmaster of the Learned School, a philologist, and some teachers in favour of the language-history stream. The main reason for maintaining European standards in mathematics education on behalf of the authorities had been to enable students to pursue studies at university level. When mathematics was no longer necessary and a language-history stream without advanced mathematics requirements became an option, it was selected for the Icelandic Learned School. There were no mathematicians to promote mathematics on the basis of a personal conviction of its utilitarian and cultural values.

The three fundamental reasons adduced by M. Niss were brought up as arguments in this case, a protracted debate lasting from 1876 until 1882. The arguments for more mathematics, presented by those, who defended the proposal of a combined stream, were that mathematics education, offered after the regulations had been put into effect, was insufficient in itself and the topics that lacked would "finalize and perfect" mathematics education in the school. More mathematics would provide students with prerequisites for further mathematical studies at institutions of higher learning, such as the Polytechnic College in Copenhagen, to become engineers and thus contribute to the technological development of society. It was also argued, that mathematics had an important role as instruction in thinking for mankind, an argument referring to contribution to society's cultural maintenance and development.

Governor Finsen, a half-Icelander who grew up in Denmark, represented the opponents, who argued that the Learned-School pupils were seeking qualifications for professional examinations in theology, medicine, law or philology, and anything else would be an extremely rare exception. Those who planned to do so had to acquire the requisite skills elsewhere. The need for engineers was not yet considered relevant, and even so, it might not have been thought unnatural that an extra year in Copenhagen would be necessary for those who were inclined to become pioneers of that kind.

The dispute ended in 1882, less than a decade before the Icelandic society stood on the threshold of a new age of technical progress, far later than its neighbouring countries. There were still neither roads, bridges, harbours nor motorized ships in that large country, and conveniences such as water systems or sewages did not exist in the growing capital of Reykjavík. Visionary national leaders might have foreseen the need for technical education, a track that demanded that the mathematics–science stream option would be maintained.

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[18] *Álitsskjal nefndarinnar í skólamálinu* (1877)
[19] National Archives of Iceland, Íslenska stjórnardeildin, Skjalasafn landshöfðingja

The basic reasons for excluding the mathematics-natural sciences stream were of an economic nature. It was not financially possible to divide the school of about 80 pupils in six age groups into two streams. The school was already a substantial item in the country's budget, which was run at a deficit, and paid for by the Danish government. As the number of hours could not be increased, some of the teachers must have been afraid that the hours for their subjects would be cut down and hence their own share of work. They were therefore also thinking of their own personal economics.

Also politically, more people would be immediately content with reducing the workload in mathematics instead of cutting back the amount of teaching in the ancient languages, Greek and Latin, even if Latin's role as *lingua franca* had declined in importance. The classical languages were considered necessary prerequisites for the most common professional occupation, the priesthood, as well as Latin for the medical studies, in addition to their renowned qualifications in training the mind. By comparison, mathematics had no immediate application. Furthermore, evidence exists that it was taught in such a manner in the 1870s at the Reykjavík Learned School that its purpose was invisible, and its popularity among pupils minimal.[20]

Iceland was without higher mathematical education from 1877 until 1919, during an important period of progress in public education and technical innovations, such as motorization of fishing boats and organized transport. The Learned School, the only one in the country until 1930, thus became an isolated institution in society. It hardly participated at all in the country's transition from a predominantly rural structure towards a modern industrial society, a sign of its conservatism and lack of sensitivity to society's needs.

MATHEMATICS STREAM AT THE REYKJAVIK HIGH SCHOOL

Gradually the number of Icelandic engineers grew to have become eleven in 1912. The relation between education and technical progress began to be generally recognized by the educational elite and the authorities, which were becoming increasingly domestic with the Home Rule in 1904 and sovereignty in 1918. Training Icelanders to become engineers was realized to be more economical than hiring foreigners, who only stayed for a short time, demanded higher salaries and were less knowledgeable about the circumstances than those native born. The recently established Association of Chartered Engineers in Iceland worked at presenting the need for domestic mathematical training for prospective engineers. This finally led to the establishment of a mathematics-science stream in 1919 at the Reykjavík High School in 1919, nearly half a century later than in comparable schools in Denmark. Thus the preparation of engineers was accepted at the Reykjavík High School after the need had been recognized but it was not undertaken there in order to enhance society's technical development.

The mathematician Ólafur Daníelsson acquired a doctoral degree from the University of Copenhagen in 1909. Dr. Daníelsson, who had been building up mathematics teaching at the Teacher Training College from 1908, was appointed to lead the mathematics stream of the Reykjavík High School. Dr. Daníelsson contributed strongly to the improvement of mathematical proficiency in Iceland by educating future teachers, engineers and mathematicians to be and by writing textbooks in arithmetic, geometry and algebra for the lower secondary level. His textbooks were an invaluable effort to raise the standards of Icelandic mathematical education. Together with his former teacher student, E. Bjarnason, Dr. Daníelsson built up a coherent system of arithmetic textbooks for primary and secondary level schools in Iceland. In the forewords to his textbooks, Dr. Daníelsson

[20] Jónsson (1883): 97–135

expressed his views that the purpose of mathematics education was to train the mind.[21] This was unique among his fellow countrymen, who considered mathematics as a tool for solving practical problems.

The reasons for establishing a mathematics stream in 1919 were utilitarian with respect to educating engineers for technical development of the country. Its implementation was entrusted to Dr. Ólafur Daníelsson, who argued for the cultural aspects of mathematics education. After his time, mathematics fell into stagnated tracks for up to a quarter of a century, until new mathematicians had acquired the status to enhance reforms.

INTERLUDE

In the 1920s, increasing interactions between the growing public primary and lower secondary education and the learned school heritage led to a relatively large number of pupils seeking attendance to the Reykjavík High School. Other educational offers remained limited, especially in Reykjavík, a rapidly expanding town, growing from 6,700 to 38,200 inhabitants in the period 1900–1940. As a result, admission to the school became restricted to 25 new pupils a year in 1928. The school and its main mathematics teacher, Dr. Daníelsson, thereby acquired a monopoly position. Even if another high school was established at that time in the northern region, the Reykjavík School in the south west was dominant. The restrictions resulted in that the growing number of primary- and lower-secondary-schools adjusted their arithmetic syllabus to the requirements of the Reykjavík School and used arithmetic textbooks by Bjarnason and Daníelsson. In a political effort to abolish the power of the Reykjavík School by separating the lower secondary grades from it by legislation in 1946, the compromise was to adopt its previous lower secondary department syllabus as a basis to its new countrywide entrance examination. This entrance examination remained in its original form into the 1960s and did not change markedly until it was abolished in 1976. Furthermore, a state textbook publishing house was established in the late 1930s in order to offer free textbooks for all children in primary schools. Bjarnason's textbooks, created in the 1920s, were chosen for arithmetic and they remained as the only standard arithmetic textbooks for the upper primary level until 1970.

The intentions of the legislator to provide university entrance to Teacher Training College graduates as a part of the 1946 educational reform were unsuccessful. Mathematical education at the Teacher Training College did not reach the level of the high school language stream. When the national entrance examination to the high school system was established in 1946, the lower secondary schools around the country had to rely on high school mathematics stream graduates to teach mathematics. Due to the entrance restrictions in 1928–1946 the number of these graduated was limited and most of them aimed at professional education. No special education for secondary level mathematics teachers was available within the country until 1951, then as a part of education of engineers at the University of Iceland. Thus the fateful decision of only implementing a language-history stream at Reykjavík Learned School in 1877 had a long lasting effect in a chronic shortage of mathematics teachers on secondary level.

The period 1920s–1960s was characterized by intellectual isolation. It was partly self-created, research focusing on literature and history of the recently independent nation, while ignoring natural sciences. It was also partly caused by the Great Depression, World War II and economic restrictions of foreign currency in the post-war period. In addition to

[21] Daníelsson (1920): iii–iv

outdated textbooks in most subjects, the educational system suffered from lack of facilities, trained mathematics teachers, and curriculum documents and it had been stretched to its limits by the mid-1960s.

MODERN MATHEMATICS IN THE 1960s

Iceland was a founding member of the OEEC. In the late 1950s theories were introduced, initiated by the OEEC, later OECD, arguing that education, especially in mathematical subjects, was central to social and economic progress. The introduction of "modern" school mathematics, stimulated by the organization, was part of a post-war awakening in science education, often associated in the West with Sputnik.

An important seminar, arranged by the OEEC, on new thinking in school mathematics was held at Royaumont, France, in November 1959. The member countries and the United States and Canada were invited to send three delegates: an outstanding mathematician, a mathematics educator or person in charge of mathematics in the Ministry of Education, and an outstanding secondary school teacher of mathematics.[22] The seminar was attended by all the invited countries except Portugal, Spain and Iceland. The Royaumont Seminar can be seen as the beginning of a common reform movement to modernize school mathematics in the world.[23]

Originally the intentions were to lay increased emphasis on applied mathematics; discrete mathematics, probability and statistics, and vectors. Influential teacher associations advocated pedagogical theories about relations between abstract algebra and logic and children's way of thinking. A quotation from the editorial of the journal *Mathematics Teaching* in April 1958 stated that "much of the psychological work of [Swiss psychologist] Piaget suggests that many of the essential notions of modern algebra (which are regarded as a university study) have to form in the pupil's mind before he is even ready to undertake the study of number … Such topics as the algebra of sets or relations might be taught with a profit not merely ... [at upper secondary level] but lower down the school as well."[24]

At the Royaumont seminar these theories won support and its final recommendations included a syllabus introducing mathematics as a unity, and that modern algebra should be the basic and unifying item in the subject of mathematics (see Sriraman & Törner, 2007). In the teaching of all secondary school mathematics, modern symbolism should be introduced as early as possible, as it represented concepts that bring clarity and conciseness to thinking and were unifying.

In the 1960s the Minister of Education in Iceland was also a Minister of Commerce and OEEC / OECD affairs. At a meeting held in Reykjavík in 1965 the director of the Educational Investment and Planning Programme of the OECD addressed the most prominent people of the Icelandic educational system. He explained that while education traditionally had been regarded as primarily serving cultural purposes, new concepts of the role of education had recently been developed which stressed that education was as much an integrated socio-economic sector of society and the national economy as traditional sectors. Thus a new view on education was amalgamating into educational discussion in Iceland in 1965.

Influences from the Royaumont seminar reached the Icelandic mathematical community due to personal contacts with Danish participants. Small scale mathematics education reform experiments were initiated on all school levels in 1964–1966 under professional and political

[22] OEEC (1961): 7
[23] Gjone, G. (1983): Vol. II, 57
[24] Cooper (1985): 76

influences from OECD initiatives. The proponents, lead by mathematics teacher Guðmundur Arnlaugsson, were for their part influenced by Piaget's pedagogical theories. They worked on the conviction that introduction of the basic concepts of set theory and logic would enhance deeper understanding of mathematics and the new concepts would be conducive to increased clarity and exactness in thought and arithmetic.[25] Guðmundur Arnlaugsson and his collaborators had the capacity to identify the needs, to introduce and present the new ideas, and to persuade and mobilize a large group of people to participate in a reform project.

In 1965–1966, a survey made for the Ministry of Education demonstrated that the syllabus of Icelandic lower secondary level schools in mathematical subjects lagged far behind that in the Nordic countries. The survey, the mathematics reform experiments and efforts on behalf of the Minister of Education led to a general reform of the Icelandic school system, launched with hitherto unprecedented generous support. The reform was developed within a framework of a new school research department of the Ministry of Education, established on the initiative of the OECD. Its main activities became school developmental projects, such as creating curricula and learning materials, and offering in-service courses and support to teachers. With official backing and high expectations of economic progress in governmental circles, Iceland joined the "modern" mathematics reform movement in a grand manner within the frame of a general reform of the educational system.

The reform of mathematics education in Iceland thus resulted from an interaction between two actors, heavily influenced by the OECD, at the professional and political level. The initiative came in 1964 from the individual leading high school mathematics teachers who, under the influence of the Royaumont seminar, redefined their upper secondary school's mathematics teaching, prepared the redefinition of the lower secondary school mathematics and provided consultation to the primary level. On the other hand, the Ministry of Education established school research in 1966 which developed into a large department working on school developmental projects, including compulsory school mathematics.

The reform enjoyed a massive support from the parliament, whose members complained in 1953 that every seventh *króna* went to a school system of a questionable quality compared to the venerable home education tradition.[26] The reasons were economical, expectations of economic gain from investment into education. It proved, however, to be a long term investment, providing revenue only many decades later. Industry did not demand mathematicians or physicists in great numbers for many decades. Those young people of the first generation of a new republic, who were influenced by the "need of the society" for science-educated manpower, were mainly exposed to teaching when and if they arrived back home from abroad with their scientific education in the 1970s or later. Gradually, openings for mathematicians developed in banks and insurance companies, and still later in genealogic and biopharmaceutical enterprises.

The implementation of "modern" mathematics in Iceland was at first aimed at elite pupils on secondary level, preparing for university and college studies. That part of the reform of mathematics education proceeded fairly successfully. The primary school experiment proved more controversial. Insufficient information about more than the first 2–3 year courses in a rather hastily chosen syllabus, translated from Danish, and an unexpectedly large group of teachers and pupils participating in the project, made it difficult for only few persons to organize. The new concepts were foreign to teachers and caused unrest among parents. The disturbing elements were radical ideas of implementing university conceptions of a unification of the various branches of mathematics, through logic and set theory, into

[25] Arnlaugsson, G., 1966
[26] Alþingistíðindi (1953)

primary school mathematics. That act proved to be the beginning of the end of the most orthodox "modern" mathematics reform, also in Iceland.[27]

MATHEMATICS EDUCATION AND INDIVIDUAL'S IMPACT- A SUMMARY

We may now turn to the question: Why did Icelanders leave out mathematics for long periods? We have seen that the most common need of the general public for mathematics, trade, was minimal. However, we have also become aware of important moments when a decision was taken to practice mathematics education for the benefit of society, supported by influential individuals who knew the capacity of mathematical education and had their vision of its cultural value. There were also moments when there were no such individuals present and mathematics was left out.

The population of Iceland is small, and so was its intellectual community for most of Iceland's 1100 years, up to the mid 1970s. The mathematical community was still smaller. In the whole of the 19th century there was only one Icelandic mathematician, Björn Gunnlaugsson, whose work as teacher and land-surveyor was an admirable and unique achievement. Dr. Ólafur Daníelsson was Björn Gunnlaugsson's successor in the 20th century, being a pupil of his grandson. Dr. Ólafur Daníelsson's influence on Icelandic mathematics education through his textbooks persisted for more than six decades. After his time, his pupil, Guðmundur Arnlaugsson, became the most influential person in mathematics education in the second half of the 20th century, and together with a colleague, took the lead in school mathematics reform activities in the late 1960s. Thus there was a long-standing tradition of individual authority in the field of mathematics education.

The impact of the presence or absence of influential individuals versus official reasons for crucial transformation of mathematics education in Iceland may be summarized as follows:

- In 1590s, Bishop Guðbrandur Þorláksson, being the most powerful person in the country, which adhered to a foreign rule, made a map of Iceland based on his scientific knowledge, but also on utilitarian aspects, that a correct map would provide Icelanders with safer trade and sailing on which the contact to European culture depended.
- In 1822, when mathematician Björn Gunnlaugsson offered himself to become mathematics teacher at Bessastaðir Learned School, the official reason for his appointment and for enhancing mathematics was to ensure the pupils' prerequisites for admission to the University of Copenhagen, while Björn Gunnlaugsson brought up utilitarian arguments and cultural aspects of mathematics education.
- In 1877, mathematics was no longer required for admission to the University of Copenhagen. No mathematician existed at the Learned School to present cultural or utilitarian arguments for the subject, and the mathematics syllabus was reduced.
- When a mathematics stream of the Reykjavík High School was established in 1919 on the urge of the Association of Engineers in Iceland and mathematician Dr. Ólafur Daníelsson, the official reason was to ensure prerequisites for engineering studies, i.e. utilitarian reasons for a rapidly industrializing society. Dr. Daníelsson's arguments for mathematics education were, however, mainly cultural, presenting mathematics as the most perfect science existing.
- In the mid-1960s when "modern" mathematics was implemented as part of the revision of the Icelandic school system, the official arguments were that education would contribute substantially to economic and social progress. The leader of the activities,

[27] Gjone, (1983), Vol. 1, 53. National Archives, 1989/S-56

> Guðmundur Arnlaugsson and his collaborator, had ideological arguments in mind, that the new concepts would be conducive to increased clarity and exactness.

A leadership of influential individuals was of crucial importance at points of transformation in Iceland. A redefinition of mathematical education could take place when both the official body that was to decide upon it and the persons that were to provide the pedagogical leadership had their own vision. They may not be identical but in all cases they may be classified among the fundamental reasons identified by M. Niss.

ICELAND'S RELATIONS TO DENMARK

Many historians have attempted to evaluate the impact of Iceland's relation to Denmark for close to five centuries.[28] Did the Danish monarchy exploit Iceland? In fact, the Danes had for most of the time indifferent attitude to Iceland. Furthermore, the Icelanders kept their relative cultural independence by maintaining Icelandic as official language in schools and churches, and most officials on behalf of the Church and the Danish Crown were Icelanders. It is indisputable, however, that the King's treasury yielded a considerable income from its properties, acquired at the introduction of the Reformation, and from the monopoly trade most of the period 1602–1786. By the 18th century the authorities had begun to realize that they had to nurture that resource for it to produce the desired profit, which explains various efforts toward modernization at that time. The loss of the compensations for the assets of the episcopal sees in the inflation arising from the Danish Napoleonic wars was also extremely detrimental to the economy of Iceland, the one learned school in particular. It took the school and the episcopal see several decades to recover from that blow.

It must not, however, be ignored that the Icelandic elite, the landowners and the officials, were very conservative. It was in their interest that farming was kept self-sustaining and that people were not allowed to settle in towns and by the coast. On those conditions neither social progress could be expected nor increase in population. There might be temporary increase which would be counteracted by epidemics or periods of cold climate causing famine. It was first when trade became free and fishing could be pursued as a whole-year activity on decked boats that the population began to grow and urbanization created basis for organized public education, including arithmetic. During the 19th century, trade became more prosperous than under the trade monopoly. The Icelanders gained self-esteem to begin their battle for independence and demand compensations for their losses, continuing far into the 20th century.

In their general indifference, the Danes usually adapted their activities according to the wishes of the most influential Icelanders, if it did not entail cost. The decision to adopt language-history stream at the Learned School in 1877 was a wish of the Icelandic heads of the school. Then the Danish authorities seized the opportunity to offer the pupils more Danish to learn, replacing mathematics, even if it was a general opinion that the students' knowledge of Danish was sufficient.

The first decades of sovereignty from 1918 and complete independence from 1944 were characterized by many kinds of teething problems. There was shortage of facilities, knowledge, experience and trained personnel in many spheres. In particular there was lack

[28] See for example Karlsson (2000); Thorsteinsson and Jónsson (1991)

of mathematics teachers. Being aware of being behind other nations in many respects had caused Icelanders a general feeling of inferiority, especially for the Danes, which took a long time to overcome. However, the 20th century has been a history of continuous, although somewhat periodic, progress and economic prosperity in Iceland. The latest novelty, that it has recently also taken off abroad, is only a continuation of that development.

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